HINTS & SOLUTION

1. *(c)* We have $f(-1) = f(1) = 2^{35}$

Here, two real numbers 1 and -1 have the same image. So, the function is not one-one and let

$$y = (x^2+1)^{35} \Rightarrow x = \sqrt{(y)^{1/35} - 1}$$

Thus, every real number has no pre image. So the function is not onto.
Thus, the function is neither one-one nor onto.
Hence, option *(c)* is correct.

2. *(b)* Since, $\vec{p}$ and $\vec{q}$ are collinear, then

$\vec{p} = k\vec{q}$ [where k is a scalar]

$\Rightarrow (x-2)\vec{a} + \vec{b} = k(x+1)\vec{a} - k\vec{b}$

On equating the coefficients:

$x - 2 = k(x+1)$ and $k = -1$

Putting value of k, we get

$x - 2 = -(x+1)$

$\Rightarrow 2x = 1$

$\Rightarrow x = \frac{1}{2}$

Hence, option *(b)* is correct.

3. *(c)* Number of ways of selecting 5 heads cut of total 12 flips $= {}^{12}C_5$

Probability of getting one head in a coin $= \frac{1}{2}$

Also, probability of getting one tail in a coin $= \frac{1}{2}$

Probability of getting 5 head $= \left(\frac{1}{2}\right)^5$

Probability of getting 7 tails $= \left(\frac{1}{2}\right)^7$

So, required probability

$$= {}^{12}C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^7 = {}^{12}C_5 \left(\frac{1}{2}\right)^{12} = \frac{C(12,5)}{2^{12}}$$

Hence, option *(c)* is correct.

4. *(a)* Given $\bar{x} = 60$ and $n = 10$ and $\sum(\bar{x} - 50)^2 = 5000$

$\Rightarrow \sum(x^2 - 100x + 2500) = 5000$

$\Rightarrow \sum x^2 - 100\sum x = -20000$

$\Rightarrow \sum x^2 = 60000 - 20000$

$\Rightarrow \sum x^2 = 40000$

Now,

$$\sigma = \sqrt{\frac{\sum x^2}{n} - (\bar{x})^2} = \sqrt{4000 - 3600} = 20$$

Hence, option *(a)* is correct.

5. *(d)* The given differential equation is

$\frac{dy}{dx} = 1 + x + y^2 + xy^2$

$\Rightarrow \frac{dy}{dx} = (1+x)(1+y^2)$

$\Rightarrow \frac{dy}{(1+y^2)} = (1+x)dx$

$\Rightarrow \tan^{-1} y = \frac{x^2}{2} + x + c$

Given that when $x = 0, y(0) = 0$. Hence $c = 0$

$\Rightarrow y = \tan\left(\frac{x^2}{2} + x\right)$

Hence, option *(d)* is correct.

6. *(d)* Since the three vectors are coplanar, so one of them is expressible as a linear combination of the other two.

$\therefore (m, -1, 2) = x(2, -3, 4) + y(1, 2, -1)$

$\Rightarrow 2x + y = m$ ....(i)

$-3x + 2y = -1$ ....(ii)

And $4x - y = 2$ ....(iii)

On solving equation (ii) and (iii), we get

$x = \frac{3}{5}$ and $y = \frac{2}{5}$

$\therefore$ from (i), $2\left(\frac{3}{5}\right)+\frac{2}{5}=m$

$\Rightarrow \frac{6}{5}+\frac{2}{5}=m \Rightarrow m=\frac{8}{5}$

Hence, option *(d)* is correct.

7. *(c)* Let $\frac{1}{x}=u$ and $\frac{1}{y}=v$

$\therefore a_1u+b_1v=c_1$ and $a_2u+b_2v=c_2$

Using the method of cross multiplication,

$$\frac{u}{b_1c_2-b_2c_1}=\frac{v}{c_1a_2-c_2a_1}=\frac{-1}{a_1b_2-a_2b_1}$$

$$\Rightarrow \frac{\frac{1}{x}}{\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}}=\frac{\frac{1}{y}}{\begin{vmatrix} c_1 & a_1 \\ c_2 & a_2 \end{vmatrix}}=\frac{-1}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

$$\Rightarrow \frac{\frac{1}{x}}{\Delta_2}=\frac{\frac{1}{y}}{\Delta_3}=-\frac{1}{\Delta_1}$$

$\therefore \frac{1}{x}=-\frac{\Delta_2}{\Delta_1}$ and $\frac{1}{y}=-\frac{\Delta_3}{\Delta_1}$

$\Rightarrow x=-\frac{\Delta_1}{\Delta_2}$ and $y=-\frac{\Delta_1}{\Delta_3}$

Hence, option *(c)* is correct.

8. *(b)* Given differential equation is

$$(x+y)(dx-dy)=dx+dy$$

$\Rightarrow (x+y)dx-(x+y)dy=dx+dy$

$\Rightarrow (x+y-1)dx=(x+y+1)dy$

$\Rightarrow \frac{dy}{dx}=\frac{x+y-1}{x+y+1}$

Let $x+y=v$ and $\frac{dy}{dx}=\frac{dv}{dx}-1$

$\therefore \frac{dv}{dx}-1=\frac{v-1}{v+1}$

$\Rightarrow \frac{dv}{dx}=\frac{v-1}{v+1}+1$

$\Rightarrow \frac{dv}{dx}=\frac{v-1+v+1}{v+1}$

$\Rightarrow \frac{v+1}{2v}dv=dx$

$\Rightarrow \frac{1}{2}\int 1dv+\frac{1}{2}\int\frac{1}{v}dv=\int 1dx$

$\Rightarrow \frac{1}{2}v+\frac{1}{2}\log v=x+c_1$

$\Rightarrow x+y+\log(x+y)=2x+c \quad [\because 2c_1=c]$

$\Rightarrow (y-x)+\log(x+y)=c$

Hence, option *(b)* is correct.

9. *(c)* We check from the given options one by one. Options *(a)* and *(b)* do not satisfy. We check option *(c)*.

Let $f(x)=\frac{x}{2}+\alpha$

$\therefore \int\frac{dx}{\frac{x}{2}+\alpha}=\int\frac{2dx}{x+2\alpha}$

$=2\log(x+2\alpha)+c_1=\log(x+2\alpha)^2+c_1$

$=\log\left(\frac{x}{2}+\alpha\right)^2+\log 2^2+c_1=\log\left(\frac{x}{2}+\alpha\right)^2+c$

Hence, option *(c)* is correct.

10. *(d)* By observing the options, let $f(t)=t^k$

Suppose $t=xy$

$\therefore f(xy)=(xy)^k=x^k\cdot y^k=f(x)f(y)$

Hence, $f(t)=t^k$, where k is a constant.

Hence, option *(d)* is correct.

11. *(d)* Let $f(x)=2^{\sin x}$

Differentiate both sides w.r.t. x, we get

$f'(x)=\frac{d}{dx}\left[2^{\sin x}\right]$

$=\ln(2)\cdot 2^{\sin x}\cdot\frac{d}{dx}[\sin x]$

$\left\{\because\left[a^{u(x)}\right]'=\ln(a)\cdot a^{u(x)}\cdot u'(x)\right\}$

$=2^{\sin x}\ln(2)\cos x$

Hence, option *(d)* is correct.

