HINTS & SOLUTION

1. *(b)* Let $A=\{x : x\in R, |x|<1\}=\{x : x\in R, -1<x<1\}$

and $B=\{x : x\in R, |x-1|\geq 1\}$

$\Rightarrow B=\{x : x\in R, x-1\geq 1 \text{ or } x-1\leq -1\}$

$\Rightarrow B=\{x : x\in R, x\geq 2 \text{ or } x\leq 0\}$

So, $A=(-1,1)$ and $B=(-\infty,0]\cup(2,\infty]$

Then, $A\cup B=(-\infty,1)\cup[2,\infty)$

$\Rightarrow A\cup B=(-\infty,\infty)-[1,2)$

$\Rightarrow A\cup B=R-[1,2)$

$\Rightarrow A\cup B=R-D$

where, $D=\{x : x\in R, 1\leq x<2\}$

Hence, option *(b)* is correct.

2. *(c)* Let α and α^2 be the roots of the equation $3x^2+px+3=0$

$\therefore$ Product of roots, $\alpha\cdot\alpha^2=\frac{3}{3}$

$\Rightarrow \quad \alpha^3=1$

$\Rightarrow \quad \alpha=(1)^{\frac{1}{3}}$

$\Rightarrow \quad \alpha=1,\omega,\omega^2 \qquad$ [cube roots of unity]

$\alpha=1$ is not possible. So, roots of the given equation are ω and ω^2.

Sum of roots, $\omega+\omega^2=-\frac{p}{3}$

$\Rightarrow \quad -1=\frac{-p}{3}$

$\Rightarrow \quad p=3$

Hence, option *(c)* is correct.

3. *(a)* Let the two numbers be a and b.

Then, $\text{AM}=A=\frac{a+b}{2}$, $\text{GM}=G=\sqrt{ab}$

and $\text{HM}=H=\frac{2ab}{a+b}$

Given, $A-G=2 \quad \text{....(i)}$

and $G-H=\frac{8}{5} \quad \text{....(ii)}$

$\because \quad G^2=AH$

$\Rightarrow G^2=(G+2)\left(G-\frac{8}{5}\right) \quad$ [from Eqs.(i) and (ii)]

$\Rightarrow G=8$

$\Rightarrow ab=64 \quad \text{....(iii)}$

From Eq.(i), $A=10$

$\Rightarrow \quad \frac{a+b}{2}=10$

$\Rightarrow \quad a+b=20 \quad \text{....(iv)}$

Solving equations (iii) and (iv), we get

$a=16, b=4$

$\therefore \quad \frac{b}{a}=\frac{1}{4}$

Hence, option *(a)* is correct.

4. *(b)* We have $\frac{z^{2n}-1}{z^{2n}+1}=\frac{(\cos\theta+i\sin\theta)^{2n}-1}{(\cos\theta+i\sin\theta)^{2n}+1}$

$=\frac{\cos(2n\theta)+i\sin(2n\theta)-1}{\cos(2n\theta)+i\sin(2n\theta)+1}$

[from Demoivre's theorem]

$=\frac{1-2\sin^2 n\theta+i(2\sin n\theta\cos n\theta)-1}{2\cos^2 n\theta-1+i(2\sin n\theta\cos n\theta)+1}$

$=\frac{2\sin n\theta(-\sin n\theta+i\cos n\theta)}{2\cos n\theta(\cos n\theta+i\sin n\theta)}$

$=i\tan(n\theta) \qquad [\because -x+iy=i(y+ix)]$

Hence, option *(b)* is correct.

5. *(d)* Since, $-\frac{\pi}{2}\leq\sin^{-1}\left\{2x\sqrt{1-x^2}\right\}\leq\frac{\pi}{2}$

$\Rightarrow -\frac{\pi}{2} \le 2\sin^{-1} x \le \frac{\pi}{2}$

$\Rightarrow -\frac{\pi}{4} \le \sin^{-1} x \le \frac{\pi}{4}$

$\Rightarrow -\sin\frac{\pi}{4} \le x \le \sin\frac{\pi}{4}$

$\Rightarrow -\frac{1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}$

So, $x \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$

Hence, option *(d)* is correct.

6. *(b)* Given, differential equation is

$$\left\{\left(\frac{d^4y}{dx^4}\right)^3\right\}^{1/3} - 7x\left(\frac{d^3y}{dx^3}\right)^2 = 8$$

Order = Highest derivative = 4
Degree = Integral power of highest derivative = 3
$\therefore$ Required sum = 4 + 3 = 7
Hence, option *(b)* is correct.

7. *(b)* Four letter words are to be formed using the letters of the word FAILURE when F is to be included in each word.
This shows that we have 6 choices for 3 places as one place is occupied by F.
So, required possible words ${}^6C_3 \times 4!$
Hence, option *(b)* is correct.

8. *(c)* In ΔABC, $AB = c$, $BC = a$ and $CA = b$
$\because D$ is the midpoint of BC

$\Rightarrow BD = CD = \frac{a}{2}$

In ΔABC, $\angle ADB = 90°$

So, $\cos B = \frac{BD}{AB} = \frac{a}{2c}$

Hence, option *(c)* is correct.

9. *(a)* We have $p = \frac{1}{2} \Rightarrow q = \frac{1}{2}$

Given, $P(X=4), P(X=5)$ and $P(X=6)$ are in AP.

$$P(X=4) = {}^nC_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^{n-4} = {}^nC_4\left(\frac{1}{2}\right)^n$$

$$P(X=5) = {}^nC_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^{n-5} = {}^nC_5\left(\frac{1}{2}\right)^n$$

$$P(X=6) = {}^nC_6\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^{n-6} = {}^nC_6\left(\frac{1}{2}\right)^n$$

$\Rightarrow {}^nC_4, {}^nC_5$ and ${}^nC_6$ are also in AP.

$\Rightarrow 2\,{}^nC_5 = {}^nC_4 + {}^nC_6$

$$\Rightarrow 2\cdot\frac{n!}{5!(n-5)!} = \frac{n!}{4!(n-4)!} + \frac{n!}{6!(n-6)!}$$

Clearly, this equation is satisfied for $n = 7$
Hence, option *(a)* is correct.

10. *(c)* Given, $f(x) = 2x^2 - 3x$

$\Rightarrow f'(x) = 4x - 3$

$= 4\left(x - \frac{3}{4}\right)$

Clearly, $f'(x) > 0\ \forall\ x > \frac{3}{4}$

And $f'(x) < 0\ \forall\ x < \frac{3}{4}$

So, $f(x)$ is strictly increasing in the interval $\left(\frac{3}{4}, \infty\right)$

and strictly decreasing in the interval $\left(-\infty, \frac{3}{4}\right)$

Thus, both the statements I and II are correct.
Hence, option *(c)* is correct.

11. *(b)* We have $f(x) = \int \frac{x^2 dx}{(1+x^2)\left(1+\sqrt{1+x^2}\right)}$

Let $x = \tan\theta \Rightarrow \sec^2\theta\, d\theta = dx$

$\Rightarrow f(x)=\int \frac{\tan^2\theta\sec^2\theta\,d\theta}{(1+\tan^2\theta)\left(1+\sqrt{1+\tan^2\theta}\right)}$

$=\int \frac{\tan^2\theta}{1+\sec\theta}d\theta \qquad \{\because 1+\tan^2\theta=\sec^2\theta\}$

$=\int \frac{\sin^2\theta}{\cos\theta(1+\cos\theta)}d\theta$

$=\int \frac{1-\cos^2\theta}{\cos\theta(1+\cos\theta)}d\theta$

$=\int \frac{1-\cos\theta}{\cos\theta}d\theta$

$=\int(\sec\theta-1)\,d\theta$

$=\log(\sec\theta+\tan\theta)-\theta+C$

Given $f(0)=0\Rightarrow C=0$

So, $f(x)=\log(\sec\theta+\tan\theta)-\theta$

$\Rightarrow f(x)=\log\left(\sqrt{1+x^2}+x\right)-\tan^{-1}x$

$\Rightarrow f(1)=\log\left(\sqrt{2}+1\right)-\frac{\pi}{4}$

Hence, option *(b)* is correct.

12. *(c)* Given, $A=\begin{bmatrix} i & 0 & 0\\ 0 & i & 0\\ 0 & 0 & i\end{bmatrix}, i=\sqrt{-1}$

$\Rightarrow \quad A=i\begin{bmatrix} 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$

$\Rightarrow \quad A=iI$

So, $A^n=(iI)^n=i^nI^n=i^nI$

Given, $A^n=I$

This is possible only when $i^n=1\Rightarrow n=4,8,12,....$

or $n=4p$

Hence, option *(c)* is correct.

13. *(b)* Given, $|a+b|=|a-b|$

$\Rightarrow |a+b|^2=|a-b|^2$

$\Rightarrow |a|^2+|b|^2+2a\cdot b=|a|^2+|b|^2-2a\cdot b$

$\Rightarrow a\cdot b=0$

$\Rightarrow a$ and b are orthogonal

So, statement I is correct.

Now, let $|a+b|=|a|+|b|$

$\Rightarrow |a+b|^2=(|a|+|b|)^2$

$\Rightarrow |a|^2+|b|^2+2a\cdot b=|a|^2+|b|^2+2|a||b|$

$\Rightarrow a\cdot b=ab$

$\Rightarrow ab\cos\theta=ab \Rightarrow \cos\theta=1$

$\Rightarrow \theta=0°$

$\Rightarrow a\,||\,b$

So, statement II is not correct.

Now, let $|a+b|^2=|a|^2+|b|^2$

$\Rightarrow |a|^2+|b|^2+2a\cdot b=|a|^2+|b|^2$

$\Rightarrow a\cdot b=0$

$\Rightarrow a$ and b are orthogonal

$\Rightarrow a\,||\,b$

So, statement III is correct.

Thus, only I and III statements are correct.

Hence, option *(b)* is correct.

14. *(d)* Number 4-digit number formed from the digits 0, 1, 2, 3, 4, 5, 6, 7 when repetition is not allowed

$=7\times7\times6\times5=1470 \quad [\because 0 \text{ cannot come at first place}]$

Number of 4-digit number formed from the digits 0, 2, 3, 4, 5, 6, 7 when repetition is not allowed

$=6\times6\times5\times4=720 \quad [\because 0 \text{ cannot come at first place}]$

So, total numbers of 4-digit number such that each number contain digit 1 is

$=1470-720=750$

Hence, option *(d)* is correct.

15. *(b)* Given, $25\cos^2\alpha+5\cos\alpha-12=0$

$\Rightarrow (5\cos\alpha+4)(5\cos\alpha-3)=0$

$\Rightarrow \cos\alpha=-\frac{4}{5}$ or $\cos\alpha=\frac{3}{5}$

$\because \alpha$ is the root of the above equation such that

$\pi<\alpha<\frac{3\pi}{2}$. So, $\cos\alpha=-\frac{3}{5}$

Hence, $\tan\alpha=\frac{3}{4}$

Hence, option *(b)* is correct.

16. *(c)* Given curve is $y^2=4(x+2)$

$\Rightarrow 2y\frac{dy}{dx}=4$

$\Rightarrow \frac{dy}{dx}=\frac{2}{y}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(2,4)}=\frac{2}{4}=\frac{1}{2}$

Tangent at $(2,4)$ is given by

$y-4=\frac{1}{2}(x-2)$

$\Rightarrow x-2y+6=0$

Now, length of perpendicular from $(0,0)$ on this tangent

$=\left|\frac{0-2(0)+6}{\sqrt{1+4}}\right|=\frac{6}{\sqrt{5}}$

Hence, option *(c)* is correct.

17. *(d)* Here, $a=BC=\sqrt{0^2+(12-0)^2}=12$

$b=AC=\sqrt{(0-8)^2+(6-0)^2}=10$

$c=AB=\sqrt{8^2+6^2}=10$

$\therefore$ In-centre is given by

$\left(\frac{ax_1+bx_2+cx_3}{a+b+c},\frac{ay_1+by_2+cy_3}{a+b+c}\right)$

$=\left(\frac{12\times0+10\times8+10\times8}{12+10+10},\frac{12\times6+10\times12+10\times0}{12+10+10}\right)$

$=\left(\frac{160}{32},\frac{192}{32}\right)$

$=(5,6)$

Hence, option *(d)* is correct.

