HINTS & SOLUTION

1. *(c)* As given, $\sin\theta$ and $\cos\theta$ are the roots of the equation $ax^2+bx+c=0$

So, sum of roots $=\sin\theta+\cos\theta=-\frac{b}{a}$ ....(i)

and product of roots $=\sin\theta\cos\theta=\frac{c}{a}$ ....(ii)

On squaring both sides in Eq.(i) and (ii), we get

$$(\sin\theta+\cos\theta)^2=\frac{b^2}{a^2}$$

$$\Rightarrow \sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta=\frac{b^2}{a^2}$$

$$\Rightarrow 1+\frac{2c}{a}=\frac{b^2}{a^2} \quad [\text{using Eq.(ii)}]$$

$$\Rightarrow a^2+2ca=b^2$$

$$\Rightarrow a^2-b^2+2ac=0$$

Hence, option *(c)* is correct.

2. *(b)* We have $1001.01=1\times2^3+1\times2^0+1\times2^{-2}$

Corresponding to number of base 10

$$=8+1+\frac{1}{4}=\frac{37}{4}=9.25$$

and $11.1=1\times2^1+1\times2^0+1\times2^{-1}$

$$=2+1+\frac{1}{2}=\frac{7}{2}=3.5$$

Corresponding to the number of base 10

$\therefore 1001.01\times11.1=9.25\times3.5=32.375$

From decimal to binary, we have

$(32)_{10}=(100000)_2$ and $(.375)_{10}=0.25+0.125$

$$=\frac{1}{4}+\frac{1}{8}=1\times2^{-2}+1\times2^{-3}=(0.011)_2$$

$$\therefore (32.375)_{10}=(100000.011)_2$$

Hence, option *(b)* is correct.

3. *(a)* As given the series is

$$S=1+\frac{1}{8}+\frac{1\cdot3}{8\cdot16}+\frac{1\cdot3\cdot5}{8\cdot16\cdot24}+\cdots\infty$$

On comparing this series with

$S=(1+x)^n=1+nx+\frac{n(n-1)}{2!}x^2+\cdots\infty$, we get

$$nx=\frac{1}{8} \quad ....(i)$$

and $\frac{n(n-1)}{2!}x^2=\frac{1\cdot3}{8\cdot16}$ ....(ii)

from Eqs. (i) and (ii), we get

$$\frac{\frac{n(n-1)}{2!}x^2}{n^2x^2}=\frac{\frac{1\cdot3}{8\cdot16}}{\frac{1}{8}\cdot\frac{1}{8}}$$

$$\Rightarrow \frac{n-1}{2n}=\frac{3}{2}$$

$$\Rightarrow n-1=3n$$

$$\Rightarrow n=-\frac{1}{2}$$

On putting this value in Eq. (i)

$$\Rightarrow \left(-\frac{1}{2}\right)x=\frac{1}{8}$$

$$\Rightarrow x=-\frac{1}{4}$$

But, $S=(1+x)^n=\left(1-\frac{1}{4}\right)^{-1/2}$

$$=\left(\frac{3}{4}\right)^{-1/2}=\frac{2}{\sqrt{3}}$$

Hence, option *(a)* is correct.

4. *(c)* k men selected out of 5 and $(5-k)$ women out of 5.

These are ${}^5C_k$ and ${}^5C_{5-k}$

According to problem:

$${}^5C_k \times {}^5C_{5-k} = 100$$

$$\Rightarrow \frac{5!}{k!(5-k)!} \times \frac{5!}{(5-k)!5!} = 100$$

$$\Rightarrow \left(\frac{5}{k!(5-k)!}\right)^2 = 100$$

$$\Rightarrow \frac{5!}{k!(5-k)!} = 10$$

This is true for $k = 2$ or 3

Hence, option *(c)* is correct.

5. *(c)* Consider $\left(\frac{\sqrt{3}+i}{\sqrt{3}-i}\right) = \frac{(\sqrt{3}+i)(\sqrt{3}+i)}{(\sqrt{3}-i)(\sqrt{3}+i)}$

$$= \frac{(\sqrt{3}+i)^2}{(\sqrt{3})^2 - (i)^2} = \frac{3 + i^2 + 2\sqrt{3}i}{3 - i^2}$$

$$= \frac{2 + 2\sqrt{3}i}{4} = \frac{1+\sqrt{3}i}{2} = -\omega^2 \qquad [\because i^2 = -1]$$

Now, $\left(\frac{\sqrt{3}+i}{\sqrt{3}-i}\right)^6 = (-\omega^2)^6 = \omega^{12} = 1 \quad [\because \omega^3 = 1]$

Hence, option *(c)* is correct.

6. *(b)* Radius of circle $= 6$, centre $= (3,5)$

$\therefore$ Equation of circle is

$$S \equiv (x-3)^2 + (y-5)^2 = (6)^2$$

$$\Rightarrow S \equiv (x-3)^2 + (y-5)^2 - 36$$

Now, consider all four options:

(a) $(-2,-1)$

Put it in S

$$S \equiv (-2-3)^2 + (-1-5)^2 - 36 = 25 + 36 = 25 > 0$$

$\Rightarrow (-2,-1)$ is outside the circle.

(b) $(0,1)$

$$S \equiv (0-3)^2 + (1-5)^2 - 36$$

$$= 9 + 16 - 36 = 25 - 36 = -11 < 0$$

Hence, $(0,1)$ lies inside a circle.

Hence, option *(b)* is correct.

7. *(d)* Consider $\left(x^2 - \frac{1}{x}\right)^9$

$$t_{r+1} = {}^9C_r \left(x^2\right)^{9-r} \left(-\frac{1}{x}\right)^r$$

$$= {}^9C_r x^{18-2r} (-1)^r \cdot x^{-r}$$

$$= {}^9C_r (x)^{18-3r} (-1)^r \qquad \text{....(i)}$$

Term will be independent of x when

$18 - 3r = 0 \Rightarrow r = 6$

Put $r = 6$ in (i), we get

$$t_7 = {}^9C_6 (-1)^6 = \frac{9!}{6!3!} = 84$$

Hence, option *(d)* is correct.

8. *(a)* Since, a, $2a+2, 3a+3$ are in GP

$\therefore (2a+2)^2 = a(3a+3)$

$\Rightarrow 4a^2 + 4 + 8a = 3a^2 + 3a \Rightarrow a^2 + 5a + 4 = 0$

$\Rightarrow (a+4)(a+1) = 0 \Rightarrow a + 4 = 0$ or $a + 1 = 0$

$\Rightarrow a = -4$ or $a = -1$

Let the fourth term be x

$$\therefore \frac{a}{2a+2} = \frac{3a+3}{x}$$

$$\Rightarrow x = \frac{(3a+3)(2a+2)}{a}$$

When $a = -4 \Rightarrow x = -13.5$

And $a = -1 \Rightarrow x = 0$

So, the fourth term is -13.5

Hence, option *(a)* is correct.

9. *(b)* The given number $\sqrt[3]{2+\sqrt{5}} + \sqrt[3]{2-\sqrt{5}}$ can be written as:

$$=\left(2+\sqrt{5}\right)^{1/3}+\left(2-\sqrt{5}\right)^{1/3}$$

$$=2^{1/3}\left[1+\frac{1}{2}\sqrt{5}\right]^{1/3}+2^{1/3}\left[1-\frac{1}{2}\sqrt{5}\right]^{1/3}$$

$$=2^{1/3}\left[1+\frac{1}{6}\sqrt{5}+\ldots+1-\frac{1}{6}\sqrt{5}+\ldots\right]$$

Thus the given number is a rational number but not an integer.
Hence, option *(b)* is correct.

10. *(d)* As given, number of rows $= x$ and number of seats in each row $= x$

Total number of seats in the hall $= x^2$
Revised number of rows $= 2x$
Revised number in each row $= x-10$

Thus revised number of seats $= 2x(x-10) = 2x^2 - 20x$

According to the question,

$$2x^2 - 20x = 300 + x^2$$

$$\Rightarrow x^2 - 20x - 300 = 0$$

$$\Rightarrow x^2 - 30x + 10x - 300 = 0$$

$$\Rightarrow (x-30)(x+10) = 0$$

$$\Rightarrow x = 30 \qquad (\because x \neq -10)$$

Hence, option *(d)* is correct.

11. *(b)* Given, equation of circle $x^2 + y^2 = a^2$ ....(i)

Equation of chord, $x + y = a$ ....(ii)

$$(\text{i}) \Rightarrow x^2 + (a-x)^2 = a^2$$

$$\Rightarrow x^2 + a^2 + x^2 = 2ax = a^2$$

$$\Rightarrow 2x^2 = 2ax$$

$$\Rightarrow x = 0, a$$

When, $x = 0, y = a$ and when $x = a, y = 0$

$\therefore$ Points of intersection are $(0,a)$ and $(a,0)$

$\therefore$ Equation of circle with chord as diameter is

$$(x-0)(x-a)+(y-a)(y-0)=0$$

$$\Rightarrow x(x-a)+y(y-a)=0$$

$$\Rightarrow x^2 - ax + y^2 - ay = 0$$

$$\Rightarrow x^2 + y^2 - ax - ay = 0$$

Hence, option *(b)* is correct.

12. *(c)* Let binary number $0.1111111 = x$

$$\Rightarrow x = 2^{-1} + 2^{-2} + 2^{-3} + \ldots\infty = \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots\infty$$

This is an infinite G.P. series with first term $= \frac{1}{2}$ and common ratio $= \frac{1}{2}$

$$\Rightarrow x = \frac{\frac{1}{2}}{1-\frac{1}{2}} = \frac{1/2}{1/2} = 1$$

Hence, option *(c)* is correct.

13. *(a)* We have $2\log_8 2 - \frac{\log_3 9}{3} = 2\log_{2^3} 2 - \frac{\log_3 3^2}{3}$

$$= \frac{2}{3}\log_2 2 - \frac{\log_3 3}{3}$$

$$= \frac{2}{3} - \frac{2}{3} = 0 \qquad (\because \log_a a = 1)$$

Hence, option *(a)* is correct.

14. *(a)* Given equation can be written as

$$\frac{x^2}{36}+\frac{y^2}{16}=1$$

This is an ellipse

$$\Rightarrow a^2 = 36,\ b^2 = 16$$

$$\therefore e = \sqrt{1-\frac{b^2}{a^2}} = \sqrt{1-\frac{16}{36}} = \sqrt{\frac{20}{36}} = \frac{\sqrt{5}}{3}$$

Hence, option *(a)* is correct.

15. *(c)* Given, $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Hyperbola passes through $\left(3\sqrt{5},1\right)$

$$\therefore \frac{\left(3\sqrt{5}\right)^2}{a^2}-\frac{1}{b^2}=1$$

$$\Rightarrow \frac{45}{a^2}-\frac{1}{b^2}=1 \qquad ....(i)$$

Now length of latus rectum $=\frac{2b^2}{a}$

$$\Rightarrow \frac{4}{3}=\frac{2b^2}{a}$$

$$\Rightarrow \frac{2}{3}=\frac{b^2}{a} \Rightarrow a=\frac{3b^2}{2} \qquad ....(ii)$$

Putting the value of a from equation (ii) in equation (i)

$$\Rightarrow \frac{45\times 4}{9b^4}-\frac{1}{b^2}=1$$

$$\Rightarrow \frac{20}{b^4}-\frac{1}{b^2}=1$$

$$\Rightarrow 20-b^2=b^4$$

$$\Rightarrow b^4+b^2-20=0$$

$$\Rightarrow b^4+5b^2-4b^2-20=0$$

$$\Rightarrow b^2\left(b^2+5\right)-4\left(b^2+5\right)=0$$

$$\Rightarrow \left(b^2-4\right)\left(b^2+5\right)=0$$

$$\Rightarrow b^2=4,\ b^2=-5$$

$$\therefore b^2=4 \Rightarrow b=2$$

Now length of conjugate axis $=2b=2\left(2\right)=4$

Hence, option *(c)* is correct.

16. *(a)* No. of diagonals in a polygon $={}^nC_2-n$.

$$\Rightarrow 44={}^nC_2-n$$

$$\Rightarrow 44=\frac{n!}{2!\left(n-2\right)!}-n$$

$$\Rightarrow 44=\frac{n\left(n-1\right)}{2}-n$$

$$\Rightarrow 44=\frac{n\left(n-3\right)}{2}$$

$$\Rightarrow n^2-3n-88=0$$

$$\Rightarrow \left(n-11\right)\left(n+8\right)=0$$

$$\Rightarrow n=11 \qquad \left(\because n\neq -8\right)$$

Hence, option *(a)* is correct.