12. *(b)* Given velocity is $v=s+1$

Since, velocity $=\frac{ds}{dt}$

$\therefore\ \frac{ds}{dt}=s+1\Rightarrow\frac{ds}{s+1}=dt$

Integrate both sides, we get

$\log(s+1)=t$

As $s=9\text{m}$

$t=(\log 10)\text{second}$

Hence, option *(b)* is correct.

13. *(a)* Given $A=\begin{bmatrix}\omega & 0\\ 0 & \omega\end{bmatrix}$

Now, $A^2=\begin{bmatrix}\omega & 0\\ 0 & \omega\end{bmatrix}\begin{bmatrix}\omega & 0\\ 0 & \omega\end{bmatrix}=\begin{bmatrix}\omega^2 & 0\\ 0 & \omega^2\end{bmatrix}$

And, $A^2=\begin{bmatrix}\omega^2 & 0\\ 0 & \omega^2\end{bmatrix}\begin{bmatrix}\omega & 0\\ 0 & \omega\end{bmatrix}=\begin{bmatrix}\omega^3 & 0\\ 0 & \omega^3\end{bmatrix}$

Similarly, $A^{100}=\begin{bmatrix}\omega^{100} & 0\\ 0 & \omega^{100}\end{bmatrix}$

$=\begin{bmatrix}(\omega^3)^{33}\cdot\omega^1 & 0\\ 0 & (\omega^3)^{33}\cdot\omega^1\end{bmatrix}$

$=\begin{bmatrix}\omega & 0\\ 0 & \omega\end{bmatrix}=A\qquad\{\because\omega^3=1\}$

Hence, option *(a)* is correct.

14. *(c)* Let $O(0,0),A(x_1,y_1)$ and $B(x_2,y_2)$ be three points

$OA=\sqrt{x_1^2+y_1^2},\ OB=\sqrt{x_2^2+y_2^2}$

$\therefore\ AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

In ΔAOB

$\cos\angle AOB=\frac{OA^2+OB^2-AB^2}{2(OA)(OB)}$

$\Rightarrow(OA)(OB)\cos\angle AOB=\frac{OA^2+OB^2-AB^2}{2}$

$=\frac{x_1^2+y_1^2+x_2^2+y_2^2-\left\{(x_2-x_1)^2+(y_2-y_1)^2\right\}}{2}$

$\Rightarrow(OA)(OB)\cos\angle AOB=\frac{OA^2+OB^2-AB^2}{2}$

$=\frac{x_1^2+y_1^2+x_2^2+y_2^2-\left\{x_2^2+x_1^2-2x_1x_2+y_2^2+y_1^2-2y_1y_2\right\}}{2}$

$=\frac{2(x_1x_2+y_1y_2)}{2}$

$=x_1x_2+y_1y_2$

Hence, option *(c)* is correct.

15. *(d)* The given equation is:

$\log_{10}\left\{999+\sqrt{x^2-3x+3}\right\}=3$

$\Rightarrow 999+\sqrt{x^2-3x+3}=10^3=1000$

$\Rightarrow\sqrt{x^2-3x+3}=1000-999$

$\Rightarrow\sqrt{x^2-3x+3}=1$

$\Rightarrow x^2-3x+3=1$

$\Rightarrow x^2-3x+2=0$

$\Rightarrow(x-1)(x-2)=0$

$\Rightarrow x=1,2$

Hence, option *(d)* is correct.

16. *(d)* Given that $4a-2b+c=0$

Substitute 2 in the equation, $ax^2+bx+c=0$

$\Rightarrow a(2)^2+b(2)+c=0$

$\Rightarrow 4a+2b+c=0$

So, $(x-2)$ is not the factor.

Substitute -2 in the equation, $ax^2+bx+c=0$

$\Rightarrow a(-2)^2 + b(-2) + c = 0$

$\Rightarrow 4a - 2b + c = 0$

So, $(x+2)$ is the factor.

$\therefore$ Only statement 1 is true.

Hence, option *(d)* is correct.

17. *(a)* We have $0.3 + 0.33 + 0.333 + \cdots n$

$$= \frac{3}{10} + \frac{33}{100} + \frac{333}{1000} + \cdots$$

$$= 3\left(\frac{1}{10} + \frac{11}{100} + \frac{111}{1000} + \cdots\right)$$

$$= \frac{3}{9}\left(\frac{9}{10} + \frac{9}{100} + \frac{999}{1000} + \cdots\right)$$

$$= \frac{1}{3}\left[\left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{100}\right) + \left(1 - \frac{1}{1000}\right) + \cdots\right]$$

$$= \frac{1}{3}\left[n - \left(\frac{1}{10} + \frac{1}{10^2} + \frac{1}{10^3} + \cdots + \frac{1}{10^n}\right)\right]$$

$$= \frac{1}{3}\left[n - \frac{\frac{1}{10}\left(1 - \frac{1}{10^n}\right)}{1 - \frac{1}{10}}\right]$$

$$\left(\text{Since, sum of } n \text{ terms of GP} = \frac{a(1-r^n)}{1-r}\right)$$

$$= \frac{1}{3}\left[n - \frac{10^n - 1}{9(10^n)}\right] = \frac{1}{3}\left[n - \frac{1}{9}\left(1 - \frac{1}{10^n}\right)\right]$$

Hence, option *(a)* is correct.

18. *(a)* We have $i^{1000} + i^{1001} + i^{1002} + i^{1003}$

$= i^{1000}(1 + i + i^2 + i^3)$

$= i^{1000}(1 + i - 1 - i)$ $\quad [\because i^2 = -1 \Rightarrow i^3 = -i]$

$= i^{1000}(0)$

$= 0$

Hence, option *(a)* is correct.

19. *(b)* Consider $(x-y)^n, n \geq 5$

General term, $T_{r+1} = {}^nC_r x^{n-r}(-y)^r$

Since, $T_5 + T_6 = 0$ (given)

$\Rightarrow \left[{}^nC_4 x^{n-4}(-y)^4\right] + \left[{}^nC_5 x^{n-5}(-y)^5\right] = 0$

$\Rightarrow {}^nC_4 x^{n-4} y^4 - {}^nC_5 x^{n-5} y^5 = 0$

$\Rightarrow {}^nC_4 x^{n-4} y^4 = {}^nC_5 x^{n-5} y^5$

$$\Rightarrow \frac{x^{n-4-n+5}}{y} = \frac{{}^nC_5}{{}^nC_4}$$

$$\Rightarrow \frac{x}{y} = \frac{n!}{5!(n-5)!} \times \frac{4!(n-4)!}{n!}$$

$$\Rightarrow \frac{x}{y} = \frac{n!}{5 \times 4!(n-5)!} \times \frac{4!(n-4)(n-5)!}{n!}$$

$$\Rightarrow \frac{x}{y} = \frac{n-4}{5}$$

Hence, option *(b)* is correct.

20. *(a)* As given $\sin A = \sin B$ and $\cos A = \cos B$

$$\Rightarrow \frac{\sin A}{\cos A} = \frac{\sin B}{\cos B}$$

$\Rightarrow \tan A = \tan B$ or $\tan B = \tan A = \tan(n\pi + A)$

$\Rightarrow B = n\pi + A$

Hence, option *(a)* is correct.