18. *(a)* Let the given numbers be a and b.

Then, AM $=2$

$\Rightarrow \frac{a+b}{2}=2\Rightarrow a+b=4$ ....(i)

According to the question,

$\sqrt{a(b+1)}=2$

$\Rightarrow ab+a=4$ ....(ii)

On solving Eqs. (i) and (ii), we get

$a=1$ and $b=3$

So, HM $=\frac{2ab}{a+b}=\frac{2(3)}{1+3}=\frac{3}{2}$

Hence, option *(a)* is correct.

19. *(b)* Given, SD of $x=\sigma$

Let new variable y

Then, $y=\frac{ax+b}{c}$ (given)

$\because$ SD is affected by change of scale but not by change of origin $\left\{\text{SD}=\sqrt{\frac{\sum(x_i-\bar{x})^2}{n}}\right\}$

So, SD for $y=\left|\frac{a}{c}\right|\sigma$

Hence, option *(b)* is correct.

20. *(c)* Let $I=\int_{\pi/4}^{\pi/2}e^x(\log\sin x+\cot x)dx$

$\because \int e^x\{f(x)+f'(x)\}dx=e^xf(x)+c$

$\left[\because \frac{d}{dx}(\log\sin x)=\frac{\cos x}{\sin x}=\cot x\right]$

Therefore,

$$I = \left[e^{x}(\log \sin x)\right]_{\pi/4}^{\pi/2}$$

$$= e^{\frac{\pi}{2}} \log(1) - e^{-\frac{\pi}{4}} \log\left(\frac{1}{\sqrt{2}}\right)$$

$$= -e^{\frac{\pi}{4}} \log(2)^{-\frac{1}{2}} \qquad [\because \log 1 = 0]$$

$$= \frac{1}{2} e^{\frac{\pi}{4}} \log(2)$$

Hence, option *(c)* is correct.

21. *(b)* An event having no sample point is called impossible event while an event having one sample point is called an elementary event.
Hence, option *(b)* is correct.

22. *(d)* Let the roots of the equation $x^2 - 12x + a = 0$ be α and 3α.
Sum of roots, $\alpha + 3\alpha = -(-12) \Rightarrow \alpha = 3$

Product of roots, $\alpha(3\alpha) = a$

$\Rightarrow \qquad a = 3(3 \times 3)$

$\Rightarrow \qquad a = 27$

Hence, option *(d)* is correct.

23. *(c)* Let $a = 1,\ b = 2,\ c = 3$

Clearly, a, b, c are in AP but a^2, b^2, c^2 i.e., $1, 4, 9$ are not in AP.
So statement I is not correct.
Now, let a, b, c be in GP.

$\Rightarrow \qquad b^2 = ac$

$\Rightarrow \qquad b^4 = a^2c^2$

Clearly a^2, b^2, c^2 are also in GP.
So statement II is correct.
Now, again a, b, c are in GP.

$\Rightarrow \qquad b^2 = ac$

$\Rightarrow \quad 2\log b = \log a + \log c$

$\Rightarrow \quad \log a,\ \log b,\ \log c$ are in AP

$\Rightarrow \quad \log a + 1,\ \log b + 1,\ \log c + 1$ are in AP

$\Rightarrow \quad \dfrac{1}{1+\log a}, \dfrac{1}{1+\log b}, \dfrac{1}{1+\log c}$ are in AP

So statement III is also correct.
Hence, option *(c)* is correct.

24. *(c)* Since, $\sin A - \cos B = \cos C$

$\Rightarrow \sin A = \cos B + \cos C$

$$= 2\cos\frac{B+C}{2}\cos\frac{B-C}{2}$$

$\Rightarrow 2\sin\dfrac{A}{2}\cos\dfrac{A}{2} = \cos\left(90° - \dfrac{A}{2}\right)\cos\dfrac{B-C}{2}$

$[\because A + B + C = 90°$ and $\cos(90° - \theta) = \sin\theta]$

$\Rightarrow \cos\dfrac{A}{2} = \cos\dfrac{B-C}{2} \Rightarrow \dfrac{A}{2} = \dfrac{B-C}{2}$

$\Rightarrow A + C = B \qquad \qquad ....(i)$

$\because A + B + C = 180°$

$\Rightarrow B = 90° = \dfrac{\pi}{2}$

Hence, option *(c)* is correct.

25. *(d)* Consider, $\cos^{-1}\sqrt{\dfrac{2}{3}} - \cos^{-1}\dfrac{\sqrt{6}+1}{2\sqrt{3}}$

$$= \tan^{-1}\left(\frac{1}{\sqrt{2}}\right) - \tan^{-1}\left(\frac{\sqrt{3}-\sqrt{2}}{1+\sqrt{6}}\right)$$

$$\left\{\because \cos^{-1}x = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)\right\}$$

$$= \cot^{-1}(\sqrt{2}) - \tan^{-1}\left(\frac{\sqrt{3}-\sqrt{2}}{1+\sqrt{2}\sqrt{3}}\right)$$

$$= \cot^{-1}(\sqrt{2}) - \left\{\tan^{-1}(\sqrt{3}) - \tan^{-1}(\sqrt{2})\right\}$$

$$= \left\{\cot^{-1}(\sqrt{2}) + \tan^{-1}(\sqrt{2})\right\} - \tan^{-1}(\sqrt{3})$$

$$= \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$$

$\therefore \frac{\pi}{k} = \frac{\pi}{6} \Rightarrow k = 6$

Hence, option *(d)* is correct.

26. *(d)* Let $a = \hat{i} + \hat{j} + 2\hat{k}$, $b = \hat{i} + 2\hat{j} + \hat{k}$ and $c = \hat{i} + \hat{j} + \hat{k}$

Then, vector perpendicular to c and coplanar to a and b is given by

$= c \times (a \times b) = (c \cdot b)a - (c \cdot a)b$

$= (1+2+1)(\hat{i} + \hat{j} + 2\hat{k}) - (1+1+2)(\hat{i} + 2\hat{j} + \hat{k})$

$= -4\hat{j} + 4\hat{k}$

$= 4(-\hat{j} + \hat{k})$

This is satisfied by option *(d)*.

27. *(b)* Out of 100 students, 70 are boys.

So remaining 30 are girls.

Average marks of boys $= 75$

Average marks of complete class $= 72$

Let the average marks of girls $= x$

Then, $72 = \frac{75 \times 70 + x \times 30}{100}$

$\Rightarrow 7200 = 5250 + 30x$

$\Rightarrow x = \frac{195}{3} = 65$

Hence, option *(b)* is correct.

28. *(d)* Probability that a prediction is correct $= \frac{1}{3}$ (either win, draw or defeat)

$\Rightarrow p = \frac{1}{3}$

$\Rightarrow q = 1 - \frac{1}{3} = \frac{2}{3}$

Here, total 8 matches are played.

So, $n = 8$

Probability that 8 predictions are correct $= P(X = 4)$

$= {}^8C_4 (p)^4 q^4$

$= \frac{8!}{4!4!}\left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^4$

$= \frac{1120}{3^8}$

Hence, option *(d)* is correct.

29. *(c)* Given, $\begin{vmatrix} x^2+2x & 2x+1 & 1 \\ 2x+1 & x+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (x-1)^k$

On expanding, we get

$\Rightarrow (x^2+2x)(x+2-3) - (2x+1)(2x+1-3)$
$+1(6x+3-3x-6) = (x-1)^k$

$\Rightarrow (x^2+2x)(x-1) - (2x+1)(2x-2) + (3x-3) = (x-1)^k$

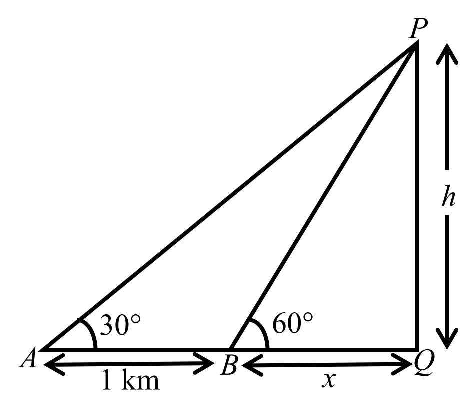
$\Rightarrow x^3 - 3x^2 + 3x - 1 = (x-1)^k$

$\Rightarrow (x-1)^3 = (x-1)^k$

$\Rightarrow k = 3$

Hence, option *(c)* is correct.

30. *(b)* Let P be the balloon at height h from the ground and from point A, its elevation angle is $30°$. After walking 1 km to point B, elevation angle of P is $60°$.



Then, in ΔPAQ, $\tan 30° = \frac{h}{1 + BQ}$

Let $x = BQ$

$\Rightarrow 1 + x = h\sqrt{3}$

$\Rightarrow \quad x = h\sqrt{3} - 1 \quad$ ....(i)

And, in ΔPBQ, $\tan 60° = \frac{h}{x}$

$\Rightarrow \quad h = \sqrt{3}x \quad \text{....(ii)}$

From equations (i) and (ii), we get

$$h = \sqrt{3}\left(h\sqrt{3} - 1\right)$$

$\Rightarrow \quad h = 3h - \sqrt{3}$

$\Rightarrow \quad h = \frac{\sqrt{3}}{2}\text{km}$

Hence, option *(b)* is correct.

31. *(b)* Relative order of vowels and consonant remain unchanged.

Therefore, vowels will occupy only vowel's place and consonants will occupy consonants place.

Now, 4 consonants can be arranged in $\frac{4!}{2!}$ and 3 vowels can be arranged in $\frac{3!}{2!}$

$\therefore$ Required number $= \frac{4!}{2!} \times \frac{3!}{2!} = 36$

Hence, option *(b)* is correct.

32. *(c)* Coefficient of correlation

$$\rho = \frac{\text{Cov}(x, y)}{\sqrt{\text{var}(x)}\sqrt{\text{var}(y)}}$$

$$= \frac{\frac{\sum x_i y_i}{n} - \left(\frac{\sum x_i}{n}\right)\left(\frac{\sum y_i}{n}\right)}{\sigma_x \sigma_y}$$

$$= \frac{\frac{12}{10} - (0)(0)}{2 \times 3} \quad \left[\because \bar{x} = \frac{\sum x_i}{n} = 0, \bar{y} = \frac{\sum y_i}{n} = 0\right]$$

$$= \frac{2}{10} = 0.2$$

Hence, option *(c)* is correct.

33. *(c)* Given, $x\hat{i} + y\hat{j} + z\hat{k}$ is a unit vector.

So, $x^2 + y^2 + z^2 = 1 \quad \text{....(i)}$

Also, given $x : y : z = \sqrt{3} : 2 : 3$

$\Rightarrow x - \sqrt{3}k,\ y = 2k,\ z = 3k \quad [\text{let}]$

From Eq.(i),

$$3k^2 + 4k^2 + 9k^2 = 1$$

$\Rightarrow 16k^2 = 1$

$\Rightarrow k = \pm\frac{1}{4}$

So, $z = 3k = \pm\frac{3}{4}$

$\therefore \quad z = \frac{3}{4}$

Hence, option *(c)* is correct.

34. *(b)* Given two lines

$$3x - 4y + 12 = 0 \quad \text{....(i)}$$

and $3x - 4y = 6 \quad \text{....(ii)}$

are parallel lines.

Let equation of line mid-way between two given lines is

$$3x - 4y + k = 0 \quad \text{....(iii)}$$

Let (α, β) be the arbitrary point equidistant from lines (i) and (ii).