17. *(a)* Let $z=\frac{1+2i}{2-i}-\frac{2-i}{1+2i}$

$$z=\frac{\left(1+2i\right)^2-\left(2-i\right)^2}{\left(2-i\right)\left(1+2i\right)}=\frac{1+4i^2+4i-4-i^2+4i}{2+4i-i-2i^2}$$

$$=\frac{-3-4+8i+1}{4+3i}=\frac{-6+8i}{4+3i}=\frac{\left(-6+8i\right)\left(4-3i\right)}{16+9}$$

$$=\frac{-24+18i+32i-24i^2}{25}$$

$$=\frac{50i}{25}=2i$$

Consider $z^2+z\bar{z}=\left(2i\right)^2+\left(2i\right)\left(-2i\right)=4i^2-4i^2=0$

Hence, option *(a)* is correct.

18. *(c)* Let on the set of real numbers, R is a relation defined by xRy if and only if $3x+4y=5$

Consider, $3x+4y=5$

(I) Put $x=0$ and $y=1$, we get

$$\text{LHS}=3\left(0\right)+4\left(1\right)=4\neq 5\left(=\text{RHS}\right)$$

Hence, 0 is not related to 1.

(, we get

(II) Now, put $x=1$ and $y=\frac{1}{2}$, we get

$$\text{LHS}=3\left(1\right)+4\times\frac{1}{2}=5\left(=\text{RHS}\right)$$

Hence, 1 is related to $\frac{1}{2}$

(III) Similarly, $\frac{2}{3}$ is related to $\frac{3}{4}$.

Hence, both statements II and III are correct.
Hence, option *(c)* is correct.

19. *(a)* As given, roots of the equation $x^2+kx+1=0$ are α and β

$\therefore \alpha+\beta=-k$ and $\alpha\beta=1$

Given expression

$$(\alpha+\beta)\left(\alpha^{-1}+\beta^{-1}\right)=(\alpha+\beta)\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)$$

$$=(\alpha+\beta)\left(\frac{\alpha+\beta}{\alpha\beta}\right)=\frac{(\alpha+\beta)^2}{\alpha\beta}=\frac{(-k)^2}{1}=k^2$$

Hence, option *(a)* is correct.

20. *(d)* Foci of an ellipse are $(4,0)$ and $(-4,0)$. (given)

$\therefore 2ae=8 \Rightarrow ae=4$

and semi minor axis is 3, $\therefore b=3$

We know that, $e=\sqrt{1-\frac{b^2}{a^2}}$

$$\Rightarrow\left(\frac{4}{a}\right)^2=\left(1-\frac{9}{a^2}\right) \qquad \left(\because e=\frac{4}{a}, b=3\right)$$

$$\Rightarrow\frac{16}{a^2}=\frac{a^2-9}{a^2}$$

$$\Rightarrow 16=a^2-9 \Rightarrow a^2=25 \Rightarrow a=5$$

Now, standard equation of an ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Thus, the equation of an ellipse is

$$\frac{x^2}{25}+\frac{y^2}{9}=1$$

which is satisfied by $(5,0)$. Hence the ellipse passes through $(5,0)$.

Hence, option *(d)* is correct.

21. *(a)* Let, $\tan\theta=\sqrt{m}$, where m is a non-square natural number.

$\Rightarrow \sin\theta=\sqrt{m}\cos\theta$

Consider, $\sec 2\theta=\frac{1}{\cos 2\theta}=\frac{1}{\cos^2\theta-\sin^2\theta}$

$$=\frac{1}{\cos^2\theta-m\cos^2\theta}=\frac{1}{\cos^2\theta(1-m)}$$

$$=\frac{\sec^2\theta}{(1-m)}=\frac{1+\tan^2\theta}{(1-m)}=\frac{1+m}{1-m}$$

$$=\frac{(1+m)(1-m)}{(1-m)(1-m)}=\frac{\left(1-m^2\right)}{(1-m)^2}$$

Numerator will always be negative and denominator will always be positive.

Hence, $\sec 2\theta=\frac{1-m^2}{(1-m)^2}$ is a negative number.

Hence, option *(a)* is correct.

22. *(d)* Since, α and β be the roots of

$$2x^2-2\left(1+n^2\right)x+\left(1+n^2+n^4\right)=0$$

$$\therefore \alpha+\beta=-\left[\frac{-2\left(1+n^2\right)}{2}\right]=\left(n^2+1\right)$$

and $\alpha\beta=\frac{1+n^2+n^4}{2}$

Now, consider $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$$=\left(n^2+1\right)^2-\left(1+n^2+n^4\right)$$

$$=n^4+1+2n^2-1-n^2-n^4$$

$$=n^2$$

Hence, option *(d)* is correct.

23. *(c)* We have $C(20,n+2)=C(20,n-2)$

$$\Rightarrow \frac{20!}{(n+2)!(20-n-2)!} = \frac{20!}{(n-2)!(20-n+2)!}$$

$$\Rightarrow \frac{(22-n)!}{(18-n)!} = \frac{(n+2)!}{(n-2)!}$$

$$\Rightarrow (22-n)(21-n)(20-n)(19-n)$$
$$= (n+2)(n+1)\cdot n\cdot(n-1)$$

For $n=10$

$$\Rightarrow (22-10)(21-10)(20-10)(19-10)$$
$$= (10+2)(10+1)\cdot 10\cdot(10-1)$$
$$\Rightarrow 12\cdot 11\cdot 10\cdot 9 = 12\cdot 11\cdot 10\cdot 9$$

Thus, $n=10$

Hence, option *(c)* is correct.

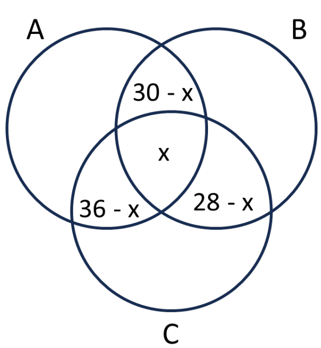
24. *(b)* Let the people who read all three papers A, B, and C be x%.

So, people who read only A and B but not C = $(30-x)\%$

So, people who read only B and C but not A = $(28-x)\%$

So, people who read only A and C but not B = $(36-x)\%$

The Venn diagram is shown below:



Remaining numbers in circles can be filled as shown below:

People who read only A =

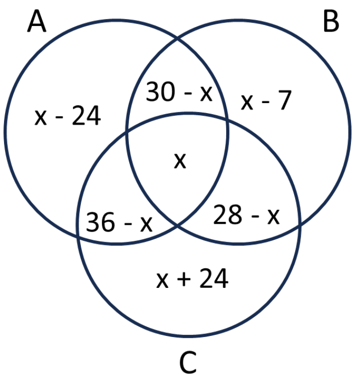
$$42-(30-x+x+36-x) = x-24$$

People who read only B =

$$51-(30-x+x+28-x) = x-7$$

People who read only C =

$$68-(36-x+x+28-x) = x+4$$



Let x % people read all the three newspaper. Since 8% people do not read any newspapers.

$$(x-24)+(x-7)+(x+4)+(30-x)+(28-x)$$
$$+(36-x)+x = 92$$
$$\Rightarrow x+98-31 = 92$$
$$\Rightarrow \quad x+67 = 92$$
$$\Rightarrow \quad x = 92-67$$
$$\Rightarrow \quad x = 25$$

25. *(c)* As we know

$$|x| = \begin{cases} x & \text{if } x>0 \\ -x & \text{if } x<0 \end{cases}$$

$$\therefore\ f(x) = \frac{|x|}{x} = \begin{cases} \frac{x}{x} & \text{if } x>0 \\ \frac{-x}{x} & \text{if } x<0 \end{cases}$$

$$= \begin{cases} 1 & \text{if } x>0 \\ -1 & \text{if } x<0 \end{cases}$$

Hence, Range $= \{-1, 1\}$

Hence, option *(c)* is correct.

26. *(a)* Let x^2, y^2, z^2 are in AP

$$\Rightarrow y^2 - x^2 = z^2 - y^2$$
$$\Rightarrow 2y^2 = x^2 + z^2$$

From the given option, we have:

(a) Suppose $y+z, z+x$ and $x+y$ are in AP

$\therefore\ (z+x)-(y+z)=(x+y)-(z+x)$

$\Rightarrow 2(z+x)=(y+z)+(x+y)$

$\Rightarrow 2z+2x=2y+z+x \Rightarrow z+x=2y$

$\Rightarrow x, y$ and z are in AP, which is true.

(b) Let $y+z, z+x, x+y$ are in HP.

$\therefore\ z+x=\dfrac{2(y+z)(x+y)}{y+z+x+y}$

$\Rightarrow z+x=\dfrac{2(y+z)(x+y)}{2y+z+x}$

$\Rightarrow 2yz+z^2+zx+2xy+xz+x^2=2yx+2y^2+2zx+2yz$

$\Rightarrow z^2+x^2=2y^2$

$\Rightarrow x^2, y^2$ and z^2 are in AP, which is true.

Thus, $y+z, z+x$ and $x+y$ are in AP.

Hence, option *(a)* is correct.

27. *(c)* We have $n(T)=50$

$n(D)=30$

$n(H)=40$

$n(T)=n(D)+n(H)-n(D\cap H)$

$\therefore\ 50=30+40-n(D\cap H)$

$n(D\cap H)=70-50=20$

Number of people having diabetes and high blood pressure $=20$

Hence, option *(c)* is correct.

28. *(a)* We have $z=\left(\dfrac{\sqrt{3}}{2}+\dfrac{i}{2}\right)^{107}+\left(\dfrac{\sqrt{3}}{2}-\dfrac{i}{2}\right)^{107}$

$\because \cos\dfrac{\pi}{6}=\dfrac{\sqrt{3}}{2}$ and $\sin\dfrac{\pi}{6}=\dfrac{1}{2}$

$\Rightarrow z=\left[\cos\dfrac{\pi}{6}+i\sin\dfrac{\pi}{6}\right]^{107}+\left[\cos\dfrac{\pi}{6}-i\sin\dfrac{\pi}{6}\right]^{107}$

Also, $(\cos\theta+i\sin\theta)^n=\cos n\theta+i\sin n\theta$

$\Rightarrow z=\cos\dfrac{107\pi}{6}+i\sin\dfrac{107\pi}{6}+\cos\dfrac{107\pi}{6}-i\sin\dfrac{107\pi}{6}$

$\Rightarrow \text{Im}(z)=0$

Hence, option *(a)* is correct.

29. *(b)* Foci of an ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ are given as $(ae,0)$ and $(-ae,0)$.

Since, two foci are at the end of the diameter.

$\therefore$ Equation of circle, is

$(x-ae)(x+ae)+(y-0)(y-0)=0$

$\Rightarrow x^2-a^2e^2+y^2=0$

$\Rightarrow x^2+y^2-a^2\left(1-\dfrac{b^2}{a^2}\right)=0 \qquad \left(\because e=\sqrt{1-\dfrac{b^2}{a^2}}\right)$

Hence, option *(b)* is correct.

30. *(c)* Given that $\tan\theta=m$ and $\tan 2\theta=n$

We know from fundamentals that

$\Rightarrow \tan 3\theta=\dfrac{\tan\theta+\tan 2\theta}{1-\tan\theta\tan 2\theta}$

Since, $\tan 3\theta=\tan\theta+\tan 2\theta ...(\text{as given})$

$\Rightarrow \tan\theta+\tan 2\theta=\dfrac{\tan\theta+\tan 2\theta}{1-\tan\theta\tan 2\theta}$

$\Rightarrow (\tan\theta+\tan 2\theta)(1-\tan\theta\tan 2\theta)-(\tan\theta+\tan 2\theta)=0$

$\Rightarrow (\tan\theta+\tan 2\theta)\{1-\tan\theta\tan 2\theta-1\}=0$

$\Rightarrow (\tan\theta+\tan 2\theta)(\tan\theta\tan 2\theta)=0$

$\Rightarrow (m+n)mn=0$

$\Rightarrow m+n=0 \qquad [\text{since}, m\neq 0 \text{ and } n\neq 0]$

Hence, option *(c)* is correct.