21. *(d)* We have $\lim\limits_{x \to 0} \frac{x}{\sqrt{1 - \cos x}} = \lim\limits_{x \to 0} \frac{x}{\sqrt{1 - \left(1 - 2\sin^2 \frac{x}{2}\right)}}$

$$= \lim_{x \to 0} \frac{x}{\sqrt{2\sin^2 \frac{x}{2}}} = \frac{1}{\sqrt{2}} \lim_{x \to 0} \frac{x}{\left|\sin \frac{x}{2}\right|}$$

$$\text{L.H.L.} = f(0-0) = \lim_{h \to 0} \frac{x}{\left|\sin \frac{x}{2}\right|}$$

$$= -\frac{1}{\sqrt{2}} \lim_{h \to 0} \frac{2\left(\frac{h}{2}\right)}{\sin \frac{h}{2}}$$

$$= -\frac{1}{\sqrt{2}} \times 2 \times 1 = -\sqrt{2} \qquad \left[\because \lim_{\theta \to 0} \frac{\theta}{\sin \theta} = 1\right]$$

R.H.L. $= f(0+0) = \lim_{h\to 0} f(0+h)$

$= \frac{1}{\sqrt{2}} \lim_{h\to 0} \frac{2\left(\frac{h}{2}\right)}{\sin\frac{h}{2}} = \frac{1}{\sqrt{2}} \times 2 \times 1 = \sqrt{2}$

L.H.L. $\neq$ R.H.L. $= \sqrt{2}$

Therefore, limit does not exist.

Hence, option *(d)* is correct.

22. *(c)* Given equations are

$2x+3y+4=0$ ....(i)

$4x+3y+2=0$ ....(ii)

On solving (i) and (ii), the coordinates of the intersecting point are $(1,-2)$

Now, $\sqrt{(0-1)^2+(0-(-2))^2} = d$

$\Rightarrow d = \sqrt{1+4} = \sqrt{5}$

Hence, option *(c)* is correct.

23. *(a)* We know, for two sets A and B

$A-B = A-(A\cap B)$

$\therefore n(A-B) = n(A)-n(A\cap B)$

Given, $n(A)=115$, $n(B)=326$ and $n(A-B)=47$

$\Rightarrow 47 = 115 - n(A\cap B)$

$\Rightarrow n(A\cap B) = 68$

Consider $n(A\cup B) = n(A)+n(B)-n(A\cap B)$

$\Rightarrow n(A\cup B) = 115+326-68 = 373$

Hence, option *(a)* is correct.

24. *(c)* The given expression $\left[\left(a^m\right)^m - \left(\frac{1}{m}\right)\right]^{\frac{1}{m+1}}$

$= \left[\left(a^m\right)^{\frac{m^2-1}{m}}\right]^{\frac{1}{m+1}} = \left[\left(a^m\right)^{\frac{(m-1)(m+1)}{m}}\right]^{\frac{1}{m+1}}$

$= \left[a^m\right]^{\frac{m-1}{m}} = a^{m-1}$

Its $(m-1)^{\text{th}}$ root $= \left(a^{m-1}\right)^{1/m-1} = a$

Hence, $(m-1)^{\text{th}}$ root of $\left[\left(a^m\right)^m - \left(\frac{1}{m}\right)\right]^{\frac{1}{m+1}}$

$= (m-1)^{\text{th}}$ root of $a^{m-1} = a$.

Hence, option *(c)* is correct.

25. *(b)* Given that $f(x) = \sqrt{x+\sqrt{x+\sqrt{x+\sqrt{\cdots\infty}}}}$

$\Rightarrow f(x) = \sqrt{x+f(x)}$

$\Rightarrow [f(x)]^2 = x+f(x)$

On differentiating both sides w.r.t. x, we get

$\Rightarrow 2f(x)f'(x) = 1+f'(x)$

$\Rightarrow f'(x)\{2f(x)-1\} = 1$

$\Rightarrow f'(x) = \frac{1}{2f(x)-1}$

Hence, option *(b)* is correct.

26. *(a)* Let $y = \ln(x+\sin x)$

$\therefore \frac{dy}{dx} = \frac{1}{(x+\sin x)}(1+\cos x) = \frac{(1+\cos x)}{(x+\sin x)}$

Let $x+\cos x = z$ (say)

$\therefore \frac{dz}{dx} = (1-\sin x)$

Derivative of $\ln(x+\sin x)$ w.r.t. $(x+\cos x)$ is

$\frac{dy}{dz} = \frac{1+\cos x}{(x-\sin x)(1-\sin x)}$

Hence, option *(a)* is correct.

27. *(b)* Given curve is $y = 4x-x^2-3$

Since, area bounded by x-axis

$\therefore y = 0$

$\Rightarrow 4x-x^2-3=0 \Rightarrow x^2-4x+3=0$

$\Rightarrow x^2-3x-x+3=0 \Rightarrow (x-3)(x-1)=0$

$\Rightarrow x = 1, 3$

$\therefore$ Required area $= \int_1^3 \left(4x - x^2 - 3\right) dx$

$= \left.\frac{4x^2}{2} - \frac{x^3}{3} - 3x\right|_1^3 = \left(\frac{36}{2} - \frac{27}{3} - 9\right) - \left(\frac{4}{2} - \frac{1}{3} - 3\right)$

$= (18 - 9 - 9) - \left(2 - \frac{10}{3}\right) = \frac{4}{3}$ sq unit

Hence, option *(b)* is correct.

28. *(a)* Since, $f(x)$ is an increasing function on $[-1,1]$ and it has a root in $(-1,1)$.

$\therefore$ Only statement I is correct.

Hence, option (a) is correct.

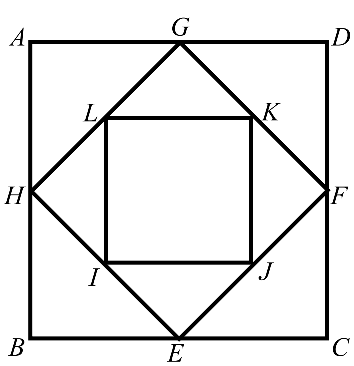
29. *(b)* Let $ABCD, EFGH$ and $IJKL$ be squares.

Let the side of square $ABCD = 16$

Let side of square $ABCD = (16)^2$

Now, area of $EFGH = \frac{(16)^2}{2}$

Area of $IJKL = \frac{(16)^2}{4}$. So on…



$\therefore$ Required sum is

$= (16)^2 + \frac{1}{2}(16)^2 + \frac{1}{4}(16)^2 + \ldots\infty$

$= (16)^2 \left\{1 + \frac{1}{2} + \frac{1}{4} + \ldots\infty\right\}$ ....(i)

Now, $1 + \frac{1}{2} + \frac{1}{4} + \ldots\infty$ is a GP

$\therefore$ Sum $= \frac{a}{1-r}$ where $a = 1$ and $r = \frac{1}{2}$

$\therefore$ From equation (i), we get

Required sum $= 256\left\{1 + \frac{1}{2} + \frac{1}{4} + \ldots\infty\right\}$

$= 256\left\{\frac{1}{1 - \frac{1}{2}}\right\} = 256 \times 2$

$= 512$ sq cm

Hence, option *(b)* is correct.

30. *(a)* Let $A = \{n^2 : n \in N\}$ and $B = \{n^3 : n \in N\}$

$\therefore A = \{1, 4, 9, 16, \ldots\}$ and $B = \{1, 8, 27, 64, \ldots\}$

Now, $A \cap B = \{1\}$ which is a finite set.