Then, d_1 = perpendicular distance of (α, β) from the line $3x - 4y + 12 = 0$

$\Rightarrow \quad d_1 = \left|\frac{3\alpha - 4\beta + 12}{\sqrt{9+16}}\right|$

And d_2 = perpendicular distance of (α, β) from line $3x - 4y - 6 = 0$

$\Rightarrow \quad d_2 = \left|\frac{3\alpha - 4\beta - 6}{\sqrt{9+16}}\right|$

Now, $d_1 = d_2$

$\Rightarrow \quad |3\alpha - 4\beta + 12| = |3\alpha - 4\beta - 6|$

$\Rightarrow \quad 3\alpha - 4\beta + 12 = (3\alpha - 4\beta - 6)$

or $\quad 3\alpha - 4\beta + 12 = -(3\alpha - 4\beta - 6)$

$\Rightarrow \quad 6\alpha - 8\beta + 18 = 0$

$\Rightarrow \quad 3\alpha - 4\beta + 9 = 0$

Thus, required line is

$3x-4y+9=0$

$\{\because (\alpha,\beta)\text{satisfies Eq.(iii)}, 3\alpha-4\beta+k=0 \Rightarrow k=9\}$

Hence, option *(b)* is correct.

35. *(a)* Family of lines is

$(a+2b)x+(a-3y)y+a-8b=0$

$\Rightarrow a(x+y+1)+b(2x-3y-8)=0$

Given, above lines are concurrent at point of intersection of lines $x+y+1=0$ and $2x-3y-8=0$ which is $A(1,-2)$.

Now, the line through $A(1,-2)$ which is farthest from point $B(2,2)$ is perpendicular to *AB*.

Slope of $AB=\frac{2-(2)}{2-1}=4$

So, required equation of line *L* is

$y-(-2)=-\frac{1}{4}(x-1)$

$\Rightarrow x+4y+7=0$

Hence, option *(a)* is correct.

36. *(c)* From the solution of previous question, line L meets the coordinate axes at $C(-7,0)$ and $D\left(0,-\frac{7}{4}\right)$

So, area enclosed by line L and coordinate axes

$=\frac{1}{2}\times(-7)\times\left(-\frac{7}{4}\right)=\frac{49}{8}\text{sq.units}$

Hence, option *(c)* is correct.

37. *(a)* Let $|a|=1, |b|=1, |c|=1$ and $a\times(b\times c)=\frac{\sqrt{3}}{2}(b+c)$

$\Rightarrow (a\cdot c)b-(a\cdot b)c=\frac{\sqrt{3}}{2}b+\frac{\sqrt{3}}{2}c$

$\Rightarrow a\cdot c=\frac{\sqrt{3}}{2}$ and $a\cdot b=-\frac{\sqrt{3}}{2}$

Let the angle between a and b be θ

Then, $a\cdot b=-\frac{\sqrt{3}}{2}$

$\Rightarrow |a||b|\cos\theta=-\frac{\sqrt{3}}{2}$

$\Rightarrow \cos\theta=-\frac{\sqrt{3}}{2}$ $\quad[\because |a|=|b|=1]$

$\Rightarrow \theta=\frac{5\pi}{6}$

Hence, option *(a)* is correct.

38. *(b)* Let the equation of the ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

If foci S and S' are $(\pm ae,0)$

Let $P(x,y)$ be any point on the ellipse.

The sum of focal radii $=PS+PS'$

$=\sqrt{(x-ae)^2+y^2}+\sqrt{(x+ae)^2+y^2}$

$=2a$

$=\text{Length of major axis}$

Hence, option *(b)* is correct.

39. *(d)* Since, *A* and *B* are two independent events having probabilities $P(A)=\frac{1}{2}$ and $P(B)=\frac{1}{5}$

So, $P(A\cap B)=P(A)\cdot P(B)$

$=\frac{1}{2}\times\frac{1}{5}=\frac{1}{10}$

$\Rightarrow P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$=\frac{1}{2}+\frac{1}{5}-\frac{1}{10}=\frac{3}{5}$

Now, $P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}=\frac{1}{2}$,

$P\left(\frac{A}{A\cup B}\right)=\frac{P(A\cap(A\cup B))}{P(A\cup B)}$

$=\frac{P(A)}{P(A\cup B)}=\frac{5}{6}$

And, $P\left(\frac{A\cap B}{A'\cup B'}\right)=\frac{P((A\cap B)\cap(A'\cup B'))}{P(A'\cup B')}$

$$=\frac{P\left(\left(A\cap B\right)\cap\left(A\cap B\right)'\right)}{P\left(A\cap B\right)'}=0$$

Hence, all the statements are correct.
Hence, option *(d)* is correct.

40. *(d)* We have $\int \frac{e^x}{x}\left(1+x\log x\right)dx$

$$=\int e^x\left(\frac{1}{x}+\log x\right)dx$$

$$=\int e^x \log x dx+\int e^x\left(\frac{1}{x}\right)dx$$

$$=\left(\log x\right)\left(e^x\right)-\int\left(\frac{1}{x}\right)e^x dx+\int e^x\left(\frac{1}{x}\right)dx+C$$

$$=\left(\log x\right)\left(e^x\right)+C$$

Now, $\int e^x\left[f\left(x\right)+f'\left(x\right)\right]dx$

$$=\int e^x f\left(x\right)dx+\int e^x f'\left(x\right)dx$$

$$=e^x f\left(x\right)-\int f'\left(x\right)e^x dx+\int e^x f'\left(x\right)dx$$

$$=e^x f\left(x\right)+C$$

Hence, assertion and reason both are correct and reason is the correct explanation of assertion.
Hence, option *(d)* is correct.

41. *(b)* Given, $A=\{x:-1\le x\le 1\}$ and $B=\{y:1\le y\le 2\}$

$\because\ -1\le x\le 1$

$\Rightarrow\ 0\le x^2\le 1$

$\Rightarrow\ 1\le 1+x^2\le 2$

$\Rightarrow\ 1\le y\le 2$

Clearly, the given function is onto but not one-one as $1+x^2$ takes the same value for both positive and negative values of x.
Therefore, it is surjective but not injective.
Hence, option *(b)* is correct.

42. *(a)* Given $z\left(2-2\sqrt{3}i\right)^2=i\left(\sqrt{3}+i\right)^4$

$$\Rightarrow z=\frac{i\left(\sqrt{3}+i\right)^4}{2\left(1-\sqrt{3}i\right)^2}$$

$$=\frac{i\left(\sqrt{3}+i\right)^4}{2\left(i^2\right)\left(\sqrt{3}+i\right)^2}=-\frac{i}{2}\left(\sqrt{3}+1\right)^2$$

$$=\frac{i}{2}\left(2+2i\sqrt{3}\right)=-\sqrt{3}+i$$

Clearly, z lies in the II quadrant.
So, $\arg\left(z\right)=\pi-\theta$

$$=\pi-\tan^{-1}\left|\frac{1}{-\sqrt{3}}\right|$$

$$=\pi-\frac{\pi}{6}=\frac{5\pi}{6}$$

Hence, option *(a)* is correct.

43. *(c)* We have $f\left(x\right)=\int_1^x t\left(t^2-3t+2\right)dt$

$\Rightarrow f'\left(x\right)=x\left(x^2-3x+2\right)=x\left(x-1\right)\left(x-2\right)$

$\Rightarrow f'\left(x\right)>0\ \forall x\in\left[2,3\right]$

and $\Rightarrow f'\left(x\right)<0\ \forall x\in\left[1,2\right]$

$\Rightarrow f\left(x\right)$ is increasing in the interval $\left[2,3\right]$ and decreasing in the interval $\left[1,2\right]$.

So, minimum value of $f\left(x\right)=f\left(2\right)$ and maximum value of $f\left(x\right)=\max\{f\left(1\right),f\left(3\right)\}$

Now,

$$f\left(x\right)=\left[\frac{t^4}{4}-t^3+t^2\right]_1^x$$

$$\Rightarrow f\left(x\right)=\frac{x^4}{4}-x^3+x^2-\frac{1}{4}$$

$$\Rightarrow f\left(1\right)=0, f\left(2\right)=-\frac{1}{4}\text{ and } f\left(3\right)=2$$

$\therefore$ Range of $f\left(x\right)=\left[-\frac{1}{4},2\right]$

So, $\left[-k,2\right]=\left[-\frac{1}{4},2\right]$

$\therefore\ k=\frac{1}{4}$

Hence, option *(c)* is correct.

44. *(b)* When a particular bowler plays, then two batsmen will not play. So rest of 10 payers can be selected from 17 other players. This is done in ${}^{17}C_{10}$ ways.

If a particular bowler does not play, then number of selections is ${}^{19}C_{11}$

If all the three players do not play, then number of selections is ${}^{17}C_{11}$.

So, total number of selections

$={}^{17}C_{10}+{}^{19}C_{11}+{}^{17}C_{11}$

Hence, option *(b)* is correct.

45. *(b)* If the particular batsman is selected, then rest of 10 players can be selected in ${}^{18}C_{10}$ ways.

If particular wicket keeper is selected, then rest of 10 players can be selected in ${}^{18}C_{10}$ ways.

If both are not selected, then number of ways $={}^{18}C_{11}$

So, total number of ways

$=2\,{}^{18}C_{10}+{}^{18}C_{11}={}^{19}C_{11}+{}^{18}C_{10}$

Hence, option *(b)* is correct.

46. *(b)* We have $\frac{dN(t)}{dt}=\alpha N(t)$

$\Rightarrow\ \int\frac{dN(t)}{N(t)}=\alpha\int dt$

$\Rightarrow\ \log\{N(t)\}=\alpha t+\log c$

$\Rightarrow\ \log\left\{\frac{N(t)}{c}\right\}=\alpha t$

$\Rightarrow\ \frac{N(t)}{c}=e^{\alpha t}\ \Rightarrow\ N(t)=ce^{\alpha t}$

Comparing with $N(t)=ce^{kt}$, we get

$\alpha=k$

Hence, option *(b)* is correct.

47. *(d)* Given, average salary of 30 men is ₹4050 and average salary of total 50 employees is ₹3550.

Let average salary of 20 women be x, therefore

$50\times3550=30\times4050+20\times x$

Then,

$\Rightarrow x=\frac{50\times3550-30\times4050}{20}$

$\Rightarrow x=\frac{177500-121500}{20}=2800$

So, average salary of women is ₹2800

Hence, option *(d)* is correct.

48. *(c)* Middle term in the expansion of $(1+\alpha x)^4$ is T_3

So, $T_3=T_{2+1}={}^4C_2(\alpha x)^2$

Middle term in the expansion of $(1-\alpha x)^6$ is T_4

So, $T_4=T_{3+1}={}^6C_3(-\alpha x)^2$

According to the question, coefficients of both the middle terms are equal.

So, ${}^4C_2(\alpha^2)={}^6C_3(-\alpha^3)$

$\Rightarrow 6\alpha^2=-20\alpha^3$

$\Rightarrow \alpha^2(3+10\alpha)=0$

$\Rightarrow \alpha=-\frac{3}{10}\qquad[\because\alpha\neq0]$

Hence, option *(c)* is correct.

49. *(c)* Since, $f:R\to R$ such that $f(x)=3^{-x}$

Let x_1 and x_2 be two elements of $f(x)$ such that

$y_1=y_2$

$\Rightarrow\ 3^{-x_1}=3^{-x_2}\ \Rightarrow\ x_1=x_2$

Since, if two images are equal, then their elements are equal, therefore it is ono-one function.

Since, $f(x)$ is positive for every value of x_1, therefore $f(x)$ is into.

On differentiating w.r.t. x, we get

$\frac{dy}{dx}=-3^{-x}\log3<0$, for every value of x.

$\therefore$ It is decreasing function.

$\therefore$ Statements I and III are true.
Hence, option *(c)* is correct.

50. *(a)* Let $f(x)=\begin{cases} x^2-4x+3, & x<3 \\ x-4, & x\ge 3 \end{cases}$ and

$g(x)=\begin{cases} x-3, & x<4 \\ x^2+2x+2, & x\ge 4 \end{cases}$

So, $(f+g)(3.5)=f(3.5)+g(3.5)$
$=\{(3.5)-4\}+\{(3.5)-3\}$
$=\{-0.5\}+\{-0.5\}$
$=0$

Hence, option *(a)* is correct.