31. *(d)* As given, $\sin^{-1}x-\cos^{-1}x=\dfrac{\pi}{6}$ ....(i)

And we know that $\sin^{-1}x+\cos^{-1}x=\dfrac{\pi}{2}$ ....(ii)

On adding Eqs. (i) and (ii), we get

$$2\sin^{-1}x=\frac{\pi}{6}+\frac{\pi}{2}=\frac{2\pi}{3}$$

$$\Rightarrow \sin^{-1}x=\frac{\pi}{3}$$

$$\Rightarrow x=\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$$

Hence, option *(d)* is correct.

32. *(c)* As given, in ΔABC, $AC=b=\sqrt{3}\,\text{cm}$, $AB=c=1\,\text{cm}$ and $\angle A=30°$.

From cosine formulae

$$\cos A=\frac{b^2+c^2-a^2}{2bc}=\frac{(\sqrt{3})^2+(1)^2-a^2}{2\sqrt{3}\cdot 1}$$

$$\Rightarrow \frac{\sqrt{3}}{2}=\frac{3+1-a^2}{2\sqrt{3}} \Rightarrow 3=4-a^2$$

$$\Rightarrow a^2=4-3=1 \Rightarrow a=1\,\text{cm}$$

Hence, option *(c)* is correct.

33. *(d)* Given $\sin^{-1}x+\sin^{-1}y=\frac{\pi}{2}$ ....(i)

and $\cos^{-1}x-\cos^{-1}y=0$

$$\Rightarrow \left(\frac{\pi}{2}-\sin^{-1}x\right)-\left(\frac{\pi}{2}-\sin^{-1}x\right)=0$$

$$\Rightarrow \sin^{-1}y-\sin^{-1}x=0$$

$$\Rightarrow \sin^{-1}y=\sin^{-1}x \quad ....(ii)$$

From Eqs. (i) and (ii), we get

$$2\sin^{-1}x=\frac{\pi}{2} \Rightarrow \sin^{-1}x=\frac{\pi}{4} \Rightarrow x=\frac{1}{\sqrt{2}}$$

From equation (ii), we get

$$y=\frac{1}{\sqrt{2}}$$

Hence, option *(d)* is correct.

34. *(a)* Let $f(x)=\frac{(x-1)^2}{|x-1|}=\begin{cases}(x-1), & x\ge 1\\ -(x-1), & x<1\end{cases}$

Now, $\text{LHL}=\lim_{h\to 0} f(1-h)$

$$=\lim_{h\to 0}\left[-(1-h-1)\right]=\lim_{h\to 0} h=0$$

and $\text{RHL}=\lim_{h\to 0} f(1+h)$

$$=\lim_{h\to 0}(1+h-1)=\lim_{h\to 0} h=0$$

$\therefore$ LHL = RHL

$\therefore \lim_{x\to 0} f(x)=\text{LHL}=\text{RHL}$

$$\Rightarrow \lim_{x\to 1}\frac{(x-1)^2}{|x-1|}=0$$

Hence, option *(a)* is correct.

35. *(c)* Consider $\tan 15°+\cot 15°=\frac{\sin 15°}{\cos 15°}+\frac{\cos 15°}{\sin 15°}$

$$=\frac{\sin^2 15°+\cos^2 15°}{\cos 15°\sin 15°}$$

$$=\frac{2\times 1}{2\cos 15°\sin 15°} \quad \left[\because \sin^2\theta+\cos^2\theta=1\right]$$

$$=\frac{2}{\sin 30°}$$

$$=4 \quad \left[\because \sin 30°=\frac{1}{2}\right]$$

Hence, option *(c)* is correct.

36. *(a)* We have $f(x)=[x]\sin(\pi x)$ at $x=k$

Left hand derivative, $\lim_{h\to 0}\frac{f(k)-f(k-h)}{h}$

$$=\lim_{h\to 0}\frac{[k]\sin(k\pi)\text{-}[k-h]\sin(k-h)\pi}{h}$$

$$=\lim_{h\to 0}\frac{-(k-1)\sin(k-h)\pi}{h}$$

$\sin k\pi=0$ and $\sin(k\pi-\theta)=(-1)^{k-1}\sin\theta$

$$=\lim_{h\to 0}\frac{-(k-1)-(1)^{k-1}\sin\pi}{h\pi}\times\pi$$

$$=\lim_{h\to 0}\frac{-(k-1)(1)^{k-1}\sin\pi}{h\pi}\times\pi$$

$$=\pi(k-1)(-1)^k$$

Hence, option *(a)* is correct.

37. *(c)* Consider $\sum_{n=2}^{11}\left(i^n+i^{n+1}\right)$

We know, $i+i^2+i^3+i^4=i-1-i+1=0$

Also, $i^3+i^4+i^5+i^6=0$

The sum of 4 consecutive powers of i is always 0.

$$\therefore \sum_{n=2}^{11}\left(i^n+i^{n+1}\right)=i^2+i^3+i^3+i^4+i^4+i^5+i^5+i^6+i^6$$
$$+i^7+i^7+i^8+i^8+i^9+i^9+i^{10}+i^{10}+i^{11}$$
$$+i^{11}+i^{12}$$
$$=\left(i^2+i^3+i^4+i^5+i^6+i^7+i^8+i^9+i^{10}+i^{11}\right)+$$
$$\left(+i^3+i^4+i^5+i^6+i^7+i^8+i^9+i^{10}+i^{11}+i^{12}\right)$$
$$=\left(i^2+i^3+0\right)+\left(i^3+i^4+0\right)$$
$$=i^2+i^3+i^3+i^4$$
$$=i^2+2i^3+i^4$$
$$=-1+2(-1)+1$$
$$=-2i$$

Hence, option *(c)* is correct.

38. *(d)* We have $a=n(n!)=(n+1)n!-n!$

$$=(n+1)!-n!$$

Now,

$a_1=2!-1!$

$a_2=3!-2!$

$a_3=4!-3!$

$a_{10}=11!-...10!$

Adding all above equations, we get

$a_1+a_2+a_3+...+a_{10}=11!-1!$

Hence, option *(d)* is correct.

39. *(a)* Given, $f(x_1)-f(x_2)=f\left(\frac{x_1-x_2}{1-x_1x_2}\right)$

Then, $f(x)=\log\frac{(1-x)}{(1+x)}$

$f(x_1)=\log\frac{(1-x_1)}{(1+x_1)}$ and $f(x_2)=\log\frac{(1-x_2)}{(1+x_2)}$

$$f(x_1)-f(x_2)=\log\frac{(1-x_1)}{(1+x_1)}-\log\frac{(1-x_2)}{(1+x_2)}$$
$$=\log\frac{(1-x_1)}{(1+x_1)}\times\frac{(1+x_2)}{(1-x_2)}$$
$$=\log\frac{(1-x_1+x_2-x_1x_2)}{(1+x_1-x_2-x_1x_2)}$$
$$=\log\frac{(1-x_1x_2)-(x_1-x_2)}{(1-x_1x_2)+(x_1-x_2)}$$
$$=\log\frac{1-\left(\frac{x_1-x_2}{1-x_1x_2}\right)}{1+\left(\frac{x_1-x_2}{1-x_1x_2}\right)}$$

$$\therefore f(x)=\ln\left(\frac{1-x}{1+x}\right)$$

Hence, option *(a)* is correct.

40. *(a)* Given equation is $2(y+2)^2-5(y+2)=12$

Let $y+2=a$

So, quadratic equation can be rewritten as

$2a^2-5a-12=0$

$\Rightarrow 2a^2-8a+3a-12=0$

$\Rightarrow 2a(a-4)+3(a-4)=0$

$\Rightarrow (2a+3)(a-4)=0$

$\Rightarrow 2a+3=0$ or $a-4=0$

$\Rightarrow a=-\frac{3}{2}$ or $a=4$

$\therefore\ y+2=-\dfrac{3}{2}$ or $y+2=4$

$\Rightarrow y=-2$ or $y=2$

$\Rightarrow y=-\dfrac{7}{2}$ or 2

Hence, option *(a)* is correct.

41. *(c)* Here, $R=\{(1,3),(1,5),(2,3),(2,5),(3,5),(4,5)\}$

$\Rightarrow R^{-1}=\{(3,1),(5,1),(3,2),(5,2),(5,3),(5,4)\}$

$\Rightarrow R\circ R^{-1}=\{(3,3),(3,5),(5,3),(5,5)\}$

Hence, option *(c)* is correct.

42. *(b)* Given, $(\sin x+\operatorname{cosec} x)^2+(\cos x+\sec x)^2$

$=k+\tan^2 x+\cot^2 x$

$\Rightarrow \sin^2 x+\operatorname{cosec}^2 x+2\sin x\operatorname{cosec} x+\cos^2 x+\sec^2 x$

$+2\sec x\cos x=k+\tan^2 x+\cot^2 x$

$\Rightarrow \sin^2 x+\operatorname{cosec}^2 x+2+\cos^2 x+\sec^2 x+2$

$k+\tan^2 x+\cot^2 x$ $\quad[\because \sin x\operatorname{cosec} x=1 \text{ and } \sec x\cos x=$

$\Rightarrow 1+\operatorname{cosec}^2 x-\cot^2 x+\sec^2 x-\tan^2 x+4=k$

$\Rightarrow 1+1+1+4=k$

$\Rightarrow k=7$

Hence, option *(b)* is correct.

43. *(d)* We have given that

$$\sin^{-1}\left(\frac{2a}{1-a^2}\right)-\cos^{-1}\left(\frac{1-b^2}{1+b^2}\right)=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

$\therefore\ 2\tan^{-1}a-2\tan^{-1}b=2\tan^{-1}x$

$\Rightarrow \tan^{-1}a-\tan^{-1}b=\tan^{-1}x$

$\Rightarrow \tan^{-1}\left(\dfrac{a-b}{1+ab}\right)=\tan^{-1}x$

$\Rightarrow x=\dfrac{a-b}{1+ab}$

Hence, option *(d)* is correct.

44. *(b)* We have $f(x)=\dfrac{4x+x^4}{1+4x^3}$ and $g(x)=\ln\left(\dfrac{1+x}{1-x}\right)$

$$g\left(\frac{e-1}{e+1}\right)=\ln\left(\frac{1+\left(\frac{e-1}{e+1}\right)}{1-\left(\frac{e-1}{e+1}\right)}\right)=\ln\left(\frac{e+1+e-1}{e+1-e+1}\right)$$

$$=\ln\left(\frac{2l}{2}\right)=\ln e=1$$

$$\therefore\ f\circ g\left(\frac{e-1}{e+1}\right)=f(1)=\frac{4(1)+(1)^4}{1+4(1)^3}=\frac{4+1}{1+4}=\frac{5}{5}=1$$

Hence, option *(b)* is correct.

45. *(d)* Consider $\cos\left(\dfrac{\pi}{9}\right)+\cos\left(\dfrac{\pi}{3}\right)+\cos\left(\dfrac{5\pi}{9}\right)+\cos\left(\dfrac{7\pi}{9}\right)$

$=\cos\left(\dfrac{\pi}{9}\right)+\dfrac{1}{2}+\cos\left(\dfrac{5\pi}{9}\right)+\cos\left(\dfrac{7\pi}{9}\right)\quad\left[\because \cos\dfrac{\pi}{3}=\dfrac{1}{2}\right]$

$=\dfrac{1}{2}+\left[\cos\dfrac{\pi}{9}+\cos\dfrac{5\pi}{9}\right]+\cos\left(\dfrac{7\pi}{9}\right)$

$=\dfrac{1}{2}+\left[2\cos\dfrac{6\pi}{18}\cos\dfrac{4\pi}{18}\right]+\cos\left(\dfrac{7\pi}{9}\right)$

$\left[\because \cos A+\cos B=2\cos\left(\dfrac{A+B}{2}\right)\cos\left(\dfrac{A-B}{2}\right)\right]$

$=\dfrac{1}{2}+\left[2\cos\dfrac{\pi}{3}\cos\dfrac{2\pi}{9}\right]+\cos\left(\dfrac{7\pi}{9}\right)$

$=\dfrac{1}{2}+\left[2\cdot\dfrac{1}{2}\cos\dfrac{2\pi}{9}\right]+\cos\left(\dfrac{7\pi}{9}\right)$

$=\dfrac{1}{2}+\cos\dfrac{2\pi}{9}+\cos\left(\dfrac{7\pi}{9}\right)$

$=\dfrac{1}{2}+2\cos\left(\dfrac{9\pi}{18}\right)\cos\left(\dfrac{5\pi}{18}\right)$

$=\dfrac{1}{2}+2\cos\left(\dfrac{\pi}{2}\right)\cos\left(\dfrac{5\pi}{18}\right)=\dfrac{1}{2}\qquad\left[\because \cos\dfrac{\pi}{2}=0\right]$

Hence, option *(d)* is correct.