Also, $A \cup B = \{1, 4, 8, 9, 27, \ldots\}$

So, complement of $A \cup B$ is infinite set.

Hence, $A \cup B \neq N$

Hence, option *(a)* is correct.

31. *(a)* Given that α and β are the root of the equation $lx^2 - mx + m = 0$

$\therefore$ Sum of roots, $\alpha + \beta = \frac{m}{l}$ and,

Product of roots, $\alpha\beta = \frac{m}{l}$

Now $\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} = \frac{\alpha + \beta}{\sqrt{\alpha\beta}} = \frac{m/l}{\sqrt{m/l}}$

$\Rightarrow \sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} = \sqrt{\frac{m}{l}}$

$\Rightarrow \sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} - \sqrt{\frac{m}{l}} = 0$

Hence, option *(a)* is correct.

32. *(c)* Given that $\operatorname{cosec}^2\theta = 3\sqrt{3}\cot\theta - 5$

$\Rightarrow 1 + \cot^2\theta - 3\sqrt{3}\cot\theta + 5 = 0$

$\left[\text{since, } \operatorname{cosec}^2\theta = 1 + \cot^2\theta\right]$

$\Rightarrow \cot^2\theta - 3\sqrt{3}\cot\theta + 6 = 0$

Work with options, we find that

This equation is satisfied by $\theta = \frac{\pi}{6}$

Hence, option *(c)* is correct.

33. *(b)* In $\Delta ABC, a=2, b=3$ and $\sin A = \frac{2}{3}$

We know, $\frac{\sin A}{a} = \frac{\sin B}{b}$

$\Rightarrow \frac{\frac{2}{3}}{2} = \frac{\sin B}{3}$

$\Rightarrow \frac{2}{6} = \frac{\sin B}{3} \Rightarrow \sin B = \frac{6}{6} = 1$

$\Rightarrow B = \sin^{-1}(1) = \frac{\pi}{2}$

Hence, option *(b)* is correct.

34. *(c)* Given that $y = e^{x^2} \sin 2x$

$\frac{dy}{dx} = 2e^{x^2} \cdot \cos 2x + 2xe^{x^2} \cdot \sin 2x$

$= 2e^{x^2}(\cos 2x + x \sin 2x)$

$\left.\frac{dy}{dx}\right|_{x=\pi} = 2e^{\pi^2}(\cos 2\pi + \pi \sin 2\pi)$

$= 2e^{\pi^2}(1+0)$

$= 2e^{\pi^2}$

Hence, option *(c)* is correct.

35. *(b)* Since, (p,q) is the point on the x-axis.

$\therefore q = 0$

Let $P = (p,0), A = (1,2)$ and $B = (2,3)$

Given $PA = PB$

$\Rightarrow PA^2 = PB^2$

$\Rightarrow (1-p)^2 + 4 = (2-p)^2 + p$

$\Rightarrow 1 + p^2 - 2p - 4 - p^2 + 4p = 5$

$\Rightarrow 2p = 8$

$\Rightarrow p = 4$

Thus, $p = 4, q = 0$

Hence, option *(b)* is correct.

36. *(c)* Let $f(x) = \cos 2x - \sin 2x$

$\Rightarrow f(x) = \sqrt{2}\left[\frac{1}{\sqrt{2}}\cos 2x - \frac{1}{\sqrt{2}}\sin 2x\right]$

$\Rightarrow f(x) = \sqrt{2}\left[\cos\frac{\pi}{4}\cos 2x - \sin\frac{\pi}{4}\sin 2x\right]$

$\Rightarrow f(x) = \sqrt{2}\left[\cos\left(\frac{\pi}{4} + 2x\right)\right]$

We know,

$-1 \le \cos\left(\frac{\pi}{4} + 2x\right) \le 1$

$\Rightarrow -\sqrt{2} \le \sqrt{2}\cos\left(\frac{\pi}{4} + 2x\right) \le \sqrt{2}$

$\Rightarrow -\sqrt{2} \le f(x) \le \sqrt{2}$

$\therefore$ Range of $f(x) = \left[-\sqrt{2}, \sqrt{2}\right]$

Hence, option *(c)* is correct.

37. *(b)* Consider $16\left(\frac{a-x}{a+x}\right)^3 = \frac{a+x}{a-x}$

$\Rightarrow \left(\frac{a-x}{a+x}\right)^3 \times \left(\frac{a+x}{a-x}\right) = \frac{1}{16}$

$\Rightarrow \left(\frac{a-x}{a+x}\right)^4 = \left(\frac{1}{2}\right)^4$

$\Rightarrow \frac{a-x}{a+x} = \frac{1}{2}$

$\Rightarrow 2a - 2x = a + x$

$\Rightarrow a = 3x$

$\Rightarrow x = \frac{a}{3}$

Hence, option *(b)* is correct.

38. *(a)* We know that $|x| = \begin{cases} x, & x \ge 0 \\ -x, & x < 0 \end{cases}$

For domain, $|x| - x > 0$

Case 1: $x > 0 \Rightarrow x - x = 0$ (not possible)

Case 2: $x < 0$

$-x-x>0$
$\Rightarrow -2x>0$
$\Rightarrow x<0$
So, $x\in(-\infty,0)$
Hence, option *(a)* is correct.

39. *(a)* Let $I=\int e^{e^x}e^x dx$
Put $e^x=y \Rightarrow e^x dx=dy \Rightarrow dx=\frac{dy}{e^x}$
$\therefore I=\int e^y e^x\cdot\frac{dy}{e^x}=\int e^y dy=e^y+c$
$\Rightarrow I=e^{e^x}+c \qquad \{\because y=e^x\}$
Hence, option *(a)* is correct.

40. *(a)* Let $A=\begin{bmatrix}1&p&q\\0&x&0\\0&0&1\end{bmatrix}$ and $A^{-1}=\begin{bmatrix}1&-p&-q\\0&1&0\\0&0&1\end{bmatrix}$

Since, $AA^{-1}=I$
$\Rightarrow\begin{bmatrix}1&p&q\\0&x&0\\0&0&1\end{bmatrix}\begin{bmatrix}1&-p&-q\\0&1&0\\0&0&1\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$
$\Rightarrow\begin{bmatrix}1&0&0\\0&x&0\\0&0&1\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$
$\Rightarrow x=1$
Hence, option *(a)* is correct.

41. *(b)* Let $A(2,6),B(3,4),C(4,5)$ and $D(-2,5)$ are the given points. Let O be the origin, i.e. $O(0,0)$
$OA=\sqrt{(2-0)^2+(6-0)^2}=\sqrt{40}=2\sqrt{10}$ units
$OB=\sqrt{(3-0)^2+(4-0)^2}=\sqrt{25}=5$ units
$OC=\sqrt{(4-0)^2+(5-0)^2}=\sqrt{16+25}=\sqrt{41}$ units
$OD=\sqrt{(-2-0)^2+(5-0)^2}=\sqrt{4+25}=\sqrt{29}$ units
So, $q=OB=5$ units
Hence, option *(b)* is correct.

42. *(c)* Let on the set or real numbers, R is a relation defined on xRy if and only if $3x+4y=5$
Consider, $3x+4y=5$
(I) Put $x=0$ and $y=1$, we get
$\text{LHS}=3(0)+4(1)=4\neq5(=\text{RHS})$
Hence, 0 is not related to 1.