51. *(b)* We have $f(g(3))=f(3-3)=f(0)$
$=0^2-4(0)+3=3$

Hence, option *(b)* is correct.

52. *(a)* Let $x=\omega-\omega^2-2$

$\Rightarrow x+2=\omega-\omega^2$

$\Rightarrow (x+2)^2=(\omega-\omega^2)^2$

$\Rightarrow x^2+4x+4=\omega^2+\omega^4-2\omega^3$

$\Rightarrow x^2+4x+4=\omega^2+\omega-2 \quad [\because \omega^3=1]$

$\Rightarrow x^2+4x+4=-3 \quad [\because 1+\omega+\omega^2=0]$

$\Rightarrow x^2+4x+7=0$

Now, $x^4+3x^3+2x^2-11x-6$
$=(x^2+4x+7)(x^2-x-1)+1$
$=1 \quad [\text{from Eq.(i)}]$

Hence, option *(a)* is correct.

53. *(d)* Consider, $\cos^{-1}\sqrt{\dfrac{2}{3}}-\cos^{-1}\dfrac{\sqrt{6}+1}{2\sqrt{3}}$

$=\tan^{-1}\left(\dfrac{1}{\sqrt{2}}\right)-\tan^{-1}\left(\dfrac{\sqrt{3}-\sqrt{2}}{1+\sqrt{6}}\right)$

$\left\{\because \cos^{-1}x=\tan^{-1}\left(\dfrac{\sqrt{1-x^2}}{x}\right)\right\}$

$=\cot^{-1}(\sqrt{2})-\tan^{-1}\left(\dfrac{\sqrt{3}-\sqrt{2}}{1+\sqrt{2}\sqrt{3}}\right)$

$=\cot^{-1}(\sqrt{2})-\{\tan^{-1}\sqrt{3}-\tan^{-1}\sqrt{2}\}$

$=\{\cot^{-1}\sqrt{2}+\tan^{-1}\sqrt{2}\}-\tan^{-1}\sqrt{3}$

$=\dfrac{\pi}{2}-\dfrac{\pi}{3}=\dfrac{\pi}{6}$

$\therefore \dfrac{\pi}{k}=\dfrac{\pi}{6} \Rightarrow k=6$

Hence, option *(d)* is correct.

54. *(c)* We have $\displaystyle\int_{\log 2}^{x}\frac{dx}{\sqrt{e^x-1}}=\frac{\pi}{6}$

$\Rightarrow \displaystyle\int_{\log 2}^{x}\frac{e^{-x/2}}{\sqrt{1-(e^{-x/2})^2}}=\frac{\pi}{6}$

$\left\{\text{Let } e^{-x/2}=t \Rightarrow -\dfrac{1}{2}e^{-x/2}dx=dt\right\}$

$\Rightarrow -2\displaystyle\int_{\log 2}^{x}\frac{dt}{\sqrt{1-t^2}}=\frac{\pi}{6}$

$\Rightarrow -2\left[\sin^{-1}\left(e^{-(x/2)}\right)\right]_{\log 2}^{x}=\dfrac{\pi}{6}$

$\Rightarrow -2\left[\sin^{-1}\left(e^{-(x/2)}\right)\right]+2\left[\sin^{-1}\left(\dfrac{1}{\sqrt{2}}\right)\right]=\dfrac{\pi}{6}$

$\Rightarrow -2\sin^{-1}\left(e^{-(x/2)}\right)=\dfrac{\pi}{6}-\dfrac{\pi}{2}=-\dfrac{\pi}{3}$

$\Rightarrow \sin^{-1}\left(e^{-(x/2)}\right)=\dfrac{\pi}{6}$

$\Rightarrow e^{-x/2}=\dfrac{1}{2} \Rightarrow -\dfrac{x}{2}=\log_e\left(\dfrac{1}{2}\right)$

$\Rightarrow x=2\log 2=\log 4$

Hence, option *(c)* is correct.

55. *(c)* Consider $b\cos^2\frac{C}{2}+c\cos^2\frac{B}{2}$

$$=b\frac{s(s-c)}{ab}+c\frac{s(s-b)}{ac}$$

$$=\frac{1s}{a}(s-c+s-b)=s$$

So, Statement I is correct.

Now, $\cot\frac{A}{2}=\frac{b+c}{a}$

$$\Rightarrow \cot\frac{A}{2}=\frac{\sin B+\sin C}{\sin A} \quad [\text{from sine rule}]$$

$$\Rightarrow \frac{\cos\frac{A}{2}}{\sin\frac{A}{2}}=\frac{2\sin\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right)}{2\sin\frac{A}{2}\cos\frac{A}{2}}$$

$$\Rightarrow \cos^2\frac{A}{2}=\sin\left(90^\circ-\frac{A}{2}\right)\cos\left(\frac{B-C}{2}\right)$$

$$\Rightarrow \cos\frac{A}{2}=\cos\left(\frac{B-C}{2}\right) \quad \left[\because \sin\left(90^\circ-\frac{A}{2}\right)=\cos\frac{A}{2}\right]$$

$$\Rightarrow \frac{A}{2}=\frac{B-C}{2}$$

$$\Rightarrow A+C=B$$

$$\because\ A+B+C=180^\circ$$

$$\Rightarrow B=90^\circ$$

Hence, option *(c)* is correct.

56. *(b)* Given equations are

$$a_1x+b_1y+c_1z+d_1=0$$

$$a_2x+b_2y+c_2z+d_2=0$$

$$a_3x+b_3y+c_3z+d_3=0$$

For unique solution, we have

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}\neq 0$$

$$\Rightarrow \Delta(a,b,c)=0$$

From Cramer's Rule,

$$x=\frac{\begin{vmatrix} -d_1 & b_1 & c_1 \\ -d_2 & b_2 & c_2 \\ -d_3 & b_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}$$

$$\Rightarrow x=\frac{-\Delta(d,b,c)}{\Delta(a,b,c)}$$

$$\Rightarrow x=\frac{\Delta(b,d,c)}{\Delta(a,b,c)}=-\frac{\Delta(b,c,d)}{\Delta(a,b,c)}$$

Hence, option *(b)* is correct.

57. *(b)* We have $f(x)=\begin{cases} x^2-5, & x\le 3 \\ \sqrt{x+13}, & x>3 \end{cases}$

To find $\lim_{x\to 3} f(x)$, we have

$$\text{LHL}=\lim_{x\to 3^-} f(x)=\lim_{x\to 3^-} x^2-5$$

$$=\lim_{x\to 3-b}\left[(3-b)^2-5\right]$$

$$=\lim_{b\to 0}\left(9-6b+b^2-5\right)=4$$

And,

$$\text{RHL}=\lim_{x\to 3^+} f(x)=\lim_{x\to 3^+}\sqrt{x+13}$$

$$=\lim_{x\to 3+b}\sqrt{3+b+13}$$

$$=\lim_{b\to 0}\sqrt{16+b}=4$$

$$\therefore \lim_{x\to 3^-} f(x)=\lim_{x\to 3^+} f(x)=4$$

So, $\lim_{x\to 3} f(x)=4$

Hence, option *(b)* is correct.

58. *(d)* For continuous,

$$\lim_{x\to 3^-} f(x)=\lim_{x\to 3^+} f(x)=f(3)$$

$$\therefore \lim_{x\to 3^-} f(x)=\lim_{x\to 3^+} f(x)=4$$

$\Rightarrow f(x)$ is continuous at $x=4$

We have, $f(x)=x^2-5,\ x\le 3$

$$\Rightarrow f'(x)=2x \Rightarrow f'(0)=0$$

Hence, $f(x)$ is differentiable at $x=0$.

So, neither I nor II statements are correct.

Hence, option *(d)* is correct.

59. *(a)* We have to select 2 white and 1 black balls. This could be done in the following ways:

1 white from Box I , 1 white from Box II and 1 black from Box III.

Or 1 white from Box I, 1 black from Box II and 1 white from Box III.

Or 1 black from Box I, 1 white from Box II and 1 white from Box III.

So, required probability

$$=\frac{{}^3C_1\times{}^2C_1\times{}^3C_1}{{}^4C_1\times{}^4C_1\times{}^4C_1}+\frac{{}^3C_1\times{}^2C_1\times{}^1C_1}{{}^4C_1\times{}^4C_1\times{}^4C_1}+\frac{{}^1C_1\times{}^2C_1\times{}^1C_1}{{}^4C_1\times{}^4C_1\times{}^4C_1}$$

$$=\frac{3\times2\times3}{4\times4\times4}+\frac{3\times2\times1}{4\times4\times4}+\frac{1\times2\times1}{4\times4\times4}$$

$$=\frac{18+6+2}{64}=\frac{26}{64}=\frac{13}{32}$$

Hence, option *(a)* is correct.

60. *(c)* Let R be an equivalence relation on set A containing n elements. Then, R divides the given set into n equivalence classes. All the equivalence classes are either disjoint or identical.

Thus, both the statements are correct.

Hence, option *(c)* is correct.

61. *(b)* Consider the left hand integral $\int x\tan^{-1}x\,dx$

Using the integration by parts, we have:

$$\int \underset{II}{x}\underset{I}{\tan^{-1}}x\,dx=\tan^{-1}x\cdot\frac{x^2}{2}-\int\frac{d}{dx}\left(\tan^{-1}x\right)\cdot\frac{x^2}{2}dx$$

$$=\frac{x^2\tan^{-1}x}{2}-\frac{1}{2}\int\frac{x^2}{1+x^2}dx$$

$$=\frac{x^2\tan^{-1}x}{2}-\frac{1}{2}\int\frac{1+x^2-1}{1+x^2}dx$$

$$=\frac{x^2\tan^{-1}x}{2}-\frac{1}{2}\left(\int 1\,dx-\int\frac{dx}{1+x^2}\right)$$

$$=\frac{x^2\tan^{-1}x}{2}-\frac{1}{2}\left(x-\tan^{-1}x\right)+C$$

$$=\frac{x^2\tan^{-1}x}{2}-\frac{x}{2}+\frac{\tan^{-1}x}{2}+C$$

$$=\frac{1}{2}\left(x^2+1\right)\tan^{-1}x-\frac{x}{2}+C$$

On comparing with the right side, we get

$A=\frac{1}{2}$ and $B=-\frac{1}{2}$

Hence, option *(b)* is correct.

62. *(c)* We have $\lim_{x\to\frac{\pi}{2}}\frac{\cot x-\cos x}{(\pi-2x)^3}=\lim_{x\to\frac{\pi}{2}}\frac{\cot x-\cos x}{8\left(\frac{\pi}{2}-x\right)^3}$

Let $\frac{\pi}{2}-x=t \Rightarrow x=\frac{\pi}{2}-t$

So, we get $\lim_{t\to0}\frac{\tan t-\sin t}{8t^3}$

$$=\lim_{t\to0}\frac{\sin t\left\{\frac{1}{\cos t}-1\right\}}{8t^3}$$

$$=\lim_{t\to0}\left(\frac{\sin t}{t}\right)\left(\frac{1-\cos t}{t^2}\right)\left(\frac{1}{8\cos t}\right)$$

$$=(1)\left(\frac{1}{2}\right)\left(\frac{1}{8}\right)=\frac{1}{16}$$

$$\left[\because\ \lim_{x\to0}\frac{\sin x}{x}=1 \text{ and } \lim_{x\to0}\frac{1-\cos ax}{x^2}=\frac{a^2}{2}\right]$$

Hence, option *(c)* is correct.

63. *(a)* Direction ratios of a line joining the points $(3,1,4)$ and $(7,2,12)$ are $(7-3,2-1,12-4)$ i.e. $(4,1,8)$

Now, angle between the line having direction ratios $(2,2,1)$ and $(4,1,8)$ is

$$\cos\theta = \frac{2\times4+2\times1+1\times8}{\sqrt{2^2+2^2+1^2}\sqrt{4^2+1^2+8^2}}$$

$$= \frac{8+2+8}{3(9)} = \frac{18}{27} = \frac{2}{3}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{2}{3}\right)$$

Hence, option *(a)* is correct.