46. *(a)*

3	.	.
1	8	4

3 is fixed hundred place, so no. of choice $=1$

Number of choice for unit place $\{1,5,7,9\}=4$

Number of choice for tens place $=8$

$\therefore$ Number of odd number between 300 and 400

$=1\times4\times8=32$

Hence, option *(a)* is correct.

47. *(d)* The equation of given lines are

$4x+3y=12$ ....(i)

$3x+4y=12$ ....(ii)

On simplifying (i) and (ii), we get

$x=\frac{12}{7}$ and $y=\frac{12}{7}$

$\therefore$ Point of intersection of given lines $\left(\frac{12}{7},\frac{12}{7}\right)$

Hence, the equation of line passing through $(0,0)$ and $\left(\frac{12}{7},\frac{12}{7}\right)$ is

$$\frac{y-0}{x-0}=\frac{\frac{12}{7}-0}{\frac{12}{7}-0}\Rightarrow y=x$$

Hence, option (d) is correct.

48. *(c)* The numbers between 200 and 400 which are divisible by 7 are

203, 210, 217, ……., 399

This is an AP with first term $=a=203$ and common difference $=d=7$

Now, let number of terms be n.

Therefore, from the n^{th} term of AP $=a+(n-1)d$, we have

$399=203+(n-1)7$

$\Rightarrow\frac{196}{7}=(n-1)\Rightarrow n=29$

Required sum $=\frac{n}{2}[a+l]$ where $l=$ last term

Thus, required sum $=\frac{29}{2}[203+399]$

$=\frac{29\times602}{2}$

$=8729$

Hence, option *(c)* is correct.

49. *(d)* Given, $\left|x^2-x-6\right|=x+2$

$\therefore x^2-x-6=x+2$ and $x^2-x-6=-(x+2)$

$\Rightarrow x^2-2x-8=0$ and $x^2-x-6=-x-2$

$\Rightarrow x^2-4x+2x-8=0$ and $x^2=4$

$\Rightarrow x(x-4)+2(x-4)=0$ and $x=\pm2$

$\Rightarrow (x+2)(x-4)=0$ and $x=\pm2$

$\Rightarrow x=4,-2$ and $x=\pm2$

$\therefore x=-2,2,4$

Hence, option *(d)* is correct.

50. *(b)* Let $y=2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cdots\infty}}}$

$\therefore\ y=2+\frac{1}{y}$

$\Rightarrow y^2=2y+1\Rightarrow y^2-2y-1=0$

$\Rightarrow y=\frac{2\pm\sqrt{(-2)^2-4(-1)}}{2}\Rightarrow y=\frac{2\pm2\sqrt{2}}{2}=1\pm\sqrt{2}$

$\because\ y>2$

$\therefore y=1+\sqrt{2}$

Hence, option *(b)* is correct.

51. *(b)* Since, $\alpha\beta=\alpha^2\beta^2\Rightarrow\alpha\beta(1-\alpha\beta)=0$

$\therefore\ \alpha\beta=0$ or 1

When $\alpha\beta=1$ then $\alpha=\frac{1}{\beta}$

Again, from $\alpha+\beta=\alpha^2+\beta^2$

$\Rightarrow \frac{1}{\beta}+\beta=\frac{1}{\beta^2}+\beta^2 \Rightarrow \beta^2-\beta=\frac{1}{\beta}-\frac{1}{\beta^2}$

$\Rightarrow \beta(\beta-1)=\frac{(\beta-1)}{\beta^2} \Rightarrow (\beta-1)\left(\beta-\frac{1}{\beta^2}\right)=0$

$\Rightarrow (\beta-1)\left(\beta^3-1\right)=0 \Rightarrow (\beta-1)^2\left(\beta^2+\beta+1\right)=0$

$\therefore \beta=1$ and $\beta=\frac{-1\pm\sqrt{3}i}{2}$

Again, when $\beta=1$, then $\alpha=\frac{1}{\beta}=1$,

Roots $(\alpha,\beta)=(1,1)$

When $\beta=\frac{-1+\sqrt{3}i}{2}$, then $\alpha=\frac{-1-\sqrt{3}i}{2}$

Roots $(\alpha,\beta)=\left(\frac{-1\pm\sqrt{3}i}{2},\frac{-1\mp\sqrt{3}i}{2}\right)$

Thus, number of different quadratic equations $=2$
Hence, option *(b)* is correct.

52. *(d)* Let $P(x,y)$ be a point and $A=(a,0), B=(-a,0)$

Now, $PA^2=(x-a)^2+y^2$

$PB^2=(x+a)^2+y^2$

Since, the sum of the distances of the point $P(x,y)$ from the points $A(a,0)$ and $B(-a,0)$ is $2b^2$.

$\therefore\ PA^2+PB^2=2b^2$

$\Rightarrow (x-a)^2+(y)^2+(x+a)^2+(y)^2=2b^2$

$\Rightarrow x^2+a^2-2ax+y^2+x^2+a^2+2ax+y^2=2b^2$

$\Rightarrow x^2+a^2+y^2=b^2$

$\Rightarrow x^2+a^2=b^2-y^2$

Hence, option *(d)* is correct.

53. *(b)* Let $\sin^{-1}x+\cot^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{2}$

As we know $\cot^{-1}x=\sin^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$

$\therefore\ \sin^{-1}x+\cot^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{2}$

$\Rightarrow \sin^{-1}x+\sin^{-1}\left(\frac{1}{\sqrt{1+\frac{1}{4}}}\right)=\frac{\pi}{2}$

$\Rightarrow \sin^{-1}x+\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)=\frac{\pi}{2}$ ....(i)

Now, $\sin^{-1}x=\cos^{-1}\sqrt{1-x^2}$

$\therefore \sin^{-1}\left(\frac{2}{\sqrt{5}}\right)=\cos^{-1}\left(\sqrt{1-\frac{4}{5}}\right)=\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$

$\therefore$ From equation (i), we have

$\sin^{-1}x+\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)=\frac{\pi}{2}$

Since, $\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$

$\therefore x=\frac{1}{\sqrt{5}}$

Hence, option *(d)* is correct.

54. *(c)* We know geometric mean of 3 numbers x_1,x_2,x_3 is $=\sqrt[3]{x_1\cdot x_2\cdot x_3}$

Given, if observations are $x_1,x_2,12$, GM is 6.

$\Rightarrow \sqrt[3]{x_1\cdot x_2\cdot 12}=6$

$\Rightarrow x_1\times x_2\times 12=6^3=216$

$\Rightarrow x_1\times x_2=\frac{216}{12}=18$ ....(i)

Also, given that actual number is 8.

$\therefore$ Actual GM $=\sqrt[3]{x_1\cdot x_2\cdot 8}=\sqrt[3]{18\times 8}$ [from (i)]

$=\sqrt[3]{18\times 2\times 2\times 2}$

$=2\sqrt[3]{18}$

Hence, option *(c)* is correct.

55. *(c)* Number of proper subset of any set of n elements $=2^n-1$

Here given set $=\{1,2,3,4\}$

Number of proper subset $= 2^4 - 1 = 16 - 1 = 15$

Proper subset $= \{(1),(2),(3),(4),(1,2),(1,3),(1,4),$
$(2,3),(2,4),(3,4),(1,2,3),(1,2,4),$
$(1,3,4),(2,3,4),\phi\}$

Now A is a superset of B, if B is proper set of A, but B is not proper set of A.

i.e. $B \le A$ but $A \not\subset B$. Then $A \ge B$

So, superset of $\{3\}$ are $\{(3),(1,3),(2,3),(3,4),(1,2,3),$
$(1,3,4),(2,3,4)\}$

Hence, number of supersets of $\{3\} = 7$

Hence, option *(c)* is correct.

56. *(b)* Consider $\sqrt{3}\,\text{cosec}\,20° - \sec 20°$

$$= \frac{\sqrt{3}}{\sin 20°} - \frac{1}{\cos 20°} = \frac{\sqrt{3}\cos 20° - \sin 20°}{\sin 20° \cos 20°}$$

Multiply and divide by 4

$$= \frac{4}{2\sin 20° \cos 20°}\left(\frac{\sqrt{3}}{2}\cos 20° - \frac{1}{2}\sin 20°\right)$$

$$= \frac{4}{\sin 40°}\left(\frac{\sqrt{3}}{2}\cos 20° - \frac{1}{2}\sin 20°\right)$$

$$[\because \sin 2\theta = 2\sin\theta\cos\theta]$$

$$= \frac{4}{\sin 40°}(\sin 60° \cos 20° - \cos 60° \sin 20°)$$

$$= \frac{4}{\sin 40°}\sin(60° - 20°)$$

$$= 4 \qquad [\because \sin(A-B) = \sin A\cos B - \cos A \sin B]$$

Hence, option *(b)* is correct.

57. *(d)* The given lines are:

$$3x + 4y = 9 \Rightarrow y = \frac{9}{4} - \frac{3}{4}x \qquad \text{....(i)}$$

$$9x + 12y + 28 = 0 \Rightarrow y = -\frac{7}{3} - \frac{3}{4}x \qquad \text{....(ii)}$$

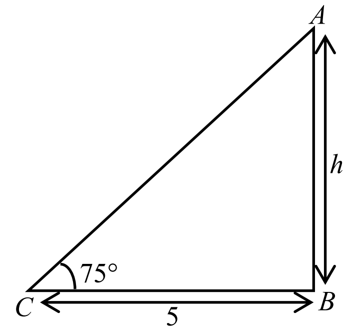
We have

$$m = -\frac{3}{4},\ C_1 = \frac{9}{4},\ C_2 = -\frac{7}{3}$$

Now, distance $= \dfrac{|C_1 - C_2|}{\sqrt{1+m^2}} = \dfrac{\left|\frac{9}{4} - \left(-\frac{7}{3}\right)\right|}{\sqrt{1 + \frac{9}{16}}} = \dfrac{11}{3}$ units

Hence, option *(d)* is correct.

58. *(c)* Let h be the height of the flag post where BC is the base.



In ΔABC,

$$\tan 75° = \frac{AB}{BC} = \frac{h}{5}$$

$$\Rightarrow \tan(45° + 30°) = \frac{h}{5}$$

$$\Rightarrow \frac{\tan 45° + \tan 30°}{1 - \tan 45° \tan 30°} = \frac{h}{5} \quad \left(\because \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}\right)$$

$$\Rightarrow \frac{\sqrt{3}+1}{\sqrt{3}-1} = \frac{h}{5} \Rightarrow h = \frac{(\sqrt{3}+1)^2}{(\sqrt{3})^2 - (1)^2} \times 5$$

$$\Rightarrow h = 5\left(\frac{3 + 1 + 2\sqrt{3}}{3-1}\right) = 5(2 + \sqrt{3})$$

$$\Rightarrow h = 5 \times 3.732$$

$$\Rightarrow h \approx 19\,\text{m}\,(\text{approx})$$

Hence, option *(c)* is correct.

59. *(a)* We have $f(x) = 2x - x^2$,

$$f(x+2) + f(x-2)$$

$$= 2(x+2) - (x+2)^2 + 2(x-2) - (x-2)^2$$

$= 4 + 4x - x^2 - 4 - 4x - x^2 - 4 + 4x - 4$

$= 8x - 2x^2 - 8$

When $x = 0$ then, $f(x+2) + f(x-2) = -8$

Hence, option *(a)* is correct.

60. *(c)* Let $\alpha + \beta = \frac{\pi}{4}$

$\Rightarrow \tan(\alpha + \beta) = \tan\frac{\pi}{4}$

$\Rightarrow \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = 1$

$\Rightarrow \tan\alpha + \tan\beta = 1 - \tan\alpha\tan\beta$

By adding 1 on both sides, we get

$\Rightarrow 1 + \tan\alpha + \tan\beta + \tan\alpha\tan\beta = 2$

$\Rightarrow (1 + \tan\alpha)(1 + \tan\beta) = 2$

Hence, option *(c)* is correct.