(II) Now, put $x=1$ and $y=\frac{1}{2}$, we get
$\text{LHS}=3(1)+4\left(\frac{1}{2}\right)=5=5(=\text{RHS})$
Hence, 1 is related to $\frac{1}{2}$.

(III) Similarly, $\frac{2}{3}$ and $\frac{3}{4}$.
Hence, both statements II and III are correct.
Hence, option *(c)* is correct.

43. *(b)* Let $x=2+2^{1/3}+2^{2/3}$
$\Rightarrow x=2+2^{1/3}+\left(2^{1/3}\right)^2=2+2^{1/3}\left(1+2^{1/3}\right)$
$\Rightarrow x-2=2+2^{1/3}\left(1+2^{1/3}\right)$
On cubing both sides, we get
$x^3-8-6x^2+12x=2\left(1+2+3\cdot2^{1/3}+3\cdot2^{2/3}\right)$
$\Rightarrow x^3-6x^2+6x=14+6\cdot2^{1/3}+6\cdot2^{2/3}-6x$
$\Rightarrow x^3-6x^2+6x=14+6\cdot2^{1/3}+6\cdot2^{2/3}-6\left(2+2^{1/3}+2^{2/3}\right)$
$\Rightarrow x^3-6x^2+6x=2$
Hence, option *(b)* is correct.

44. *(b)* Given, $p=\sin(989^\circ)\cos(991^\circ)$
which can be written as:
$=\sin(1080^\circ-91^\circ)\cos(1080^\circ-89^\circ)$
$=-\sin91^\circ\cos89^\circ$
$=-\sin(91^\circ+1^\circ)\cos89^\circ$
$=-\cos1^\circ\cos89^\circ$

As $\cos 1^\circ$ and $\cos 89^\circ$ are positive.
Therefore, their product is also positive.
Thus, $p=\sin(989^\circ)\cos(991^\circ)$ is finite and negative.
Hence, option *(b)* is correct.

45. *(a)* Let $\int_1^2 \{K^2+(4-4K)x+4x^3\}dx \le 12$

$\Rightarrow K^2x+\frac{(4-4K)x^2}{2}+\frac{4x^4}{4}\Big|_1^2 \le 12$

$\Rightarrow [2K^2+(2-2K)(4)+16]-[K^2+(2-2K)+1]\le 12$

$\Rightarrow (2K^2+8-8K+16)-(K^2-2K+3)\le 12$

$\Rightarrow K^2-6K+21\le 12$

$\Rightarrow K^2-6K+9\le 0$

$\Rightarrow (K-3)^2\le 0$

$\Rightarrow K=3$

Hence, option (a) is correct.

46. *(a)* The minimum value of any modulus is 0
Hence, option *(a)* is correct.

47. *(d)* Let $I=\int \sin^3 x\cos x\,dx$

Put $t=\sin x \Rightarrow dt=\cos x dx$

$\therefore I=\int t^3 dt=\frac{t^4}{4}+c$

$\Rightarrow I=\frac{\sin^2 x}{4}+c=\frac{(1-\cos^2)^2}{4}+c$

Hence, option (d) is correct.

48. *(b)* Given equation is line is $\frac{x-1}{2}=\frac{y-2}{3}$

$\Rightarrow 3x-3=2y-4 \Rightarrow 3x-2y+1=0$

$\Rightarrow y=\frac{3}{2}x+\frac{1}{2}$

And, equation of second line is $2x+3y=5$

$\Rightarrow y=-\frac{2}{3}x+\frac{5}{3}$

$\therefore$ Slope of first line, $m_1=\frac{3}{2}$

And slope of second line, $m_2=-\frac{2}{3}$

Since, $m_1m_2=-1$

So, two lines are perpendicular to each other.
Hence, option *(b)* is correct.

49. *(d)* Let $M=$ set of men and R is a relation 'is son of' defined on M.
Reflexive: aRa
($\because a$ can not be a son of a)
Symmetric: $aRb \Rightarrow bRa$
which is not also possible.
($\because$ If a is a son of b then b can not be son of a)
Transitive: $aRb, bRc \Rightarrow aRc$
which is not possible.
Hence, option *(d)* is correct.

50. *(a)* Let a,b be the roots of $x^2+px+q=0$

So, $a+b=-p$ and $ab=q$ ....(i)

Let c, d be the roots of $x^2+lx+m=0$

So, $c+d=-l$ and $cd=m$ ....(ii)

Given that roots of both the equations are in the same ratio

So, $\frac{a}{b}=\frac{c}{d}$ ....(iii)

$\Rightarrow \frac{b}{a}=\frac{d}{c}$ ....(iv)

Adding equations (iii) and (iv), we get

$\Rightarrow \frac{a}{b}+\frac{b}{a}=\frac{c}{d}+\frac{d}{c}$

$\Rightarrow \frac{a^2+b^2}{ab}=\frac{c^2+d^2}{cd}$

$\Rightarrow \frac{a^2+b^2}{ab}+2=\frac{c^2+d^2}{cd}+2$

$\Rightarrow \frac{a^2+b^2+2ab}{ab}=\frac{c^2+d^2+2cd}{cd}$

This further gives,

$$\frac{(a+b)^2}{ab}=\frac{(c+d)^2}{cd}$$

$$\Rightarrow \frac{(-p)^2}{q}=\frac{(-l)^2}{m} \qquad [\text{from}(\text{i})\text{and}(\text{ii})]$$

$$\Rightarrow p^2m=l^2q$$

Hence, option *(a)* is correct.

51. *(b)* Let the common difference d

Then, sum of first 5 terms

$$\Rightarrow \frac{1}{2}\cdot 5(4+4\cdot d)=\frac{1}{4}\cdot\frac{1}{2}\cdot 5\{(2+5\cdot d)+4\cdot d\}$$

$$\Rightarrow 4\times 4(1+d)=4+14d$$

$$\therefore d=-6$$

$\therefore$ Sum of first ten terms

$$=\frac{1}{2}\times 10\{2\times+9(-6)\}=-250$$

Hence, option *(b)* is correct.

52. *(d)* Given equation, $z^3+2z^2+2z+1=0$

$$\Rightarrow z^3-z^2+z+z^2-z+1+2z^2+2z=0$$

$$\Rightarrow (z+1)(z^2-z+1)+2z(z+1)=0$$

$$\Rightarrow (z+1)(z^2-z+1+2z)=0$$

$$\Rightarrow (z+1)(z^2+z+1)=0$$

$$\Rightarrow z=-1,\ \omega,\ \omega^2$$

And, $z^{2017}+z^{2018}+1=\omega+\omega^2+1=0$

$\therefore$ Common roots are $\omega,\ \omega^2$

Hence, option *(d)* is correct.

53. *(a)* Total no. of letters in BANANA = 6

No. of repeated letter N = 2

No. of repeated letter A = 2

Therefore, no. of ways that can be formed by using the words "BANANA' is

$$=\frac{6!}{3!2!}=\frac{6\times5\times4\times3!}{3!\times 2!}=60$$

Number of ways in which two N comes together

$$=\frac{5!}{3!}=20$$

$\therefore$ Required number of ways $=60-20=40$

Hence, option *(a)* is correct.