64. *(c)* Weighted Mean

$$= \frac{1(1)+2(2)+3(3)+...+16(16)}{1+2+3+...+16}$$

$$= \frac{16(17)(33)}{6(16)\left(\frac{17}{2}\right)} = 11$$

$$\left[\because \sum n = \frac{n(n+1)}{2} \text{ and } \sum n^2 = \frac{n(n+1)(2n+1)}{6}\right]$$

Hence, option *(c)* is correct.

65. *(b)* Order of matrix $A = x\times(x+5)$

Order of matrix $B = y\times(11-y)$

Given AB and BA both exist. It is possible only when $x+5=y$ and $11-y=x$

$\Rightarrow x-y=-5$ and $x+y=11$

So solving, we get $x=3$

Hence, option *(b)* is correct.

66. *(d)* In ΔABC, $A+B+C=\pi$

If angles A and B are given, then we can find C.

From sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

We can find any other side.

Now, area of the triangle

$$\Delta = \frac{1}{2}ab\sin C = \frac{1}{2}bc\sin A = \frac{1}{2}ca\sin B$$

Thus, either *(a)* or *(b)* or *(c)* option are correct.

Hence, option *(d)* is correct.

67. *(d)* Given that $\frac{dy}{dx} = \frac{x(2\log x+1)}{\sin y + y\cos y}$

$$\Rightarrow \int(\sin y + y\cos y)dy = \int x(2\log x+1)dx$$

$$\Rightarrow -\cos y + y\sin y + \cos y = 2\int x\log x dx + \int x dx$$

$$\Rightarrow y\sin y = 2\left\{(\log x)\frac{x^2}{2} - \int \frac{1}{x}\cdot\frac{x^2}{2}dx\right\} + \frac{x^2}{2} + C$$

$$\Rightarrow y\sin y = x^2\log x - \frac{x^2}{2} + \frac{x^2}{2} + C$$

$$\Rightarrow y\sin y = x^2\log x + C$$

Hence, option *(d)* is correct.

68. *(b)* Area bounded by $y=\sin^{-1}x$

$\Rightarrow x=\sin y$ and lines $x=0$ and $|y| = \frac{\pi}{2} \Rightarrow y = \pm\frac{\pi}{2}$ is

$$A = \int_{-\pi/2}^{\pi/2} \sin y dy$$

$$= \int_{-\pi/2}^{0}(-\sin y)dy + \int_0^{\pi/2}\sin y dy$$

$$= [\cos y]_{-\pi/2}^{0} + [-\cos y]_0^{\pi/2}$$

$$= [1-(0)] + [0-(-1)]$$

$$= 2 \text{ sq units}$$

Hence, option *(b)* is correct.

69. *(b)* Scalar projection of a on $b = \frac{a\cdot b}{|b|}$

$$= \frac{(\hat{i}-2\hat{j}+\hat{k})\cdot(4\hat{i}-4\hat{j}+7\hat{k})}{|4\hat{i}-4\hat{j}+7\hat{k}|}$$

$$= \frac{(4+8+7)}{\sqrt{(4)^2+(-4)^2+(7)^2}} = \frac{19}{9}$$

which is the required scalar projection of a on b.

Hence, option *(b)* is correct.

70. *(a)* The vector perpendicular to both the vectors a and $b = a\times b$

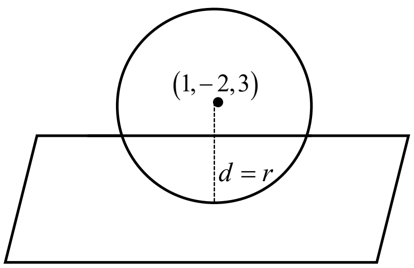
$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 1 \\ 4 & -4 & 7 \end{vmatrix}$$

$$= \hat{i}(-14+4) - \hat{j}(7-4) + \hat{k}(-4+8)$$

$$= -10\hat{i} - 3\hat{j} + 4\hat{k}$$

Hence, option *(a)* is correct.

71. *(d)* Consider the following diagram:



Radius of the sphere $=$ Distance of point $(1,-2,3)$ from the plane $6x-3y+2z-4=0$

$$= \left| \frac{ax_1+by_1+cz_1-d}{\sqrt{a^2+b^2+c^2}} \right|$$

$$= \left| \frac{6(1)+(-2)(-3)+3(2)-4}{\sqrt{6^2+3^2+2^2}} \right|$$

$$= \left| \frac{14}{\sqrt{49}} \right| = \frac{14}{7} = 2$$

So, diameter $= 2 \times \text{Radius} = 4$ units

Hence, option *(d)* is correct.

72. *(d)* Equation of the tangent at $\left(2\cos\theta, \sqrt{15}\sin\theta\right)$ on the ellipse E to the ellipse is $\frac{x}{2}\cos\theta + \frac{y}{\sqrt{15}}\sin\theta = 1$

If it represents the tangent $x-2y+8=0$, then

$$\frac{\cos\theta}{2} = \frac{\sin\theta}{-2\sqrt{15}} = \frac{-1}{8}$$

$$\Rightarrow \cos\theta = -\frac{1}{4},\ \sin\theta = \frac{\sqrt{15}}{4}$$

And the point of contact is $\left(-\frac{1}{2}, \frac{15}{4}\right)$.

Similarly, the point of contact is of the other tangent is $\left(-\frac{1}{2}, -\frac{15}{4}\right)$.

Hence, option *(d)* is correct.

73. *(c)* Given, $\sec x + \sec^2 x = 1$

$\Rightarrow \sec x = 1 - \sec^2 x = -\tan^2 x$

$\Rightarrow \sec^2 x = \tan^4 x$ [on squaring]

$\Rightarrow 1 + \tan^2 x = \tan^4 x$ $[\because \sec^2 a = 1 + \tan^2 a]$

$\Rightarrow 1 + \tan^4 x + 2\tan^2 x = \tan^8 x$ [on squaring]

$\Rightarrow \tan^8 x - \tan^4 x - 2\tan^2 x = 1$

Adding 1 to both sides, we get

$\tan^8 x - \tan^4 x - 2\tan^2 x + 1 = 1 + 1 = 2$

Hence, option *(c)* is correct.

74. *(b)* Given, $\int_a^b \left(f(x) - 3x\right)dx = a^2 - b^2$

$$\Rightarrow \int_a^b f(x)\,dx - 3\left(\frac{x^2}{2}\right)_a^b = a^2 - b^2$$

$$\Rightarrow \int_a^b f(x)\,dx - \frac{3}{2}\left(b^2 - a^2\right) = a^2 - b^2$$

$$\Rightarrow \int_a^b f(x)\,dx = \frac{b^2 - a^2}{2} \Rightarrow f(x) = x$$

$$\left[\because \int_a^b f(x)\,dx = \left(\frac{x^2}{2}\right)_a^b = \frac{b^2 - a^2}{2}\right]$$

So, $f\left(\frac{\pi}{6}\right) = \frac{\pi}{6}$

Hence, option *(b)* is correct.

75. *(b)* Given, $\dfrac{\sin^2 A - \sin^2 B}{\sin A \cos A - \sin B \cos B}$

$$= \frac{2\sin(A+B)\sin(A-B)}{\sin 2A - \sin 2B}$$

$$= \frac{2\sin(A+B)\sin(A-B)}{2\cos\left(\frac{2A+2B}{2}\right)\sin\left(\frac{2A-2B}{2}\right)}$$

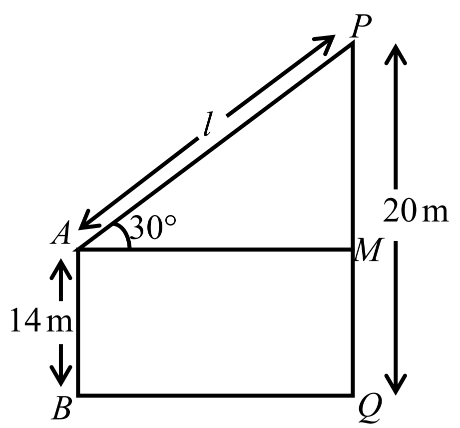
$$= \frac{2\sin(A+B)\sin(A-B)}{2\cos(A+B)\sin(A-B)}$$

$$= \frac{\sin(A+B)}{\cos(A+B)}$$

$$= \tan(A+B)$$

Hence, option *(b)* is correct.

76. *(b)* Let PQ and AB be the poles of height 20 m and 14m respectively. Let length of wire AP be l.



Then, $PM = PQ - MQ = 20 - 14 = 6\,\text{m}$

In ΔAPM, $\sin 30° = \frac{PM}{AP}$

$\Rightarrow \quad \frac{1}{2} = \frac{6}{l} \Rightarrow l = 12\,\text{m}$

Hence, option *(b)* is correct.

77. *(b)* Given, B is an idempotent matrix,

So, $B^2 = B$ ....(i)

And also given $A = I - B$

Then, $A^2 = (I-B)^2 = (I-B)(I-B)$

$= I^2 - IB - BI + B^2$

$= I - B - B + B$ [from Eq.(i)]

$= I - B$

$= A$

So, option *(a)* is correct and option *(b)* is not correct.

Now, $AB = (I-B)B$

$= IB - B^2$

$= B - B$ [from Eq.(i)]

$= 0$

Similarly, $BA = B(I-B) = BI - B^2$

$= B - B = 0$

Hence, option *(b)* is correct.

78. *(b)* Given, $\lim_{x\to\infty}\left(\frac{x+c}{x-c}\right)^x = 4$

$\Rightarrow \lim_{x\to\infty}\left(1+\frac{2c}{x-c}\right)^x = 4$

$\Rightarrow e^{\lim_{x\to\infty}\left(\frac{2c}{x-c}\right)(x)} = 4$

$\Rightarrow e^{2c} = 2^2$

$\Rightarrow (e^c)^2 = 2^2$

$\Rightarrow e^c = 2$

$\Rightarrow \log e^c = \log 2$

$\Rightarrow c\log e = \log 2$

$\Rightarrow c = \log 2$ [$\because \log e = 1$]

Hence, option *(b)* is correct.

79. *(a)* Given, $x + \frac{1}{x} = 2\cos\theta$

$\Rightarrow x^2 - 2x\cos\theta + 1 = 0$

$\Rightarrow x = \cos\theta \pm i\sin\theta$

$\Rightarrow x^n = (\cos\theta \pm i\sin\theta)^n$

$\Rightarrow x^n = \cos n\theta \pm i\sin n\theta$

$\Rightarrow \frac{1}{x^n} = \cos n\theta \mp i\sin n\theta$

$\therefore x^n + \frac{1}{x^n} = 2\cos n\theta$

Hence, option *(a)* is correct.

80. *(b)* Let $U = \{1, 2, 3, \ldots\ldots 20\}$

$A =$ Set of all natural numbers which are perfect square

$= \{1, 4, 9, 16\}$

$B =$ Set of all natural numbers which are multiples of 5 $= \{5, 10, 15, 20\}$

$C =$ Set of all natural numbers which are divisible by 2 and $3 = \{6, 12, 18\}$

Here, $A \cup B = \{1, 4, 9, 16, 5, 10, 15, 20\}$

$\Rightarrow n(A \cup B) = 8$

$\Rightarrow n(A \cup B)' = 20 - 8 = 12$

Thus, both Statements I and III are correct.
Hence, option *(b)* is correct.

81. *(d)* Let $f(x) = \dfrac{1+x}{1-x}$ (given)

Then, $\dfrac{f(x) \cdot f(x^2)}{1 + [f(x)]^2} = \dfrac{\left(\dfrac{1+x}{1-x}\right)\left(\dfrac{1+x^2}{1-x^2}\right)}{1 + \left(\dfrac{1+x}{1-x}\right)^2}$

$$= \frac{\dfrac{1+x^2}{(1-x)^2}}{\dfrac{(1-x)^2 + (1+x)^2}{(1-x)^2}} = \frac{1+x^2}{2(1+x^2)} = \frac{1}{2}$$

Hence, option *(d)* is correct.