61. *(a)* Given that $R = \{(x, y) : x^2 - 5xy + 4y^2 = 0, x, y \in N\}$

$x^2 - 5xy + 4y^2 = 0$

$\Rightarrow x^2 - 4xy - xy + 4y^2 = 0$

$\Rightarrow x(x - 4y) - y(x - 4y) = 0$

$\Rightarrow (x - y)(x - 4y) = 0$

For reflexive

$(x - x)(x - 4y) = 0$

$\therefore (x, x) \in R$

So, it is reflexive.

For symmetric

Let $(x, y) \in R$

$x^2 - 5xy + 4y^2 = 0$ but $y^2 - 5xy + 4x^2$ may be equal to zero.

So, it is not symmetric.

For transitive

Let $(x, y) \in R$

$x^2 - 5xy + 4y^2 = 0$ ....(i)

and $(y, z) \in R$

$y^2 - 5y^2 + 4z^2 = 0$ ....(ii)

From (i) and (ii), we get

$x^2 - 5xy + 4y^2 = y^2 - 5yz + 4z^2$

$x^2 + 3y^2 - 4z^2 - 5xy + 5yz = 0$

$(x^2 - 5xz + 4z^2) + (3y^2 - 4z^2 + 5xz - 5xy + 5yz) = 0$

$\therefore 3y^2 - 4z^2 + 5xz - 5xy + 5yz \neq 0$

$\therefore x^2 - 5xz + 4z^2 \neq 0$

$\therefore (x, z) \notin R$

So, it is not transitive

Hence, option *(a)* is correct.

62. *(c)* The given lines are:

$x + 2y - 3 = 0$ ....(i)

$2x - y + 5 = 0$ ....(ii)

Multiply equation (i) by 2 and subtract them

$2(x + 2y - 3) - (2x - y + 5) = 0$

$\Rightarrow 5y - 11 = 0$

$\Rightarrow y = \frac{11}{5}$

From equation (i), we get

$x + 2\left(\frac{11}{5}\right) - 3 = 0 \Rightarrow x + \frac{7}{5} = 0 \Rightarrow x = -\frac{7}{5}$

Given, the required line is parallel to $y - x + 10 = 0$

$\Rightarrow y = x - 10$

$\Rightarrow y = (1)x + (-10)$

$\therefore$ Slope$(m) = 1$

$\therefore$ Required line with slope 1 and passing through $\left(-\frac{7}{5}, \frac{11}{5}\right)$ is

$y - y_1 = m(x - x_1)$

$\Rightarrow y - \frac{11}{5} = 1\left(x + \frac{7}{5}\right)$

$\Rightarrow 5y - 11 = 5x + 7$

$\Rightarrow 5x - 5y + 18 = 0$

Hence, option *(c)* is correct.

63. *(d)* $\because f(x)=\sqrt{1-(x-1)^2}$

$\therefore 1-(x-1)^2 \geq 0 \Rightarrow 1-x^2+2x-1\geq 0$

$\Rightarrow -x(x-2)\geq 0 \Rightarrow x(x-2)\leq 0$

$\therefore x\in[0,2]$

Hence, option *(d)* is correct.

64. *(a)* Consider $\sin 15°\cos 75°$

$=\sin(45°-30°)\cos(45°+30°)$

$=(\sin 45°\cos 30°-\cos 45°\sin 30°)(\sin 45°\cos 30° + \cos 45°\sin 30°)$

$=\left(\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2}-\frac{1}{\sqrt{2}}\cdot\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}\cdot\frac{1}{2}\right)$

$=\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)$

$=\frac{3-1}{8}=\frac{2}{8}=\frac{1}{4}$

Hence, option *(a)* is correct.

65. *(d)* We have $4f(x)-f\left(\frac{1}{x}\right)=\left(2x+\frac{1}{x}\right)\left(2x-\frac{1}{x}\right)$

$4f(x)-f\left(\frac{1}{x}\right)=4x^2-\frac{1}{x^2}$ ....(i)

Relpace x by $\frac{1}{x}$, we get

$4f\left(\frac{1}{x}\right)-f(x)=\frac{4}{x^2}-x^2$ ....(ii)

From $4\times(\text{i})+(\text{ii})$

$16f(x)-4f\left(\frac{1}{x}\right)=16x^2-\frac{4}{x^2}$

$4f\left(\frac{1}{x}\right)-f(x)=\frac{4}{x^2}-x^2$

$15f(x)=15x^2$

$\therefore f(x)=x^2$

Now, $f(2)=2^2=4$

Hence, option *(d)* is correct.

66. *(b)* Let $f(x)=\frac{x^2-9}{x^2-2x-3}$, $x\neq 3$

Given, the function is continuous at $x=3$

$\therefore f(3)=\lim_{x\to 3}\frac{x^2-9}{x^2-2x-3}=\lim_{x\to 3}\frac{(x+3)(x-3)}{(x-3)(x+1)}$

$=\lim_{x\to 3}\frac{(x+3)}{(x+1)}=\frac{3+3}{3+1}$

$=\frac{6}{4}=1.5$

Hence, option *(b)* is correct.

67. *(a)* Let $x=\sin\theta+\cos\theta$ and $y=\sin\theta\cdot\cos\theta$

Now, $x^4-4x^2y-2x^2+4y^2+4y+1$

$=(\sin\theta+\cos\theta)^4-4(\sin\theta+\cos\theta)^2 y -2(\sin\theta+\cos\theta)^2+4y^2+4y+1$

$=\left(\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta\right)^2 -4\left(\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta\right)y -2\left(\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta\right)+4y^2+4y+1$

$=(1+2y)^2-4(1+2y)y-2(1+2y)+4y^2+4y+1$

$=1+4y^2+4y-4y-8y^2-2-4y+4y^2+4y+1$

$=0$

Hence, option *(a)* is correct.

68. *(c)* Let $f(x)=2x^3-3x^2-12x+1$

$\Rightarrow f'(x)=6x^2-6x-12$

Put $f'(x)=0$ for maxima or minima

$\Rightarrow 6x^2-6x-12=0 \Rightarrow x^2-x-2=0$

$\Rightarrow (x-2)(x+1)=0$

$\Rightarrow x=2,\ x=-1$

Now, $f''(x)=12x-6$

$\Rightarrow f''(2)=24-6=18>0$

Since, $f''(x)$ is +ve at $x=2$

$\therefore f(x)$ is minimum at $x=2$

Thus, minimum value is

$f(2)=2(2)^3-3(2)^2-12(2)+1=-19$

Hence, option *(c)* is correct.

69. *(c)* Let volume $=V=\frac{4}{3}\pi r^3$ ....(i)

And surface area $=S=4\pi r^2$ ....(ii)

Now, $(\text{i})\Rightarrow \frac{dv}{dt}=\frac{4}{3}\times 3\pi r^2\times\frac{dr}{dt}$

$=4\pi r^2\frac{dr}{dt}$ ....(iii)

$(\text{ii})\Rightarrow \frac{ds}{dt}=4\pi\times 2\times r\frac{dr}{dt}=\frac{8\pi r^2}{r}\frac{dr}{dt}$

$\therefore \frac{dv}{dt}=\frac{2}{r}\left[4\pi r^2\frac{dr}{dt}\right]=\frac{2}{r}\frac{dr}{dt}$ (from (iii))

Hence, option *(c)* is correct.

70. *(d)* Given function is $y=\sin^{-1}x+\sin^{-1}\sqrt{1-x^2}$

On differentiating w.r.t. *x*, we get

$\frac{dy}{dx}=\frac{1}{\sqrt{1-x^2}}+\frac{1}{\sqrt{1-1+x^2}}\cdot\frac{1}{2\sqrt{1-x^2}}(-2x)$

$=\frac{1}{\sqrt{1-x^2}}-\frac{1}{\sqrt{1-x^2}}$

$=0$

Hence, option *(d)* is correct.

71. *(c)* Let $f(\theta)=16\sin\theta-12\sin^2\theta$

$\Rightarrow f'(\theta)=16\cos\theta-24\sin\theta\cos\theta$

$\therefore f'(\theta)=0\Rightarrow\cos\theta(16-24\sin\theta)=0$

$\Rightarrow\cos\theta=0$ or $16-24\sin\theta=0$

$\Rightarrow\theta=\frac{\pi}{2}$ or $\sin\theta=\frac{2}{3}$

$\therefore f\left(\frac{\pi}{2}\right)=16\sin\frac{\pi}{2}-12\sin^2\frac{\pi}{2}=16-12=4$

$f\left(\sin\theta=\frac{2}{3}\right)=16\left(\frac{2}{3}\right)-12\left(\frac{2}{3}\right)^2=\frac{32}{3}-\frac{16}{3}=\frac{16}{3}$

Hence, option *(c)* is correct.

72. *(d)* Consider $\lim_{h\to 0}\frac{\sqrt{2x+3h}-\sqrt{2x}}{2h}$

Rationalise the numerator,

$=\lim_{h\to 0}\left(\frac{\sqrt{2x+3h}-\sqrt{2x}}{2h}\times\frac{\sqrt{2x+3h}+\sqrt{2x}}{\sqrt{2x+3h}+\sqrt{2x}}\right)$

$=\lim_{h\to 0}\frac{2x+3h-2x}{2h\left(\sqrt{2x+3h}+\sqrt{2x}\right)}$

$=\frac{3}{2\left(\sqrt{2x+0}+\sqrt{2x}\right)}=\frac{3}{4\sqrt{2x}}$

Hence, option *(d)* is correct.

73. *(b)* Let $f(x)=\ln\left(\sqrt{x^2+1}-x\right)$

$f(-x)=\ln\left(\sqrt{(-x)^2+1}-(-x)\right)=\ln\left(\sqrt{x^2+1}+x\right)$

$=\ln\left(\frac{\left(\sqrt{x^2+1}+x\right)\left(\sqrt{x^2+1}-x\right)}{\sqrt{x^2+1}-x}\right)$

$=\ln\left(\frac{x^2+1-x^2}{\sqrt{x^2+1}-x}\right)=\ln\left(\frac{1}{\sqrt{x^2+1}-x}\right)$

$=-\ln\left(\sqrt{x^2+1}-x\right)=-f(x)$

So, $f(x)$ is odd function.

Hence, option *(b)* is correct.

74. *(d)* Consider, $x^y = e^{x-y}$

Taking log both sides, we get

$\Rightarrow y\log x = x - y$

$\Rightarrow y = \dfrac{x}{1+\log x}$

On differentiating w.r.t. x, we get

$$\Rightarrow \frac{dy}{dx} = \frac{(1+\log x) - x\left(0+\frac{1}{x}\right)}{(1+\log x)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(1+\log x)-1}{(1+\log x)^2} = \frac{\log x}{(1+\log x)^2}$$

Hence, option *(d)* is correct.

75. *(b)* We have $\int\left(\sqrt{x}+x\right)^{-1}dx = \int\frac{1}{\left(x+\sqrt{x}\right)}dx$

$$= \int\frac{1}{\left(\sqrt{x}\cdot\sqrt{x}+\sqrt{x}\right)}dx$$

Let $\sqrt{x}+1 = t$

Then, $\dfrac{1}{2\sqrt{x}}dx = dt \Rightarrow \dfrac{1}{\sqrt{x}}dx = dt$

$$\therefore \int\frac{1}{\sqrt{x}\left(\sqrt{x}+x\right)}dx = \int\frac{2dt}{t} = 2\log t + C$$

$$= 2\log\left(1+\sqrt{x}\right) + C$$

Hence, option *(b)* is correct.

76. *(c)* Consider $\int\dfrac{x^4+1}{x^2+1}dx = \int\dfrac{x^4+2-1}{x^2+1}dx$

$$= \int\left(\frac{x^4-1}{x^2+1}+\frac{2}{x^2+1}\right)dx$$

$$= \int\left[\frac{\left(x^2-1\right)\left(x^2+1\right)}{x^2+1}+\frac{2}{x^2+1}\right]dx$$

$$= \int\left[x^2-1+\frac{2}{x^2+1}\right]dx$$

$$= \frac{x^3}{2} - x + 2\tan^{-1}x + C$$

Hence, option *(c)* is correct.