54. *(c)* Two circles touch each other iff distance between two centres equal to the sum of radius of two circles.

$$\sqrt{a^2+b^2}=\sqrt{a^2-c}+\sqrt{b^2-c}$$

On squaring both sides, we get

$$a^2+b^2=a^2-c+b^2-c+2\sqrt{(a^2-c)(b^2-c)}$$

$$\Rightarrow c=\sqrt{(a^2-c)(b^2-c)}$$

Again, squaring both sides, we get

$$c^2=a^2b^2-a^2c-b^2c+c^2$$

$$\Rightarrow a^2b^2=(a^2+b^2)c$$

$$\Rightarrow \frac{1}{c}=\frac{1}{a^2}+\frac{1}{b^2}$$

Hence, option *(b)* is correct.

55. *(b)* Let $\cos A+\cos B=m$ ....(i)

And $\sin A+\sin B=n$ ....(ii)

Consider $\sin(A+B)=\dfrac{(m^2+n^2)\sin(A+B)}{(m^2+n^2)}$

$$= \frac{[2+2\cos(A-B)]\sin(A+B)}{2+2\cos(A-B)} \quad (\text{from (i) and (ii)})$$

$$= \frac{2\sin(A+B)+\sin(A+B+A-B)+\sin(A+B-A+B)}{1+1+2\cos(A-B)}$$

$$= \frac{2\sin(A+B)+\sin 2A+\sin 2B}{1+1+2\cos(A-B)}$$

$$= \frac{2(\cos A+\cos B)(\sin A+\sin B)}{\sin^2 A+\cos^2 A+\sin^2 B+\cos^2 B+2\cos A\cos B+2\sin A\sin B}$$

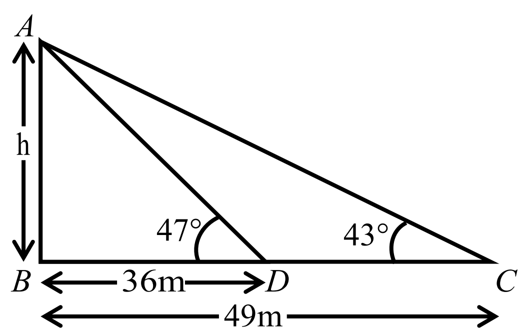
$$= \frac{2(\cos A+\cos B)(\sin A+\sin B)}{(\sin A+\sin B)^2+(\cos A+\cos B)^2}$$

$$= \frac{2mn}{m^2+n^2}$$

56. *(b)* Consider the following figure.

Let $AB = h$ (height of the tower)

$BD = 36\text{m}$, $BC = 49\text{m}$, $\angle D = 47°$ and $\angle C = 43°$



Now, in ΔABD

$$\tan 47° = \frac{h}{36} \quad ....(i)$$

And, in ΔABC

$$\tan 43° = \frac{h}{49}$$

$$\tan(90°-47°) = \frac{h}{49}$$

$$\therefore \cot 47° = \frac{h}{49} \quad ....(ii)$$

Multiplying equations (i) and (ii), we get

$$\tan 47° \cot 47° = \frac{h}{36} \times \frac{h}{49}$$

$$\Rightarrow 1 = \frac{h^2}{36\times 49}$$

$$\Rightarrow h = 6\times 7 = 42\text{m}$$

Hence, option *(b)* is correct.

57. *(c)* Let $B=\{2,3\}$ and $C=\{3,4\}$

Now, $B\cup C=\{2,3,4\}$

$\therefore A\times(B\cup C)=\{x,y\}\times\{2,3,4\}$

$A\times(B\cup C)=\{(x,2),(x,3),(x,4),(y,2),(y,3),(y,4)\}$

Hence, number of element in $A\times(B\cup C)=6$

Hence, option *(c)* is correct.

58. *(b)* Given, α and β are the roots of the equation $1+x+x^2=0$

Solving for x, we get

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-1\pm\sqrt{(1)^2-4(1)(1)}}{2(1)}$$

$$= \frac{-1\pm\sqrt{-3}}{2} = \frac{-1\pm\sqrt{3}i}{2}$$

$\therefore$ roots are $\frac{-1+\sqrt{3}i}{2}$ and $\frac{-1-\sqrt{3}i}{2}$

i.e. $\alpha=\omega$ and $\beta=\omega^2$

$$\therefore \begin{bmatrix}1 & \beta\\ \alpha & \alpha\end{bmatrix}\begin{bmatrix}\alpha & \beta\\ 1 & \beta\end{bmatrix} = \begin{bmatrix}\alpha+\beta & \beta+\beta^2\\ \alpha^2+\alpha & \alpha\beta+\alpha\beta\end{bmatrix}$$

$$= \begin{bmatrix}\omega+\omega^2 & \omega+\omega^4\\ \omega^2+\omega & \omega^3+\omega^3\end{bmatrix}$$

$$= \begin{bmatrix}-1 & \omega^2+\omega\\ -1 & 2\omega^3\end{bmatrix}$$

$$= \begin{bmatrix}-1 & -1\\ -1 & 2\end{bmatrix}$$

Hence, option *(b)* is correct.

59. *(b)* Let θ be the required angle.

$\therefore\ \theta = \frac{\text{arc}}{\text{radius}} = \frac{1}{3}$ radians

Now, $1\text{radian} = \frac{180^\circ}{\pi}$

$\therefore\ \frac{1}{3}\text{radian} = \frac{180^\circ}{\pi} \times \frac{1}{3} = \frac{60^\circ}{\pi}$

Hence, required angle $= \theta = \frac{60^\circ}{\pi}$

Hence, option *(b)* is correct.

60. *(c)* We have $\lim_{x\to\frac{\pi}{2}} \frac{\sin x}{x} = l$ and $\lim_{x\to\infty} \frac{\cos x}{x} = m$

$\lim_{x\to\frac{\pi}{2}} \frac{\sin x}{x} = \frac{1}{\frac{\pi}{2}} = \frac{2}{\pi}$; $\lim_{x\to\infty} \frac{\cos x}{x} = 0$

$\therefore\ l = \frac{2}{\pi},\ m = 0$

61. *(a)* Let $y = xe^x$

Differentiate both sides w.r.t. x

$\Rightarrow \frac{dy}{dx} = e^x + xe^x = e^x(1+x)$

Put $\frac{dy}{dx} = 0$

$\Rightarrow e^x(1+x) = 0 \Rightarrow x = -1$

Now, $\frac{d^2y}{dx^2} = e^x + e^x(1+x) = e^x(x+2)$

$\left(\frac{d^2y}{dx^2}\right)_{x=-1} = \frac{1}{e} + 0 > 0$

Thus, $y = xe^x$ is minimum function and $y_{\min} = -\frac{1}{e}$

Hence, option *(a)* is correct.

62. *(c)* We know, the perpendicular distance d from point (x_1, y_1) to line $ax+by+c=0$ is $\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

Let us find a point on line $2x+11y-5=0$

For $x=0,\ 2(0)+11y-5=0$

$\Rightarrow 11y-5=0$

$\Rightarrow y = \frac{5}{11}$

So, $(x,y) = \left(0, \frac{5}{11}\right)$

Let us find perpendicular distances of this point to the given lines.