82. *(d)* Let GP of n terms be $a, ar, ar^2, \ldots, ar^{n-1}$.

Then, $T_1 + T_n = 66$

$\Rightarrow a + ar^{n-1} = 66$ ....(i)

Now, $T_2 \times T_{n-1} = 128$

$\Rightarrow ar \times ar^{n-2} = 128$

$\Rightarrow a^2 r^{n-1} = 128$

$\Rightarrow ar^{n-1} = \dfrac{128}{a}$

From equation (i), we get

$a + \dfrac{128}{a} = 66$

$\Rightarrow a^2 - 66a + 128 = 0$

$\Rightarrow (a-2)(a-64) = 0$

$\Rightarrow a = 2$ or $a = 64$

Put the value of $a = 2$ and $a = 64$ in Eqs (i), we get

$r^{n-1} = 32$ and $r^{n-1} = \dfrac{1}{32}$

For increasing GP, $r > 1$

So, $r^{n-1} = 32$ and $a = 2$ are correct values.

Also, given sum of terms $= 126$

$\Rightarrow \dfrac{a(r^n - 1)}{r - 1} = 126$

$\Rightarrow \dfrac{2(r^n - 1)}{r - 1} = 126$

$\Rightarrow \dfrac{r^n - 1}{r - 1} = 63$

$\Rightarrow \dfrac{r^{n-1}(r) - 1}{r - 1} = 63$

$\Rightarrow \dfrac{32r - 1}{r - 1} = 63$

$\Rightarrow r = 2$

$\Rightarrow r^{n-1} = 32$

$\Rightarrow 2^{n-1} = 32 = 2^5$

$\Rightarrow n - 1 = 5$

$\Rightarrow n = 6$

Hence, option *(d)* is correct.

83. *(c)* Given, $\sin\dfrac{\pi}{18}\sin\dfrac{5\pi}{18}\sin\dfrac{7\pi}{18}$

$= \dfrac{1}{2}\left\{\sin\dfrac{\pi}{18}\right\}\left\{2\sin\dfrac{5\pi}{18}\sin\dfrac{7\pi}{18}\right\}$

$= \dfrac{1}{2}\sin\dfrac{\pi}{18}\left\{\cos\left(\dfrac{5\pi}{18} - \dfrac{7\pi}{18}\right) - \cos\left(\dfrac{5\pi}{18} + \dfrac{7\pi}{18}\right)\right\}$

$= \dfrac{1}{2}\sin\dfrac{\pi}{18}\left\{\cos\dfrac{\pi}{9} - \cos\dfrac{2\pi}{3}\right\}$

$= \dfrac{1}{4}\left(2\sin\dfrac{\pi}{18}\cos\dfrac{2\pi}{18}\right) - \dfrac{1}{2}\cos\left(\pi - \dfrac{\pi}{3}\right)\sin\dfrac{\pi}{18}$

$= \dfrac{1}{4}\left\{\sin\left(\dfrac{\pi}{18} + \dfrac{2\pi}{18}\right) + \sin\left(\dfrac{\pi}{18} - \dfrac{2\pi}{18}\right)\right\} + \dfrac{1}{2}\cos\dfrac{\pi}{3}\sin\dfrac{\pi}{18}$

$= \dfrac{1}{4}\left\{\sin\dfrac{\pi}{6} - \sin\dfrac{\pi}{18}\right\} + \dfrac{1}{4}\sin\dfrac{\pi}{18}$

$= \frac{1}{4}\sin\frac{\pi}{6} = \frac{1}{8}$

Hence, option *(c)* is correct.

84. *(d)* Let $I = \int \frac{e^x - 1}{e^x + 1}dx = \int \frac{e^x + 1 - 2}{e^x + 1}dx$

$= \int \left(1 - \frac{2}{e^x + 1}\right)dx$

$= \int 1\,dx - 2\int \frac{e^{-x}}{1 + e^{-x}}dx$

$\left[\text{let } 1 + e^{-x} = t \Rightarrow e^{-x}dx = -dt\right]$

$= x - 2\int \frac{(-dt)}{t}$

$= x + 2\log(t) + c$

$= x + 2\log\left(e^{-x} + 1\right) + c$

$= x + 2\log\left(\frac{1 + e^x}{e^x}\right) + c$

$= x + 2\log\left(1 + e^x\right) - 2\log\left(e^x\right) + c$

$= x + 2\log\left(1 + e^x\right) - 2x + c$

$= 2\log\left(1 + e^x\right) - x + c$

On comparing with $\int \frac{e^x - 1}{e^x + 1}dx = f(x) + c$, we get

$f(x) = 2\log\left(1 + e^x\right) - x$

Hence, option *(d)* is correct.

85. *(c)* Given, $\frac{dy}{dx}(x\log_e x) + y = 2\log_e x$

$\Rightarrow \frac{dy}{dx} + \frac{1}{(x\log_e x)}y = 2$

Comparing with $\frac{dy}{dx} + Py = Q$, we have

$P = \frac{1}{x\log x}$ and $Q = 2$

$\therefore IF = e^{\int Pdx} = e^{\int \frac{1}{x\log x}dx}$

$= e^{\log_e(\log_e x)} = \log_e x$

Hence, option *(c)* is correct.

86. *(d)* We have $\sin 65° + \sin 43° - (\sin 29° + \sin 7°)$

$= 2\sin\frac{65° + 43°}{2}\cos\frac{65° - 43°}{2}$

$- 2\sin\frac{29° + 7°}{2}\cos\frac{29° - 7°}{2}$

$= 2\sin 54°\cos 11° - 2\sin 18°\cos 11°$

$= 2\cos 11°(\sin 54° - \sin 18°)$

$= 2\cos 11°\left(\frac{\sqrt{5} + 1}{4} - \frac{\sqrt{5} - 1}{4}\right)$

$= \cos 11°$

Hence, option *(d)* is correct.

87. *(d)* Let $\Delta = \begin{vmatrix} 1 + \omega & \omega^2 & -\omega \\ 1 + \omega^2 & \omega & -\omega^2 \\ \omega + \omega^2 & \omega & -\omega^2 \end{vmatrix}$

$= \begin{vmatrix} -\omega^2 & \omega^2 & -\omega \\ -\omega & \omega & -\omega^2 \\ -1 & \omega & -\omega^2 \end{vmatrix} \qquad \left[\because 1 + \omega + \omega^2 = 0\right]$

$= \begin{vmatrix} \omega^2 & \omega^2 & \omega \\ \omega & \omega & \omega^2 \\ 1 & \omega & \omega^2 \end{vmatrix}$

$= \omega^2\left(\omega^3 - \omega^3\right) - \omega^2\left(\omega^3 - \omega^2\right) + \omega\left(\omega^2 - \omega\right)$

$= 0 - \omega^2\left(1 - \omega^2\right) + \omega\left(\omega^2 - \omega\right)$

$= -\omega^2 + \omega^4 + \omega^3 - \omega^2$

$= -2\omega^2 + \omega + 1$

$= -2\omega^2 - \omega^2 \qquad \left[\because 1 + \omega + \omega^2 = 0 \Rightarrow 1 + \omega = -\omega^2\right]$

$= -3\omega^2$

Hence, option *(d)* is correct.

88. *(c)* Equation of line joining the points $A(5,1,6)$ and $B(3,4,1)$ is

$$\frac{x-5}{3-5}=\frac{y-1}{4-1}=\frac{z-6}{1-6}$$

$$\Rightarrow \frac{x-5}{-2}=\frac{y-1}{3}=\frac{z-6}{-5}=\lambda \text{ (say)}$$

Let $P(-2\lambda+5, 3\lambda+1, -5\lambda+6)$ be any point on this line which crosses the YZ-plane.

So, put x-coordinate equals to zero.

$$\Rightarrow \quad -2\lambda+5=0 \Rightarrow \lambda=\frac{5}{2}$$

Hence, required point is

$$P\left(0, \frac{15}{2}+1, -\frac{25}{2}+6\right)=P\left(0, \frac{17}{2}, -\frac{13}{2}\right)$$

Hence, option *(c)* is correct.

89. *(c)* Vectors $-\lambda^2\hat{i}+\hat{j}+\hat{k}$, $\hat{i}-\lambda^2\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}-\lambda^2\hat{k}$

Then, $\begin{vmatrix} -\lambda^2 & 1 & 1 \\ 1 & -\lambda^2 & 1 \\ 1 & 1 & -\lambda^2 \end{vmatrix}=0$

$$\Rightarrow -\lambda^2(-\lambda^4-1)-1(-\lambda^2-1)+1(1+\lambda^2)=0$$

$$\Rightarrow -\lambda^6+3\lambda^2+2=0$$

$$\Rightarrow (2-\lambda^2)(\lambda^4+2\lambda^2+1)=0$$

$$\Rightarrow \lambda=\pm\sqrt{2}$$

So, there are two distinct values of λ for which the given vectors are coplanar.

Hence, option *(c)* is correct.

90. *(b)* Given, $\cos\theta=-\frac{\sqrt{3}}{2}$ and θ does not lie in the III quadrant.

$\Rightarrow \theta$ lies in II quadrant.

$$\Rightarrow \tan\theta=-\frac{1}{\sqrt{3}} \text{ and } \cot\theta=-\sqrt{3}$$

Also, given that $\sin\alpha=-\frac{3}{5}$ and α lies in III quadrant.

$$\Rightarrow \tan\alpha=\frac{3}{4} \text{ and } \cos\alpha=-\frac{4}{5}$$

So, $\frac{2\tan\alpha+\sqrt{3}\tan\theta}{\cot^2\theta+\cos\alpha}$

$$=\frac{2\left(\frac{3}{4}\right)+\sqrt{3}\left(-\frac{1}{\sqrt{3}}\right)}{\left(-\sqrt{3}\right)^2+\left(-\frac{4}{5}\right)}=\frac{\frac{3}{2}-1}{3-\frac{4}{5}}=\frac{5}{22}$$

Hence, option *(b)* is correct.

91. *(c)* Given equation is

$$2x^2-2(k-2)x-(k+1)=0$$

Let roots of the equation be α and β

Then, $\alpha+\beta=(k-2)$ and $\alpha\beta=\frac{-(k+1)}{2}$

Now, let sum of squares of the roots be

$$A=\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$$

$$=(k-2)^2+(k+1)$$

$$=k^2-3k+5$$

$$\Rightarrow A=k^2-3k+\frac{9}{4}+5-\frac{9}{4}$$

$$\Rightarrow A=\left(k-\frac{3}{2}\right)^2-\frac{11}{4}$$

For minimum value of A

$$\left(k-\frac{3}{2}\right)^2=0 \Rightarrow k=\frac{3}{2}$$

Hence, option *(c)* is correct.

92. *(c)* Consider $I=\int_0^{2\pi}\frac{dx}{1+3\cos^2 x}$

Let $f(x)=\frac{1}{1+3\cos^2 x}$

$$\Rightarrow f(2\pi-x)=\frac{1}{1+3\cos^2(2\pi-x)}$$

$$=\frac{1}{3+\cos^2 x}=f(x)$$

$\therefore$ if $f(na-x)=f(x)$, then $\int_a^{na} f(x)\,dx=n\int_a^a f(x)\,dx$

Therefore,

$$\int_0^{2\pi}\frac{dx}{1+3\cos^2 x}=\int_0^{4(\pi/2)}\frac{dx}{1+3\cos^2 x}$$
$$=4\int_0^{\pi/2}\frac{dx}{1+3\cos^2 x}$$
$$=4\int_0^{\pi/2}\frac{\sec^2 xdx}{\sec^2 x+3}$$
$$=4\int_0^{\pi/2}\frac{\sec^2 xdx}{4+\tan^2 x}$$
$$=4\cdot\frac{1}{2}\left[\tan^{-1}\left(\frac{\tan x}{2}\right)\right]_0^{\pi/2}$$
$$=2\left[\tan^{-1}(\infty)-\tan^{-1}(0)\right]$$
$$=2(\pi/2)-0$$
$$=\pi$$

Hence, option *(c)* is correct.