77. *(d)* Let $\sqrt{1-x^2}+\sqrt{1-y^2} = a$

On differentiating w.r.t. x, we get

$$\frac{1}{2\sqrt{1-x^2}}(-2x) + \frac{1}{2\sqrt{1-y^2}}(-2y)\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{-x}{\sqrt{1-x^2}} - \frac{y}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{-x}{\sqrt{1-x^2}} = \frac{y}{\sqrt{1-y^2}}\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{x}{y}\sqrt{\frac{1-y^2}{1-x^2}}$$

Hence, option *(d)* is correct.

78. *(a)* Let $I = \int\left(\dfrac{1}{\cos^2 x} - \dfrac{1}{\sin^2 x}\right)dx$

$$\Rightarrow I = \int\left(\sec^2 x - \text{cosec}^2 x\right)dx$$

$$\Rightarrow I = \int\sec^2 x\,dx - \int\text{cosec}^2 x dx$$

$$\Rightarrow I = \tan x + \cot x + C$$

$$\Rightarrow I = \tan x + \frac{1}{\tan x} + C = \frac{\tan^2 x + 1}{\tan x} + C$$

$$\Rightarrow I = \frac{\sec^2 x + 1}{\tan x} + C = \frac{2}{2\sin x\cos x} + C$$

$$\Rightarrow I = \frac{2}{\sin 2x} + \text{C} = 2\,\text{cosec}\,2x + C$$

Hence, option *(a)* is correct.

79. *(c)* Let curve $y=-x^3+3x^2+2x-27$

$$\text{Slope}=\frac{dy}{dx}=-3x^2+6x+2$$

Again, $\frac{d^2y}{dx^2}=-6x+6$

For maximum slope $\frac{d^2y}{dx^2}=0 \Rightarrow 6(-x+1)=0$

$\therefore\ x=1$

Hence, the curve has maximum slope at $x=1$

Hence, option *(c)* is correct.

80. *(d)* Consider, $\int\left(\frac{x^{e-1}+e^{x-1}}{x^e+e^x}\right)dx$

Put $x^e+e^x=t \Rightarrow ex^{e-1}+e^x=\frac{dt}{dx}$

$\Rightarrow \left(ex^{e-1}+e^x\right)dx=dt$

$$\therefore \int\left(\frac{x^{e-1}+e^{x-1}}{x^e+e^x}\right)dx=\frac{1}{e}\int\left(\frac{ex^{e-1}+e^x}{x^e+e^x}\right)dx$$

$$=\frac{1}{e}\int\frac{dt}{t}=\frac{1}{e}\cdot\ln t+c$$

$$=\frac{1}{e}\ln\left(x^e+e^x\right)c$$

Hence, option *(d)* is correct.

81. *(a)* Let $u_1=\log_x 5$ and $u_2=\log_5 x$

$\Rightarrow u_1=\frac{\log_e 5}{\log_e x}$ and $u_2=\frac{\log_e x}{\log_e 5}$

On differentiating w.r.t. *x*, we get

$$\frac{du_1}{dx}=\left[\frac{\log_e x(0)-\left(\frac{1}{x}\right)}{\left(\log_e x\right)^2}\right]\log_e 5=-\frac{\log_e 5}{x\left(\log_e x\right)^2}$$

And $\frac{du_2}{dx}=\frac{1}{x\log_e 5}$

$$\therefore \frac{du_1}{du_2}=\frac{du_1/dx}{du_2/dx}=-\frac{\log_e 5}{x\left(\log_e x\right)^2}\times x\log_e 5$$

$$=-\left(\frac{\log_e 5}{\log_e x}\right)^2=-\left(\log_x 5\right)^2$$

$$=-\left(\log_5 x\right)^{-2}$$

Hence, option *(a)* is correct.

82. *(a)* Let $I=\int_0^{\pi/2}\log(\tan x)dx$ ....(i)

and $\int_0^{\pi/2}\log\left\{\tan\left(\frac{\pi}{2}-x\right)\right\}dx$

[By the property of definite integral which says

$\int_0^a f(x)dx=\int_0^a f(a-x)dx$]

$\Rightarrow I=\int_0^{\pi/2}\log(\cot x)dx$ ....(ii)

By adding Eqs.(i) and (ii), we get

$$2I=\int_0^{\pi/2}\log(\tan x)dx+\int_0^{\pi/2}\log(\cot x)dx$$

$\Rightarrow 2I=\int_0^{\pi/2}\log(\tan x\cot x)dx$ $[\because \log m+\log n=\log mn]$

$$\Rightarrow 2I=\int_0^{\pi/2}\log\left(\tan x\cdot\frac{1}{\tan x}\right)dx=\int_0^{\pi/2}\log(1)dx=0$$

$\Rightarrow I=0$

Hence, option *(a)* is correct.

83. *(d)* Let $\Delta=\begin{vmatrix}1-i & \omega^2 & -\omega\\ \omega^2+i & \omega & -i\\ 1-2i-\omega^2 & \omega^2-\omega & i-\omega\end{vmatrix}$

Applying $R_3\to R_1-R_2-R_3$

$$= \begin{vmatrix} 1-i & \omega^2 & -\omega \\ \omega^2+i & \omega & -i \\ 0 & 0 & 0 \end{vmatrix} = 0$$

($\because$ one row of determinant is zero)

Hence, option *(d)* is correct.

84. *(a)* Let $I = \int \frac{dx}{x\left(x^2+1\right)}$

Put $x^2+1=t \Rightarrow 2xdx = dt$

$$\therefore I = \int \frac{dt}{t(t-1)} = -\frac{1}{2}\int\left[\frac{1}{t}-\frac{1}{t-1}\right]dt$$

$$= -\frac{1}{2}\left[\int \frac{1}{t}dt - \int \frac{1}{t-1}dt\right]$$

$$= -\frac{1}{2}\left[\ln(t) - \ln(t-1)\right] + c$$

$$= \frac{1}{2}\left[\ln(t) - \ln(t-1) - \ln(t)\right] + c$$

$$= \frac{1}{2}\ln\left(\frac{t-1}{t}\right) + c$$

Hence, option *(a)* is correct.

85. *(a)* Let $\lim_{x\to 0} \frac{\sin x \log(1-x)}{x^2}$

This is in $\frac{0}{0}$ form, by using L'Hospital rule, we have:

$$= \lim_{x\to 0} \frac{\sin x \log(1-x)}{x^2}$$

$$= \lim_{x\to 0} \frac{\cos x \log(1-x) + \sin x \cdot \frac{1}{(1-x)}(-1)}{2x}$$

$$= \lim_{x\to 0} \frac{\log(1-x)\cdot(-\sin x) - \frac{\cos x}{(1-x)} - \frac{1}{(1-x)}\cos x - \frac{\sin x}{(1-x)^2}}{2}$$

$$= \lim_{x\to 0} \frac{0-1-1-0}{2} = \frac{-2}{2} = -1$$

Image formed virtual, erect, magnified and behind the mirror.

Hence, option *(a)* is correct.

86. *(c)* Since, the matrix $\begin{bmatrix} 1 & -3 & 2 \\ 2 & -8 & 5 \\ 4 & 2 & \lambda \end{bmatrix}$ is not an invertible matrix.

Therefore, its determinant is zero.

$$\Rightarrow \begin{bmatrix} 1 & -3 & 2 \\ 2 & -8 & 5 \\ 4 & 2 & \lambda \end{bmatrix} = 0$$

$$\Rightarrow 1(-8\lambda-10) + 3(2\lambda-20) + 2(4+32) = 0$$

$$\Rightarrow -8\lambda - 10 + 6\lambda - 60 + 72 = 0$$

$$\Rightarrow -2\lambda + 2 = 0 \Rightarrow \lambda = 1$$

Hence, option *(c)* is correct.

87. *(a)* Consider $\begin{vmatrix} 1 & \omega & 2\omega^2 \\ 2 & 2\omega^2 & 4\omega^3 \\ 3 & 3\omega^3 & 6\omega^4 \end{vmatrix} = \begin{vmatrix} 1 & \omega & 2\omega^2 \\ 2 & 2\omega^2 & 4 \\ 3 & 3 & 6\omega \end{vmatrix}$

$\left(\because \omega^3 = 1 \text{ and } \omega^4 = \omega\right)$

$$= 2\times 3\begin{vmatrix} 1 & \omega & 2\omega^2 \\ 1 & \omega^2 & 2 \\ 1 & 1 & 2\omega \end{vmatrix}$$

$$= 6\left[1\left(2\omega^3-2\right) - \omega(2\omega-2) + 2\omega^2\left(1-\omega^2\right)\right]$$

$$= 6\left[0 - 2\omega^2 - 2\omega + 2\omega^2 - 2\omega\right]$$

$$= 0$$

Hence, option *(a)* is correct.

88. *(c)* Let $I = \int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x + \cos^3 x}dx$ ....(i)

$$= \int_0^{\pi/2} \frac{\sin^3(\pi/2-x)}{\sin^3(\pi/2-x)+\cos^3(\pi/2-x)}dx$$

Bu using the property $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\cos^3 x}{\sin^3 x + \cos^3 x} dx \quad ....(\text{ii})$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x + \cos^3 x} dx + \int_0^{\pi/2} \frac{\cos^3 x}{\sin^3 x + \cos^3 x} dx$$

$$\Rightarrow 2I = \int_0^{\pi/2} \frac{\sin^3 x + \cos^2 x}{\sin^3 x + \cos^3 x} dx = \int_0^{\pi/2} 1 dx$$

$$\Rightarrow 2I = [x]_0^{\pi/2} = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$$

Hence, option *(c)* is correct.

89. *(a)* Let $A = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$

$$\therefore A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}\begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$$

$$= \begin{bmatrix} \alpha^2 + \beta^2 & 2\alpha\beta \\ 2\alpha\beta & \alpha^2 + \beta^2 \end{bmatrix}$$

Now, $A^2 = I$

$$\Rightarrow \begin{bmatrix} \alpha^2 + \beta^2 & 2\alpha\beta \\ 2\alpha\beta & \alpha^2 + \beta^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \alpha^2 + \beta^2 = 1 \text{ or } \alpha\beta = 0$$

$$\Rightarrow \alpha = 0, \beta = 1 \text{ or } \alpha = 1, \beta = 0$$

Hence, option *(a)* is correct.

90. *(c)* Consider $\lim\limits_{n\to\infty} \frac{a^n + b^n}{a^n - b^n}$

$$= \lim_{n\to\infty} \frac{a^n \left[1 + \left(\frac{b}{a}\right)^n\right]}{a^n \left[1 - \left(\frac{b}{a}\right)^n\right]}$$

$$= \lim_{n\to\infty} \frac{\left[1 + \left(\frac{b}{a}\right)^n\right]}{\left[1 - \left(\frac{b}{a}\right)^n\right]} = \frac{1+0}{1+0}$$

$$\therefore \lim_{n\to\infty} \frac{a^n + b^n}{a^n - b^n} = 1$$

Hence, option *(c)* is correct.

91. *(a)* Let $I = \int_{-1}^{1} |x| dx = -\int_{-1}^{0} x dx + \int_0^1 x dx$

Since, $|x| = \begin{cases} x & \text{if } x \ge 0 \\ -x & \text{if } x < 0 \end{cases}$

Therefore, $|x| = -x$ when x lies between -1 and 0, and $|x| = x$ when x lies between 0 and 1.

$$= -\left[\frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2}\right]_0^1 = -\left[-\frac{1}{2}\right] + \left[\frac{1}{2}\right] = 1$$

Hence, option *(a)* is correct.

92. *(d)* Given that, Mean$(xp) = \frac{2}{3}$ and Variance$(npq) = \frac{5}{9}$

$$\therefore \frac{\text{Variance}(npq)}{\text{mean}(np)} = q = \frac{\frac{5}{9}}{\frac{2}{3}} = \frac{5}{6}$$

So, $p = 1 - q = 1 - \frac{5}{6} = \frac{1}{6}$

Here number of trial $n \times \frac{1}{6} = \frac{2}{3} \Rightarrow n = 4$

Random Variable $X = 2$

$$\therefore \text{Probability} = {}^4C_2 (p)^{4-2} q^2 = {}^4C_2 \times \left(\frac{1}{6}\right)^2 \times \left(\frac{5}{6}\right)^2$$

$$= \frac{25}{216}$$

Hence, option *(d)* is correct.