For the straight line $24x+7y=20$, we have

$$d_1 = \frac{\left|24(0)+7\left(\frac{5}{11}\right)-20\right|}{\sqrt{(24)^2+(7)^2}} = \frac{\left|\frac{35}{11}-20\right|}{\sqrt{625}}$$

$$= \frac{\left|\frac{-185}{11}\right|}{\sqrt{625}} = \frac{185}{11\times 25} = \frac{37}{55}$$

For the straight line $4x-3y=2$, we have

$$d_1 = \frac{\left|4(0)-3\left(\frac{5}{11}\right)-2\right|}{\sqrt{(4)^2+(3)^2}} = \frac{\left|\frac{-15}{11}-2\right|}{\sqrt{25}} = \frac{37}{55}$$

$\therefore d_1 = d_2$

Hence, option *(c)* is correct.

63. *(c)* E is the universal set and $A = B \cup C$

Since, E is the universal set, $E - A = A'$

$\therefore E-(E-(E-(E-(E-A))))$

$= E-(E-(E-(E-A)))$

$= E-(E-(E-A))$

$= E-(E-A)$

$= E-A$

$= A'$

$= (B\cup C)'$

$= B' \cap C'$

Hence, option *(c)* is correct.

64. *(d)* Given α and β are the roots of the quadratic equation $x^2+\alpha x-\beta=0$

$\therefore$ Sum of roots, $\alpha+\beta=-\alpha$ ....(i)

And product of roots, $\alpha\beta=-\beta$ ....(ii)

From equation (ii), we get

$\Rightarrow \alpha\beta+\beta=0$

$\Rightarrow (\alpha+1)\beta=0$

$\Rightarrow \alpha=-1 \quad [\because \beta\neq 0]$

And from equation (i), we get

$\alpha+\beta=-\alpha$

$\Rightarrow 2\alpha+\beta=0$

$\Rightarrow 2(-1)+\beta=0$

$\Rightarrow \beta=2$

$\therefore -x^2+\alpha x+\beta=-x^2-x+2$

Greatest value $=-\frac{1+8}{-4}=\frac{9}{4}$

Hence, option *(d)* is correct.

65. *(d)* Given equation of ellipse E is $\frac{x^2}{9}+\frac{y^2}{4}=1$

$\Rightarrow \frac{4x^2+9y^2}{36}=1 \Rightarrow 4x^2+9y^2=36$

$\Rightarrow 4x^2+9y^2-36=0$ ....(i)

And C: equation of circle is $x^2+y^2=9$, which can be rewritten as

$x^2+y^2-9=0$ ....(ii)

For a point $P(1,2)$, we have

$4(1)^2+9(2)^2-36=40-36>0 \quad [\text{from(i)}]$

And $1^2+2^2-9=5-9<0 \quad [\text{from(ii)}]$

$\therefore$ Point P lies outside of E and inside of C.

Hence, option *(d)* is correct.

66. *(b)* Given, $\sin^{-1}\frac{2p}{1+p^2}+\cos^{-1}\frac{1-q^2}{1+q^2}=\tan^{-1}\frac{2x}{1-x^2}$

$\Rightarrow 2\tan^{-1}p-2\tan^{-1}q=2\tan^{-1}x$

$\Rightarrow \tan^{-1}p-\tan^{-1}q=\tan^{-1}x$

$\Rightarrow \tan^{-1}\frac{p-q}{1+pq}=\tan^{-1}x$

$\Rightarrow x=\frac{p-q}{1+pq}$

Hence, option *(b)* is correct.

67. *(b)* Since, 76rx

$\Rightarrow I_1=\int\frac{e^{2x}dx}{e^{2x}+1}$

$\therefore\ I_1+I_2=\int\frac{e^{2x}dx}{e^{2x}+1}+\int\frac{dx}{e^{2x}+1}$

$=\int\frac{e^{2x}+1}{e^{2x}+1}dx=\int 1dx=x+c$

Hence, option (b) is correct.

68. *(c)* Consider the differential equation

$$\left(\frac{d^4y}{dx^4}\right)^{3/5}-5\frac{d^3y}{dx^3}+6\frac{d^2y}{dx^2}-8\frac{dy}{dx}+5=0$$

$$\Rightarrow \left(\frac{d^4y}{dx^4}\right)^{\frac{3}{5}}=5\frac{d^3y}{dx^3}-6\frac{d^2y}{dx^2}+8\frac{dy}{dx}-5$$

$$\Rightarrow \left(\frac{d^4y}{dx^4}\right)^{3}=\left(5\frac{d^3y}{dx^3}-6\frac{d^2y}{dx^2}+8\frac{dy}{dx}-5\right)^5$$

So, higher order derivative $=4$, degree $=3$.

Hence, option *(c)* is correct.

69. *(c)* Given lines $3y+4x=1\Rightarrow 4x+3y-1=0$

$y=x+5\Rightarrow x-y+5=0$

$5y+bx=3\Rightarrow bx+5y-3=0$

Since, these lines are concurrent,

$$\begin{vmatrix}4 & 3 & -1\\ 1 & -1 & 5\\ b & 5 & -3\end{vmatrix}=0$$

$\Rightarrow 4(3-25)-3(-3-5b)-1(5+b)=0$

$\Rightarrow 4(-22)+9+15b-5-b=0$

$\Rightarrow -88+14b=0$

$\Rightarrow b=6$

Hence, option *(c)* is correct.

70. *(b)* Let $y = 2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cdots\infty}}} \Rightarrow y = 2 + \frac{1}{y}$

$\Rightarrow y^2 = 2y + 1 \Rightarrow y^2 - 2y - 1 = 0$

$\Rightarrow y = \frac{2 \pm \sqrt{(-2)^2 - 4(-1)}}{2}$

$\Rightarrow y = \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$

Since, $y > 2$, therefore $y = 1 + \sqrt{2}$

Hence, option *(b)* is correct.

71. *(b)* Given that $x^2 - 30x + 221 = 0$

$\Rightarrow x^2 - 30x + 17 \times 13 = 0$

$\Rightarrow (x-13)(x-17) = 0$

$\therefore p = 13$ and $q = 17$

$\therefore p^3 + q^3 = 2197 + 4913 = 7110$

72. *(c)* We have $\frac{\cot x + \operatorname{cosec} x - 1}{\cot x - \operatorname{cosec} x + 1}$

$= \frac{\cot x + \operatorname{cosec} x - (\operatorname{cosec}^2 x - \cot^2 x)}{\cot x - \operatorname{cosec} x + 1}$

$= \frac{\cot x + \operatorname{cosec} x - [(\operatorname{cosec} x - \cot x)(\operatorname{cosec} x + \cot x)]}{\cot x - \operatorname{cosec} x + 1}$

$= \frac{(\cot x + \operatorname{cosec} x)(1 + \cot x - \operatorname{cosec} x)}{\cot x - \operatorname{cosec} x + 1}$

$= \cot x + \operatorname{cosec} x$

$= \frac{\cos x}{\sin x} + \frac{1}{\sin x}$

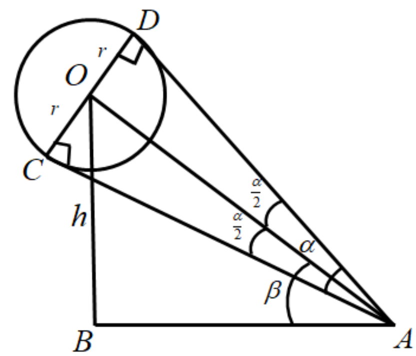
$= \frac{1 + \cos x}{\sin x}$

Hence, option *(c)* is correct.