93. *(c)* Out of the given face values $-2,-1,0,1,2,3$, there are three positive and two negative numbers. In order to get positive outcomes when die is thrown five times, there should be all five positive face values or three positive and 2 negative values or one positive and 4 negative values.

Probability of getting a positive number, $p=\frac{3}{6}$

Probability of getting a negative number, $q=\frac{2}{6}$

Hence, required probability

$$={}^5C_5\left(\frac{3}{6}\right)^5+{}^5C_2\left(\frac{3}{6}\right)^3\left(\frac{2}{6}\right)^2+{}^5C_1\left(\frac{3}{6}\right)^1\left(\frac{2}{6}\right)^4$$
$$=\frac{1}{32}+\frac{5}{36}+\frac{5}{162}$$
$$=\frac{521}{2592}$$

Hence, option *(c)* is correct.

94. *(d)* The equations are $x+2y+3z=1,\ x-y+4z=0$ and $2x+y+7z=1$

Then, $D=\begin{vmatrix}1&2&3\\1&-1&4\\2&1&7\end{vmatrix}$

$$=1(-7-4)-2(7-8)+3(1+2)$$
$$=-11+2+9=0$$

Now, $D_1=\begin{vmatrix}1&2&3\\0&-1&4\\1&1&7\end{vmatrix}$

$$=1(-7-4)+1(8+3)$$
$$=-11+11=0$$

And, $D_2=\begin{vmatrix}1&1&3\\1&0&4\\2&1&7\end{vmatrix}$

$$=1(-4)-1(7-8)+3(1)$$
$$=-4+1+3=0$$

And, $D_3=\begin{vmatrix}1&2&1\\1&-1&0\\2&1&1\end{vmatrix}$

$$=1(-1)-2(1)+1(1+2)$$
$$=-1-2+3=0$$

Clearly, system equations have infinitely many solutions.

Hence, option *(d)* is correct.

95. *(a)* Given $f(x)=x^2$ and $g(x)=x^3$

Then, $f\circ g(x)=f(g(x))=f(x^3)=x^6$

and $g\circ f(x)=g(f(x))=g(x^2)=x^6$

So, $f\circ g(x)=g\circ f(x)$

Now, let $f(x)=\log(x)$and $g(x)=x^2$

Then, $f\circ g(x)=f(g(x))=f(x^3)$

$$=\log x^2=2\log x$$

And $g\circ f(x)=g(f(x))=g(\log(x))=(\log x)^2$

Clearly, $f\circ g(x)\neq g\circ f(x)$

So, composition of functions is not commutative.

Hence, assertion is correct but reason is wrong.

Hence, option *(a)* is correct.

96. *(c)* Since, $\log 3$, $\log\left(3^x-2\right)$ and $\log\left(3^x+4\right)$ are in AP.

$\therefore \frac{\log\left(3^x+4\right)+\log 3}{2}=\log\left(3^x-2\right)$

$\Rightarrow \log\left[\left(3^x+4\right)\cdot 3\right]=2\log\left(3^x-2\right)$

$\left[\because \log(ab)=\log(a)+\log(b)\right]$

$\Rightarrow \log\left[\left(3^x+4\right)\cdot 3\right]=\log\left(3^x-2\right)^2$

$\left[\because \log a^b=b\log(a)\right]$

$\Rightarrow \left(3^x+4\right)^3=\left(3^x-2\right)^2$

Let $y=3^x$

$\therefore \left(y+4\right)^3=\left(y-2\right)^2$

$\Rightarrow 3y+12=y^2+4-4y$

$\Rightarrow y^2-7y-8=0$

$\Rightarrow y=8$ or $y=-1$

$\Rightarrow 3^x=8$ or $3^x=-1$ (not possible)

$\Rightarrow x=\log_3 8=\log_3 2^3$

Hence, option *(c)* is correct.

97. *(a)* Given $\left|z_1+z_2\right|^2=\left|z_1\right|^2+\left|z_2\right|^2$

$\Rightarrow \left|z_1\right|^2+\left|z_2\right|^2+2\,\mathrm{Re}\left(z_1\overline{z_2}\right)=\left|z_1\right|^2+\left|z_2\right|^2$

$\Rightarrow \mathrm{Re}\left(z_1\overline{z_2}\right)=0$

$\Rightarrow z_1\overline{z_2}$ is purely imaginary number ....(i)

$\arg\left(\frac{z_1}{z_2}\right)=\arg\left(\frac{z_1\overline{z_2}}{z_2\overline{z_2}}\right)$

$=\arg\left(\frac{z_1\overline{z_2}}{\left|z_2\right|^2}\right)$

$\{\because \left|z_2\right|^2$ is real and $z_1\overline{z_2}$ is purely imaginary number$\}$

$=\pm\frac{\pi}{2}$

Hence, option *(a)* is correct.

98. *(b)* We have $\left(\frac{d^3y}{dx^3}\right)^{\frac{2}{3}}+4-3\left(\frac{d^2y}{dx^2}\right)+5\left(\frac{dy}{dx}\right)=0$

$\Rightarrow \left(\frac{d^3y}{dx^3}\right)^{\frac{2}{3}}=3\left(\frac{d^2y}{dx^2}\right)-5\left(\frac{dy}{dx}\right)-4$

$\Rightarrow \left(\frac{d^3y}{dx^3}\right)^2=\left[3\left(\frac{d^2y}{dx^2}\right)-5\left(\frac{dy}{dx}\right)-4\right]^3$

Degree of differential equation = Power of the highest order derivative = 2

Hence, option *(b)* is correct.

99. *(b)* Given, $\cot^{-1}x+\cot^{-1}y+\cot^{-1}z=\frac{\pi}{2}$

$\Rightarrow \cot^{-1}x+\cot^{-1}y=\frac{\pi}{2}-\cot^{-1}z$

$\Rightarrow \tan^{-1}\left(\frac{1}{x}\right)+\tan^{-1}\left(\frac{1}{y}\right)=\tan^{-1}z$

$\Rightarrow \tan^{-1}\left(\frac{\frac{1}{x}+\frac{1}{y}}{1-\frac{1}{x}\cdot\frac{1}{y}}\right)=\tan^{-1}z$

$\Rightarrow \frac{1}{x}+\frac{1}{y}=z\left(\frac{xy-1}{xy}\right)$

$\Rightarrow x+y=xyz-z$

$\Rightarrow x+y+z=xyz$

Hence, option *(b)* is correct.

100. *(b)* Given, A and B are mutually exclusive and exhausitive events. So, $P\left(A\cup B\right)=1$ and

$P\left(A\cap B\right)=0$

$\Rightarrow P(A)+P(B)=1$

$\Rightarrow P(A)+3P(A)=1$ $\{\text{given } P(B)=3P(A)\}$

$\Rightarrow P(A)=\frac{1}{4} \Rightarrow P(B)=\frac{3}{4}$

So, $P\left(\overline{B}\right)=1-P(B)=1-\frac{3}{4}=\frac{1}{4}$

Hence, option *(b)* is correct.

101. *(b)* Equation of plane containing $x+y+z-6=0$ and $2x+3y+4z+5=0$ is given by

$$(x+y+z-6)+\lambda(2x+3y+4z+5)=0$$

$$\Rightarrow (1+2\lambda)x+(1+3\lambda)y+(1+4\lambda)z-6+5\lambda=0 \quad ....(i)$$

This plane passes through $(1,1,1)$

$$\Rightarrow 1+2\lambda+1+3\lambda+1+4\lambda-6+5\lambda=0$$

$$\Rightarrow \lambda=\frac{3}{14}$$

On putting in Eq. (i), we get

$$\left(1+\frac{6}{14}\right)x+\left(1+\frac{9}{14}\right)y+\left(1+\frac{12}{14}\right)z-6+\frac{15}{14}=0$$

$$\Rightarrow 20x+23y+26z-69=0$$

Hence, option *(b)* is correct.

102. *(b)* Let $y=\frac{1}{2-\sin 3x} \Rightarrow 2y-y\sin 3x=1$

$$\Rightarrow \quad \sin 3x=\frac{(2y-1)}{y}$$

Since, $-1\le \sin 3x\le 1$

We have, $-1\le \frac{(2y-1)}{y}\le 1$

Since, $y>0$ multiplying the inequality Eq.(i) by y, we obtain,

$-y\le 2y-1\le y$ or $1\le 3y$ and $y\le 1$

$$\Rightarrow \quad \frac{1}{3}\le y\le 1$$

Hence, option (b) is correct.

103. *(a)* Let *P* divides the line segment joined by points $A(2,3)$ and $B(-1,2)$ internally in ratio $3:4$.

Then, coordinates of point *P* are

$$P\left(\frac{3\times(-1)+4\times 2}{3+4},\frac{3\times 2+4\times 3}{3+4}\right)=P\left(\frac{5}{7},\frac{18}{7}\right)$$

This point P lies on the line $x+2y=\lambda$

So, $\frac{5}{7}+2\left(\frac{18}{7}\right)=\lambda \Rightarrow \lambda=\frac{41}{7}$

Hence, option *(a)* is correct.

104. *(b)* Given lines of regression are $x+2y-5=0$ and $x+3y-8=0$

$$\Rightarrow x=-2y+5 \text{ and } y=-\frac{1}{3}x+\frac{8}{3}$$

Comparing with $x=b_{xy}\left(y-\bar{y}\right)+\bar{x}$ and $y=b_{yx}\left(x-\bar{x}\right)+\bar{y}$, we get

$$b_{xy}=-2 \text{ and } b_{yx}=-\frac{1}{3}$$

So, coefficient of correlation is given by

$$\rho=\sqrt{b_{xy}\cdot b_{yx}}$$

$$=\sqrt{(-2)\cdot\left(-\frac{1}{3}\right)}$$

$$=\sqrt{\frac{2}{3}}=\sqrt{0.67}=0.82$$

$$\Rightarrow 100\rho=100\times 0.82=82$$

105. *(b)* We have $\alpha=\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{3}\right)$

$$=\tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\cdot\frac{1}{3}}\right)=\tan^{-1}\left(\frac{\frac{5}{6}}{\frac{5}{6}}\right)$$

$$=\tan^{-1}(1)=\frac{\pi}{4}$$

And, $\beta=\cos^{-1}\left(\frac{2}{3}\right)+\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)$

$$\Rightarrow \beta=\cos^{-1}\left(\frac{2}{3}\right)+\sin^{-1}\left(\sqrt{1-\frac{5}{9}}\right)$$

$$\Rightarrow \beta=\cos^{-1}\left(\frac{2}{3}\right)+\sin^{-1}\left(\frac{2}{3}\right)=\frac{\pi}{2}$$

And, $\gamma=\sin^{-1}\left[\sin\left(\frac{2\pi}{3}\right)\right]+\frac{1}{2}\cos^{-1}\left[\cos\left(\frac{2\pi}{3}\right)\right]$

$$\Rightarrow \gamma=\sin^{-1}\left(\sin\frac{\pi}{3}\right)+\frac{1}{2}\cos^{-1}\left[\cos\frac{2\pi}{3}\right]$$

$$\Rightarrow \gamma=\frac{\pi}{3}+\frac{1}{2}\cdot\frac{2\pi}{3}=\frac{2\pi}{3}$$

$$\therefore \cos(\alpha+\beta+\gamma)=\cos\left(\frac{\pi}{4}+\frac{\pi}{2}+\frac{2\pi}{3}\right)=\cos\left(\frac{7\pi}{12}\right)$$

106. *(d)* Let $\Delta=\begin{vmatrix}2xy & x^2 & y^2\\ x^2 & y^2 & 2xy\\ y^2 & 2xy & x^2\end{vmatrix}$

$$=\begin{vmatrix}(x+y)^2 & (x+y)^2 & (x+y)^2\\ x^2 & y^2 & 2xy\\ y^2 & 2xy & x^2\end{vmatrix}$$

$$=(x+y)^2\begin{vmatrix}1 & 1 & 1\\ x^2 & y^2 & 2xy\\ y^2 & 2xy & x^2\end{vmatrix}$$