93. *(c)* We have $\left\{\left(\frac{d^4 y}{dx^4}\right)^3\right\}^{\frac{2}{3}} - 7x\left(\frac{d^3 y}{dx^3}\right)^2 = 8$

$$\Rightarrow \left(\frac{d^4 y}{dx^4}\right)^2 - 7x\left(\frac{d^3 y}{dx^3}\right)^2 = 8$$

$\therefore$ The order and degree of the given differential equation are 4 and 2 respectively.
Hence, option *(c)* is correct.

94. *(b)* Given vectors are $\vec{a}=2\hat{i}-3\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}+4\hat{j}-2\hat{k}$

$\therefore \vec{a}+\vec{b}=\left(2\hat{i}-3\hat{j}-\hat{k}\right)+\left(\hat{i}+4\hat{j}-2\hat{k}\right)=3\hat{i}+\hat{j}-3\hat{k}$ and

$\vec{a}-\vec{b}=\left(2\hat{i}-3\hat{j}-\hat{k}\right)-\left(\hat{i}+4\hat{j}-2\hat{k}\right)=\hat{i}-7\hat{j}+\hat{k}$

$$\therefore \left(\vec{a}+\vec{b}\right)\times\left(\vec{a}-\vec{b}\right)=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 1 & -3\\ 1 & -7 & 1\end{vmatrix}$$

$$=\hat{i}\begin{vmatrix}1 & -3\\ -7 & 1\end{vmatrix}-\hat{j}\begin{vmatrix}3 & -3\\ 1 & 1\end{vmatrix}+\hat{k}\begin{vmatrix}3 & 1\\ 1 & -7\end{vmatrix}$$

$$=\hat{i}(1-21)-\hat{j}(3+3)+\hat{k}(-21-1)$$

$$=-20\hat{i}-6\hat{j}-22\hat{k}$$

$$=-2\left(10\hat{i}+3\hat{j}+11\hat{k}\right)$$

Now, $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -3 & -1\\ 1 & 4 & -2\end{vmatrix}$

$$=\hat{i}\begin{vmatrix}-3 & -1\\ 4 & -2\end{vmatrix}-\hat{j}\begin{vmatrix}2 & -1\\ 1 & -2\end{vmatrix}+\hat{k}\begin{vmatrix}2 & -3\\ 1 & 4\end{vmatrix}$$

$$=\hat{i}(6+4)-\hat{j}(-4+1)+\hat{k}(8+3)$$

$$=10\hat{i}+3\hat{j}+11\hat{k}$$

Thus, $\left(\vec{a}+\vec{b}\right)\times\left(\vec{a}-\vec{b}\right)=-2\left(\vec{a}\times\vec{b}\right)$

Hence, option *(b)* is correct.

95. *(d)* Let $f(x)=\begin{cases}1+\dfrac{x}{2k}, & 0<x<2\\ kx, & 2\le x<4\end{cases}$

Hence, $\lim\limits_{x\to 2} f(x)$ exists so we can write that

$$\lim_{x\to 2^-} f(x)=\lim_{x\to 2^+} f(x)$$

$$\Rightarrow \lim_{x\to 2^-}\left(1+\frac{x}{2k}\right)=\lim_{x\to 2^+}(kx)$$

$$\Rightarrow 1+\frac{2}{2k}=2k$$

$$\Rightarrow k+1=2k^2$$

$$\Rightarrow 2k^2-k-1=0$$

$$\Rightarrow (2k+1)(k-1)=0$$

$$\therefore k=1 \text{ or } k=-\frac{1}{2}$$

Hence, option *(d)* is correct.

96. *(d)* Given equation of planes are $2x-y-2z+1=0$

$\Rightarrow a_1=2,\ b_1=-1,\ c_1=-2$ and $d_1=1$

And, $3x-4y+5z-3=0$

$\Rightarrow a_2=3,\ b_2=-4,\ c_2=5$ and $d_2=-3$

$\therefore$ Required angle is

$$\cos\theta=\pm\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$\Rightarrow \cos\theta=\pm\frac{2(3)+(-1)(-4)+5(-2)}{\sqrt{(2)^2+(-1)^2+(5)^2}\sqrt{(3)^2+(-4)^2+(5)^2}}$$

$$\Rightarrow \cos\theta=\pm\frac{6+4-10}{\sqrt{(2)^2+(-1)^2+(5)^2}\sqrt{(3)^2+(-4)^2+(5)^2}}$$

$$\Rightarrow \cos\theta=0 \Rightarrow \theta=\frac{\pi}{2}$$

Hence, option *(d)* is correct.

97. *(d)* $P(\text{ not } A)=\dfrac{7}{10},\ P(\text{ not } B)=\dfrac{3}{10}$ and $P(A/B)=\dfrac{3}{14}$

$P(\text{ not } A)=\dfrac{7}{10},\ P(\text{ not } B)=\dfrac{3}{10}$

Now,

$$P(A/B)=\frac{P(A\cap B)}{P(B)} \Rightarrow P(A\cap B)=P(B)P(A/B)$$

$$P(A\cap B)=\left[1-P(\text{not } B)\right]P(A/B)=\left(1-\frac{3}{10}\right)\left(\frac{3}{14}\right)$$

$$=\frac{7}{10}\times\frac{3}{14}=\frac{3}{20}$$

$$\rightarrow P(B/A)=\frac{P(A\cap B)}{P(A)}=\frac{P(A\cap B)}{1-P(\text{not } A)}=\frac{\frac{3}{20}}{1-\frac{7}{10}}$$

$$=\frac{3}{20}\times\frac{10}{3}$$

$$\therefore P(B/A)=\frac{1}{2}$$

Hence, option *(d)* is correct.

98. *(a)* Consider $(3\vec{a}-\vec{b})\times(\vec{a}+3\vec{b})$

$$=(3\vec{a}-\vec{b})\times\vec{a}+(3\vec{a}-\vec{b})\times 3\vec{b}$$

$$=3\vec{a}\times\vec{a}-\vec{b}\times\vec{a}+3\vec{a}\times 3\vec{b}-\vec{b}\times 3\vec{b}$$

$$=0-(-\vec{a}\times\vec{b})+9\vec{a}\times\vec{b}-\vec{b}$$

$$=10\vec{a}\times\vec{b}$$

$\therefore\ k=10$

Hence, option *(a)* is correct.

99. *(a)* Consider $\begin{vmatrix}1-a & a-b-c & b+c\\ 1-b & b-c-a & c+a\\ 1-c & c-a-b & a+b\end{vmatrix}$

Applying $C_2\rightarrow C_2+C_3$

$$=\begin{vmatrix}1-a & a & b+c\\ 1-b & b & c+a\\ 1-c & c & a+b\end{vmatrix}$$

Applying $C_1\rightarrow C_1+C_2$

$$=\begin{vmatrix}1 & a & b+c\\ 1 & b & c+a\\ 1 & c & a+b\end{vmatrix}$$

Applying $C_3\rightarrow C_2+C_3$

$$=(a+b+c)\begin{vmatrix}1 & a & 1\\ 1 & b & 1\\ 1 & c & 1\end{vmatrix}=0$$

Hence, option *(a)* is correct.

100. *(b)* Let $I_n=\int_0^{\pi/4}\tan^n x\,dx$

Consider, $I_n+I_{n-2}=\int_0^{\pi/4}\tan^n x\,dx+\int_0^{\pi/4}\tan^{n-2} x\,dx$

$$=\int_0^{\pi/4}\tan^{n-2}x\left(\tan^2 x+1\right)dx$$

$$=\int_0^{\pi/4}\sec^2 x\tan^{n-2}x\,dx$$

Put $\tan x=t\Rightarrow\sec^2 x\,dx=dt$

When $x=0$, then $t=0$ and

When $x=\frac{\pi}{4}$, then $t=1$

$$\therefore I_n+I_{n-2}=\int_0^1 t^{n-2}dt$$

$$=\left[\frac{t^{n-2+1}}{n-2+1}\right]_o^1=\left[\frac{t^{n-1}}{n-1}\right]_o^1=\frac{1}{(n-1)}[1-0]=\frac{1}{n-1}$$

Hence, option *(b)* is correct.

101. *(d)* Given $a\left(x\frac{dy}{dx}+2y\right)=xy\frac{dy}{dx}$

$$\Rightarrow ax\frac{dy}{dx}-xy\frac{dy}{dx}=-2ay$$

$$\Rightarrow(xy-ax)\frac{dy}{dx}=2ay$$

$$\Rightarrow x(y-a)dy=2ay\,dx$$

$$\Rightarrow\frac{(y-a)}{y}dy=\frac{2a}{x}dx$$

$$\Rightarrow\left(1-\frac{a}{y}\right)dy=\frac{2a}{x}dx$$

$$\Rightarrow dy-\frac{a}{y}dy=\frac{2a}{x}dx$$

Integrate on both sides, we get

$\int dy - a\int \frac{1}{y}dy = 2a\int \frac{1}{x}dx$

$y - a\log y = 2a\log x + \log c$

$\Rightarrow y = a\log x^2 yc$

$\Rightarrow x^2 y = ke^{y/a}$ $(\because c = k = \text{constant})$

Hence, option *(d)* is correct.

102. *(d)* Here, $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$

$\Rightarrow |A| = 3-(-2) = 5$ and $|B| = -4-(-3) = -1$

$\Rightarrow A^{-1} = \frac{1}{5}\begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix}$ and $B^{-1} = -1\begin{bmatrix} -2 & -3 \\ 1 & 2 \end{bmatrix}$

$\therefore AB = \begin{bmatrix} 3 & 5 \\ 1 & 0 \end{bmatrix}$ and $A^{-1}B^{-1} = \frac{1}{5}\begin{bmatrix} 5 & 7 \\ -5 & 8 \end{bmatrix}$

$\Rightarrow AB\left(A^{-1}B^{-1}\right) = \frac{1}{5}\begin{bmatrix} -10 & -61 \\ 5 & 7 \end{bmatrix} \neq 1$

$|AB| = 0-5 = -5$

$\therefore (AB)^{-1} = -\frac{1}{5}\begin{bmatrix} 0 & -5 \\ -1 & 3 \end{bmatrix} \neq A^{-1}B^{-1}$

Thus, both the statements are not correct.

Hence, option *(d)* is correct.

103. *(d)* Let $y = \ln\sqrt{\tan x}$

Differentiate both sides w.r.t. x, we get

$\frac{dy}{dx} = \frac{1}{\sqrt{\tan x}} \cdot \frac{1}{2\sqrt{\tan x}} \cdot \sec^2 x$

Now, $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$

$= \frac{1}{\sqrt{\tan\frac{\pi}{4}}} \times \frac{1}{2\sqrt{\tan\frac{\pi}{4}}} \times \frac{1}{\cos^2\frac{\pi}{4}}$

$= \frac{1}{2} \times 1 \times \frac{1}{\left(\frac{1}{\sqrt{2}}\right)^2}$

$= \frac{1}{2} \times 1 \times 2 = 1$

Hence, option *(d)* is correct.

104. *(a)* Give $p_1 : x+y+z=1$ and $p_2 : 2x+3y+4z=7$

So, equation of plane passing through intersection of planes p_1 and p_2 is

$x+y+z-1+k(2x+3y+4z-7)=0$

$\Rightarrow x+y+z-1+2kx+3ky+4kz-7k=0$

$\Rightarrow x(1+2k)+y(1+3k)+z(1+4k)-1-7k=0$

This is perpendicular to $x-5y+3z=5$

$\Rightarrow x-5y+3z-5=0$

$\Rightarrow 1(1+2k)-5(1+3k)+3(1+4k)=0$

$\Rightarrow 1+2k-5-15k+3+12k=0$

$\Rightarrow -k-1=0 \Rightarrow k=-1$

Equation of plane $x+y+z-1-1(2x+3y+4z-7)=0$

$\Rightarrow x+y+z-1-2x-3y-4z+7=0$

$\Rightarrow -x-2y-3z+6=0$

$\Rightarrow x+2y+3z-6=0$

Hence, option *(a)* is correct.

105. *(a)* Let $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$

$\therefore A^2 = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1+4 & 2+6 \\ 2+6 & 4+9 \end{bmatrix} = \begin{bmatrix} 5 & 8 \\ 8 & 13 \end{bmatrix}$

$\therefore A^2 - kA - I_2 = \begin{bmatrix} 5 & 8 \\ 8 & 13 \end{bmatrix} - \begin{bmatrix} k & 2k \\ 2k & 3k \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 5-k-1 & 8-2k-0 \\ 8-2k-0 & 13-3k-1 \end{bmatrix}$

$= \begin{bmatrix} 4-k & 8-2k \\ 8-2k & 12-3k \end{bmatrix}$

Given, $A^2 - kA - I_2 = 0$

$\therefore 4-k=0 \Rightarrow k=4$

Hence, option *(a)* is correct.

106. *(b)* Let $y = \ln\left(e^{mx} + e^{-mx}\right)$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{e^{mx}+e^{-mx}} \cdot \frac{d}{dx}\left(e^{mx}+e^{-mx}\right)$$

$$= \frac{me^{mx}-me^{-mx}}{e^{mx}+e^{-mx}} = \frac{m\left(e^{mx}-e^{-mx}\right)}{e^{mx}+e^{-mx}}$$

$$= \frac{m\left(e^{mx}-\frac{1}{e^{mx}}\right)}{e^{mx}+\frac{1}{e^{mx}}} = \frac{m\left(e^{2mx}-1\right)}{e^{2mx}+1}$$

So, $\left.\frac{dy}{dx}\right|_{x=0} = \frac{m\left(e^{0}-1\right)}{e^{0}+1} = m(0) = 0$

Hence, option *(b)* is correct.

107. *(d)* Since, $\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$

Applying $C_1 \to C_1 + C_2 + C_3$

$$\Rightarrow \begin{vmatrix} x-(a+b+c) & c & b \\ x-(a+b+c) & b-x & a \\ x-(a+b+c) & a & c-x \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x & c & b \\ x & b-x & a \\ x & a & c-x \end{vmatrix} = 0$$

Applying $R_2 \to R_1 - R_2$ and $R_3 \to R_1 - R_3$

$$\Rightarrow \begin{vmatrix} x & c & b \\ 0 & c+x-b & b-a \\ 0 & c-a & b+x-c \end{vmatrix} = 0$$

$$\Rightarrow x\left\{(c+x-b)(b+x-c)-(b-a)(c-a)\right\} = 0$$

$$\Rightarrow x\left\{x^2-(b-c)^2-bc+ac+ab-a^2\right\} = 0$$

$$\Rightarrow x\left\{x^2-a^2-b^2-c^2+ab+bc+ca\right\} = 0$$

$$\Rightarrow x\left\{x^2-(a-b)^2-(b-c)^2-(c-a)^2\right\} = 0$$

$\therefore \; x = 0$

Hence, option *(d)* is correct.

108. *(b)* Let $\overrightarrow{d_1}$ and $\overrightarrow{d_2}$ be the two diagonals of a quadrilateral such that $\overrightarrow{d_1} = 3\hat{i}+6\hat{j}-2\hat{k}$ and $\overrightarrow{d_2} = 4\hat{i}-\hat{j}+3\hat{k}$

Now, dot product of $\overrightarrow{d_1}$ and $\overrightarrow{d_2}$ is

$\overrightarrow{d_1} \cdot \overrightarrow{d_2} = 3(4)+6(-1)-2(3) = 0$

Now, $\left|\overrightarrow{d_1}\right| = \sqrt{3^2+6^2+(-2)^2} = 7$ and

$\left|\overrightarrow{d_2}\right| = \sqrt{4^2+1^2+3^2} = \sqrt{26}$

Since, $\left|\overrightarrow{d_1}\right| \neq \left|\overrightarrow{d_2}\right|$

So, the given quadrilateral is a rhombus.

Hence, option *(b)* is correct.

109. *(b)* Let us consider triangle *ABC*. Suppose $\hat{i}$, $\hat{j}$ and $\hat{i}+\hat{j}+\lambda\hat{k}$ are the position vectors of *A, B* and *C*.

Then, $\overrightarrow{AB} = \hat{j}-\hat{i}$, $\overrightarrow{AC} = \hat{j}+\lambda\hat{k}$, $\overrightarrow{BC} = \hat{i}+\lambda\hat{k}$

$\left|\overrightarrow{AB}\right| = \sqrt{(-1)^2+(1)^2} = \sqrt{2}$

$\left|\overrightarrow{AC}\right| = \sqrt{(1)^2+(\lambda)^2} = \sqrt{1+\lambda^2}$

$\left|\overrightarrow{BC}\right| = \sqrt{(1)^2+(\lambda)^2} = \sqrt{1+\lambda^2}$

To be ΔABC is a right angled triangle, $\angle C$ should be right angle, i.e. $\overrightarrow{BC} \cdot \overrightarrow{AC} = 0$

$\Rightarrow \left(\hat{i}+\lambda\hat{k}\right)\cdot\left(\hat{j}+\lambda\hat{k}\right) = 0 \Rightarrow 0+0+\lambda^2 = 0$

$\therefore \; \lambda = 0$

Hence, option *(b)* is correct.

110. *(c)* Given $\frac{dx}{dy}+\frac{x}{y}-y^2 = 0$

$\Rightarrow \frac{dx}{dy}+\frac{x}{y} = y^2$

This is a linear differential equation of the form

$\frac{dx}{dy}+Px = Q$

Here, $P = \frac{1}{y}$ and $Q = y^2$

$\therefore$ I.F. $= e^{\int P dy} = e^{\int \frac{1}{y} dy} = e^{\log y} = y$

So, required solution is

$x \cdot y = \int y^2 \cdot y dy + c$

$xy = \int y^3 dy + c$

$xy = \frac{y^4}{4} + c$

$4xy = y^4 + c$

Hence, option *(c)* is correct.

111. *(d)* Area bounded by the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ and coordinate axes is

$A = \int_0^a y dx = \int_0^a \left(a + x - 2\sqrt{a}\sqrt{x}\right) dx$

$\left(\because \sqrt{y} = \sqrt{a} - \sqrt{x} \Rightarrow y = a + x - 2\sqrt{a}\sqrt{x}\right)$

$= ax + \frac{x^2}{2} - \frac{2\sqrt{a}x^{3/2}}{3/2}\Big|_0^a$

$= a^2 + \frac{a^2}{2} - \frac{4}{3}a^2 = \frac{3a^2}{2} - \frac{4}{3}a^2$

$= \frac{9a^2 - 8a^2}{6} = \frac{a^2}{6}$ sq units

Hence, option *(d)* is correct.

112. *(b)* Required are $= 2\int_0^{1/4} \sqrt{x} dx$

$= 2 \cdot \frac{2}{3}\left[x^{3/2}\right]_0^{1/4} = \frac{4}{3}\left[\frac{1}{8} - 0\right] = \frac{1}{6}$ sq. unit

Hence, option *(b)* is correct.

113. *(c)* Given mean of 7 observations is 10.

$\therefore \frac{x_1 + x_2 + x_3 + ... + x_7}{7} = 10$

$\Rightarrow x_1 + x_2 + x_3 + ... + x_7 = 70$ ....(i)

Also, mean of 3 observations is 5.

$\therefore \frac{x_8 + x_9 + x_{10}}{3} = 5$

$\Rightarrow x_8 + x_9 + x_{10} = 15$ ....(ii)

So, from (i) and (2), required mean is

$= \frac{x_1 + x_2 + ... + x_7 + x_8 + x_9 + x_{10}}{10} = \frac{70 + 15}{10} = \frac{85}{10} = 8.5$

Hence, option *(c)* is correct.

114. *(a)* Given $\hat{i} - \hat{j} + \hat{k}$, $2\hat{i} + \hat{j} - \hat{k}$ and $\lambda\hat{i} - \hat{j} + \lambda\hat{k}$

We know that given vectors are coplanar, if

$\begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ \lambda & -1 & \lambda \end{vmatrix} = 0$

$\Rightarrow 1(\lambda - 1) + 1(2\lambda + \lambda) + 1(-2 - \lambda) = 0$

$\Rightarrow \lambda - 1 + 3\lambda - 2 - \lambda = 0$

$\Rightarrow 3\lambda = 3 \Rightarrow \lambda = 1$

Hence, option *(a)* is correct.

115. *(b)* Let $a^{-1} + b^{-1} + c^{-1} = 0$

$\Rightarrow \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$

$\Rightarrow \frac{bc + ac + ab}{abc} = 0$

$\Rightarrow ab + bc + ca = 0$ ....(i)

Consider $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = \lambda$

$\Rightarrow (1+a)\left[(1+b)(1+c) - 1\right] - (1 + c - 1) + (1 - 1 - b) = \lambda$

$\Rightarrow (1+a)(c + b + bc) - c - b = \lambda$

$\Rightarrow bc + ac + ab + abc = \lambda$

$\Rightarrow abc = \lambda$ (using (i))

Hence, option *(b)* is correct.

116. *(c)* Total no. of observation $(n) = 20$, $\sum x_i = 1000$ and $\sum x_i^2 = 84000$

$\bar{x} = \frac{\sum x_i}{n} = \frac{1000}{20} = 5$

Variance $= sd^2$

We have $sd = \sqrt{\frac{1}{n}\sum x_i^2 - \left(\bar{x}\right)^2}$

$\therefore \left(sd\right)^2 = \frac{1}{n}\sum x_i^2 - \left(\bar{x}\right)^2 = \frac{84000}{20} - \left(50\right)^2$

$= 4200 - 2500 = 1700$

So, variance $= 1700$

Hence, option *(c)* is correct.

117. *(a)* We have $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix}$

Applying $C_3 \to C_2 + C_3$

$= \begin{vmatrix} 1 & bc & ab+bc+ac \\ 1 & ca & ab+bc+ac \\ 1 & ab & ab+bc+ac \end{vmatrix}$

$= \left(ab+bc+ac\right)\begin{vmatrix} 1 & bc & 1 \\ 1 & ca & 1 \\ 1 & ab & 1 \end{vmatrix}$

$= \left(ab+bc+ac\right) \times 0 = 0$

Hence, option *(a)* is correct.

118. *(c)* Given that $\overline{X} = 10,\ \overline{Y} = 90,\ \sigma_X = 3,\ \sigma_Y = 12$ and $r_{XY} = 0.8$

Regression equation *x* on *y* is

$x - 10 = r \cdot \frac{\sigma_x}{\sigma_y}\left(y - 90\right)$

$\Rightarrow x - 10 = 0.8 \times \frac{3}{12}\left(y - 90\right)$

$\Rightarrow x - 10 = \frac{2.4}{12}\left(y - 90\right)$

$\Rightarrow x - 10 = 0.2\left(y - 90\right)$

$\Rightarrow x - 10 = 0.2y - 18$

$\Rightarrow x = 0.2y - 8$

Hence, option *(c)* is correct.

119. *(a)* Let $\vec{b} = x\hat{i} + y\hat{j} + z\hat{k}$

Since, $\vec{b}$ is collinear with vector $\vec{a}$

$\therefore \vec{a} = k\vec{b}$, where k is a scalar.

Given, $\vec{a} = \left(2, 1, -1\right)$

$\therefore \left(2, 1, -1\right) = k\left(x, y, z\right)$

$\Rightarrow x = \frac{2}{k},\ y = \frac{2}{k},\ z = -\frac{1}{k}$

Also, $\vec{a} \cdot \vec{b} = 3$

$\Rightarrow 2x + y - z = 3$

$\Rightarrow 2\left(\frac{2}{k}\right) + \frac{1}{k} + \frac{1}{k} = 3$

$\Rightarrow \frac{6}{k} = 3 \Rightarrow k = 2$

$\therefore x = 1,\ y = \frac{1}{2}$ and $z = -\frac{1}{2}$

Thus, $\vec{b} = \left(1, \frac{1}{2}, -\frac{1}{2}\right)$

Hence, option *(a)* is correct.

120. *(c)* Consider $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

Applying $C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} a+b+c & b & c \\ a+b+c & c & a \\ a+b+c & a & b \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix}$$

$$= (a+b+c)\left[1\left(bc-a^2\right)-b\left(b-a\right)+c\left(a-c\right)\right]$$

$$= (a+b+c)\left[bc-a^2-b^2+ab+ac-c^2\right]$$

$$= (a+b+c)\left[-\left(a^2+b^2+c^2-ab-bc-ca\right)\right]$$

$$= -\frac{1}{2}(a+b+c)\left[(a-b)^2+(b-c)^2+(c-a)^2\right]$$

Thus, determinant is negative.
Hence, option *(c)* is correct.