$[\text{On applying } C_2\to C_2-C_1 \text{ and } C_3=C_3-C_1]$

$$=(x+y)^2\begin{vmatrix}1 & 0 & 0\\ x^2 & y^2-x^2 & x(2y-x)\\ y^2 & y(2x-y) & x^2-y^2\end{vmatrix}$$

$$=(x+y)^2\{(y^2-x^2)(x^2-y^2)-xy(2x-y)(2y-x)\}$$

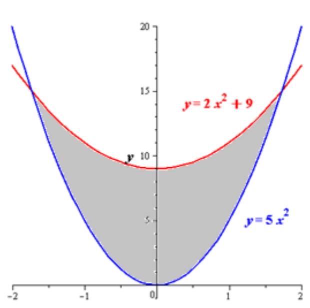
$$=(x+y)^2\{2x^2y^2-y^4-x^4-4x^2y^2+2x^3y+2xy^3-x^2y^2\}$$

$$=-(x+y)^2\{x^4+y^4+x^2y^2-2x^3y-2xy^3+2x^2y^2\}$$

$$=-(x+y)^2(x^2-xy+y^2)^2$$

$$=-(x^3+y^3)^2$$

107. *(c)* Given parabolas $5x^2-y=0$ and $2x^2-y+9=0$ intersect at the points $(-\sqrt{3},15)$ and $(\sqrt{3},15)$. (on solving both the equations)



So, required area is

$$A=2\int_0^{\sqrt{3}}\left[(2x^2+9)-5x^2\right]dx \qquad [\text{from diagram}]$$

$$=2\int_0^{\sqrt{3}}(9-3x^2)\,dx$$

$$=2\left[9x-x^3\right]_0^{\sqrt{3}}$$

$$=2\left[9\sqrt{3}-3\sqrt{3}\right]$$

$$=12\sqrt{3}\text{ sq units}$$

Hence, option *(c)* is correct.

108. *(b)* Let the side of the cube $=x$

Then, volume $(V)=x^3$ and surface area $(S)=6x^2$

$$\Rightarrow \frac{dV}{dt}=3x^2\frac{dx}{dt} \text{ and } \frac{dS}{dt}=12x\frac{dx}{dt}$$

Given, $\frac{dV}{dt}=4\,\text{cm}^3/\text{min}$ and $V=125\,\text{cm}^3$

$\Rightarrow x=5\,\text{cm}$

$$\Rightarrow 4=3(5)^2\frac{dx}{dt}$$

$$\Rightarrow \frac{dx}{dt}=\frac{4}{75}\text{cm/min}$$

Now, rate of change of surface area

$$\frac{dS}{st}=12x\frac{dx}{dt}=12(5)\times\frac{4}{75}=\frac{16}{5}\text{cm}^2/\text{min}$$

Here, we put $(-)$ve sign because of decreasing.

So, $\frac{dS}{dt}=-\frac{16}{5}\text{cm}^3/\text{min}$

Hence, option *(b)* is correct.

109. *(a)* Given, $\sin\theta+\operatorname{cosec}\theta=2$

$$\Rightarrow \sin\theta+\frac{1}{\sin\theta}=2$$

$$\Rightarrow \sin^2\theta-2\sin\theta+1=0$$

$$\Rightarrow (\sin\theta-1)^2=0$$

$$\Rightarrow \sin\theta=1$$

$$\Rightarrow \theta=90^\circ$$

So, $\sin^{10}\theta+\operatorname{cosec}^{10}\theta=\sin^{10}90^\circ+\operatorname{cosec}^{10}90^\circ$

$=1+1=2$

Hence, option *(a)* is correct.

110. *(a)* Let A be the event that a man lives 10 more years and B be the event that his wife lives 10 more years.

Then, $P(A)=1/3$

And, $P(B)=1/4$ [given]

$\Rightarrow P(\overline{A})=2/3$ and $P(\overline{B})=3/4$

Probability that at least one of them will live 10 more years $=1-$ both are dead

$=1-P(\overline{A}\cap\overline{B})$

$=1-P(\overline{A})P(\overline{B})$

[$\because A$ and B both are independent events]

$=1-\frac{2}{3}\times\frac{3}{4}=\frac{1}{2}$

Hence, option *(a)* is correct.

111. *(d)* Let equation of ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Given, major axis $=3$ (minor axis)

$\Rightarrow \quad 2a=3(2b)$

$\Rightarrow \quad \frac{a}{b}=3$ or $\frac{b}{a}=\frac{1}{3}$

So, eccentricity $e=\sqrt{1-\frac{b^2}{a^2}}$

$\Rightarrow \quad e=\sqrt{1-\frac{1}{9}}=\frac{2\sqrt{2}}{3}$

Hence, option *(d)* is correct.

112. *(c)* Equation of plane passing through point $(0,7,-7)$ is

$a(x-0)+b(y-7)+c(z+7)=0$ ....(i)

Now, the given line $\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1}$ passes through the point $(-1,3,-2)$ and its direction ratios are $(-3,2,1)$.

Therefore, plane will also contain the point $(-1,3,-2)$.

So, $-a-4b+5c=0$

$\because$ Plane contains the given line. So, it must be parallel to line

$\Rightarrow -3a+2b+c=0$ ....(ii)

On solving Eqs. (i) and (ii), we get

$$\begin{vmatrix} x-0 & y-7 & z+7 \\ -1 & -4 & 5 \\ -3 & 2 & 1 \end{vmatrix}=0$$

$\Rightarrow 14x+14y-105+14z+105=0$

$\Rightarrow x+y+z=0$

Hence, option *(c)* is correct.

113. *(d)* The sphere is $x^2+y^2+z^2+4x-2y-6z=155$

Its centre is $c(-2,1,3)$ and radius $r=13$

Given plane is $12x+y+3z-327=0$

Shortest distance between plane and sphere $=$ distance from the centre of sphere to the plane $-$ radius of the sphere

$=\left|\frac{(-2)(12)+(1)(1)+3(3)-327}{\sqrt{12^2+4^2+3^2}}\right|-13$

$=26-13$

$=13$ units

Hence, option *(d)* is correct.

114. *(b)* Given, mean $=5$ and standard deviation $=2$

Now, 5 is added to each value.

Then, new mean $=5+5=10$ and standard deviation $=2$

[$\because$ SD is not affected by change of origin]

So, coefficient of variance for new set of values

$=\frac{\text{SD}}{\text{Mean}}\times 100=\frac{2}{10}\times 100=20$

Hence, option *(b)* is correct.

115. *(d)* Probability of team A winning $= \frac{1}{3}$

And probability of team A drawing $= \frac{1}{6}$

2, 0 and 1 points are given on winning, losing or drawing respectively.
In a series of 3 one day cricket matches, cases that team A score 5 points are as follows:

$(2,2,1)$ or $(2,1,2)$ or $(1,2,2)$

i.e., (Win, Win, Draw) or (Win, Draw, Win) or (Draw, Win, Win)
So, required probability

$$= \frac{1}{3}\times\frac{1}{3}\times\frac{1}{6}+\frac{1}{3}\times\frac{1}{6}\times\frac{1}{3}+\frac{1}{6}\times\frac{1}{3}\times\frac{1}{3}$$

$$= \frac{3}{54} = \frac{1}{18}$$

Hence, option *(d)* is correct.

116. *(a)* Here $\alpha+\beta=-p$ and $\alpha\beta=q$

$$\therefore\ \tan^{-1}\alpha+\tan^{-1}\beta=\tan^{-1}\left(\frac{\alpha+\beta}{1-\alpha\beta}\right)$$

$$=\tan^{-1}\left(\frac{-p}{1-q}\right)$$

$$=\tan^{-1}\left(\frac{p}{q-1}\right)$$

Hence, option *(a)* is correct.

117. *(b)* We have, $\frac{dy}{dx}=\frac{xy}{x^2+y^2}$

Put $y=vx$

$$\Rightarrow \frac{dy}{dx}=v(1)+x\frac{dv}{dx}$$

We get $v+x\frac{dv}{dx}=\frac{x^2v}{x^2(1+v^2)}$

$$\Rightarrow x\frac{dv}{dx}=\frac{v}{1+v^2}-v=\frac{-v^3}{1+v^2}$$

$$\Rightarrow \int\frac{1+v^2}{v^3}dv=-\int\frac{dx}{x}$$

$$\Rightarrow -\frac{1}{2v^2}+\log v=-\log x+\log c$$

$$\Rightarrow -\frac{x^2}{2y^2}+\log\left(\frac{y}{x}\right)=-\log x+\log c \quad \left[\because v=\frac{y}{x}\right]$$

$$\Rightarrow -\frac{x^2}{2y^2}+\log y=\log c \quad \text{....(i)}$$

Given $y(1)=1$

$$\Rightarrow -\frac{1}{2}+\log(1)=\log c$$

$$\Rightarrow c=e^{-1/2}=\frac{1}{\sqrt{e}}$$

So, from Eq.(i), we get

$$-\frac{x^2}{2y^2}+\log y=\log\left(\frac{1}{\sqrt{e}}\right)$$

Put $y=e$

$$-\frac{x^2}{2e^2}+\log e=\log\left(\frac{1}{\sqrt{e}}\right)$$

$$\Rightarrow -\frac{x^2}{2e^2}+1=\log 1-\log e^{1/2}$$

$$\Rightarrow -\frac{x^2}{2e^2}+1=0-\frac{1}{2}$$

$$\Rightarrow -\frac{x^2}{2e^2}=-\frac{3}{2} \Rightarrow x^2=3e^2$$

$$\Rightarrow x=\sqrt{3}e$$

Hence, option *(b)* is correct.

118. *(d)* Let $I=\int_0^a\frac{1}{1+e^{f(x)}}dx \quad \text{....}(i)$

$$\Rightarrow I=\int_0^a\frac{1}{1+e^{f(a-x)}}dx \quad [\text{from property}]$$

$$\Rightarrow I=\int_0^a\frac{1}{1+e^{-f(x)}}dx$$

$$[\because f(x)+f(a-x)=0\Rightarrow f(a-x)=-f(x)]$$

$$\Rightarrow I = \int_0^a \frac{e^{f(x)}}{e^{f(x)}+1}dx \quad ....(ii)$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_0^a \frac{e^{f(x)}+1}{e^{f(x)}+1}dx = \int_0^a 1dx = a$$

$$\Rightarrow I = \frac{a}{2}$$

Hence, option *(d)* is correct.

119. *(a)* We have $f(x) = \sin x + \frac{1}{2}\cos 2x$

$$\Rightarrow f'(x) = \cos x - \sin 2x = \cos x(1-2\sin x)$$

$$\therefore \; f'(x) = 0$$

$$\Rightarrow \cos x(1-2\sin x) = 0$$

$$\Rightarrow \cos x = 0 \text{ or } \sin x = \frac{1}{2}$$

$$\Rightarrow x = \frac{\pi}{2} \text{ or } x = \frac{\pi}{6} \qquad \left[\because 0 \le x \le \frac{\pi}{2}\right]$$

Now, $f(0) = \frac{1}{2}$, $f\left(\frac{\pi}{6}\right) = \frac{3}{4}$ and $f\left(\frac{\pi}{2}\right) = \frac{1}{2}$

Out of these values, the minimum value is $\frac{1}{2} < \frac{4}{3} < \frac{3}{2}$.

Hence, option *(a)* is correct.

120. *(b)* Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$

Then,

$$z_1z_2 = (x_1+iy_1)(x_2+iy_2)$$
$$= (x_1x_2 - y_1y_2) + i(x_1y_2 + y_1x_2)$$

$$\Rightarrow \text{Im}(z_1z_2) = x_1y_2 + y_1x_2$$

$$\Rightarrow \text{Im}(z_1z_2) = \text{Re}(z_1)\text{Im}(z_2) + \text{Im}(z_1)\text{Re}(z_2)$$

Hence, option *(b)* is correct.