73. *(a)* Let A be the position of eye.

Let O be the centre of spherical balloon.

Let h be the height of centre of balloon.



From figure, in ΔOAD

$\sin\frac{\alpha}{2} = \frac{OD}{OA} = \frac{r}{OA}$

$\Rightarrow OA = \frac{r}{\sin\frac{\alpha}{2}}$ ....(i)

In ΔOAB, we have

$\sin\beta = \frac{OB}{OA} = \frac{h}{OA}$

$\Rightarrow h = OA\sin\beta$

$\Rightarrow h = \frac{r\sin\beta}{\sin\left(\frac{\alpha}{2}\right)}$ [from (i)]

74. *(b)* **Statement 1**: Given $f(x) = \begin{cases} 2 - x & \text{for } 1 \le x \le 2 \\ 3x - x^2 & \text{for } x > 2 \end{cases}$

function defined in $1 \le x < \infty$

The function is polynomial, so it is continuous and differentiable in its domain $[1, \infty] - \{2\}$.

$\text{LHL} = f(2-0) = \lim_{h \to 0} f(2-h)$

$= \lim_{h \to 0} h = 0$

$\text{RHL} = f(2+0) = \lim_{h \to 0} f(2+h)$

$= 6 - 4 = 2$

And, $f(2) = 2 - 2 = 0$

$\therefore$ LHL $\ne$ RHL

Statement 2:

$$Rf'(1.5)=\lim_{h\to 0}\frac{f(1.5+h)-f(1.5)}{h}$$
$$=\lim_{h\to 0}\frac{(2-1.5+h)-f(1.5)}{h}$$
$$=\lim_{h\to 0}\frac{-h}{h}=-1$$
$$Lf'(1.5)=\lim_{h\to 0}\frac{f(1.5+h)-f(1.5)}{-h}$$
$$=\lim_{h\to 0}\frac{(2-1.5-h)-(2-1.5)}{-h}$$
$$=\lim_{h\to 0}\frac{h}{-h}=-1$$

Therefore, the function is differentiable at $x=1.5$

Hence, option *(b)* is correct.

75. *(d)* Since, $f'(x)=\begin{cases}-1 & \text{for } 1\le x\le 2\\ 3-2x & \text{for } x>2\end{cases}$

$f(x)$ at $x=3$

$f'(3)=3-2(3)=3-6=-3$

Hence, option (d) is correct.

76. *(a)* Consider $\int_{-1}^{1} x|x|dx$

$$=\int_{-1}^{0} x|x|dx+\int_{0}^{1} x|x|dx$$
$$=\int_{-1}^{0} x(-x)dx+\int_{0}^{1} x\cdot xdx$$
$$=-\int_{-1}^{0} x^2dx+\int_{0}^{1} x^2dx$$
$$=-\left[\frac{x^3}{3}\right]_{-1}^{0}+\left[\frac{x^3}{3}\right]_{0}^{1}$$
$$=-\left[0-\frac{(-1)^3}{3}\right]+\frac{1}{3}\left[(1)^3-(0)^3\right]$$
$$=-\frac{1}{3}+\frac{1}{3}$$
$$=0$$
$$\Rightarrow K^2-6K+9\le 0$$

Hence, option (a) is correct.

77. *(a)* Let $f(x)=x\sin x+\cos x+\frac{1}{2}\cos^2 x$

$$\Rightarrow f'(x)=x\cos x+\sin x-\sin x+\sin x\cos x$$
$$=\cos x(x-\sin x)>0 \text{ in } \left(0,\frac{\pi}{2}\right)$$

Hence, option (a) is correct.

78. *(d)* Since, $-4<x<-3$

$\Rightarrow -3<x+1<-2$

$\Rightarrow [x+1]=-3$

So, $y=-3$

$\therefore \frac{dy}{dx}=0$

Hence, option *(d)* is correct.

79. *(c)* Number of proper subset of any set on n elements $=2^n-1$

Here given set $=\{1,2,3,4\}$

Number of proper subset $=2^4-1=16-1=15$

Proper subset $=\{(1),(2),(3),(4),(1,2),(1,4)$

$(2,3),(2,4),(3,4),(1,2,3)$
$(1,2,4),(1,3,4),(2,3,4),\phi\}$

Now, A is superset of B, if B is a proper set of A, but B is not proper set of A.

i.e. $B \le A$ but $A \not\subset B$. Then $A \ge B$

So, superset of $\{3\}$ are $\{(3),(1,3),(2,3),(3,4),(1,2,3),(1,3,4),(2,3,4)\}$

Hence, number of superset of $\{3\}=7$

Hence, option *(c)* is correct.

80. *(c)* Given equation $2\sin x = 2k+1$

$\Rightarrow \sin x = k+\frac{1}{2}$ as $\sin x \in [-1,1]$

$\therefore -1 \le k+\frac{1}{2} \le 1 \Rightarrow -\frac{3}{2} \le k \le \frac{1}{2}$

Hence, number of integer values of k that satisfy, are 2 and that are (-1 and 0)

Hence, option *(c)* is correct.

81. *(c)* Number of ways $=\frac{8!8!}{4!6!}\times 10!$

$=8!\times 8!\times\frac{10\times 9\times 8\times 7\times 6!}{4!6!}$

$=8!\times 8!\times\frac{10\times 9\times 8\times 7}{4!}$

$=8!\times 8!\times\frac{10\times 9\times 8\times 7}{4\times 3\times 2\times 1}$

$=8!\times 8!\times 210$

Hence, option *(c)* is correct.

82. *(c)* Length of minor axis $=2b$ and latus rectum $=\frac{2b^2}{a}$

According to the given condition, $\frac{2b^2}{a}=b$

$\Rightarrow 2b=a$

Now, $e=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{b^2}{4b^2}}=\sqrt{\frac{3}{4}}=\frac{\sqrt{3}}{2}$

83. *(d)* $\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$

We have, $\cos\frac{7\pi}{8}=\cos\left(\pi-\frac{\pi}{8}\right)=-\cos\frac{\pi}{8}$

And $\cos\frac{5\pi}{8}=\cos\left(\cos\pi-\frac{3\pi}{8}\right)=-\cos\frac{3\pi}{8}$

$\therefore =\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1-\cos\frac{\pi}{8}\right)\left(1-\cos\frac{3\pi}{8}\right)$

$=\left(1-\cos^2\frac{\pi}{8}\right)\left(1-\cos^2\frac{3\pi}{8}\right)$

$=\sin^2\frac{\pi}{8}\cdot\sin^2\frac{3\pi}{8}$

$=\frac{1}{4}\left[2\sin^2\frac{\pi}{8}\cdot 2\sin^2\frac{3\pi}{8}\right]$

$=\frac{1}{4}\left[\left(1-\cos\frac{\pi}{4}\right)\left(1-\cos\frac{3\pi}{4}\right)\right]$ $\left\{\because 1-\cos\theta=2\sin^2\frac{\theta}{2}\right\}$

$=\frac{1}{4}\left[\left(1-\frac{1}{\sqrt{2}}\right)\left(1+\frac{1}{\sqrt{2}}\right)\right]=\frac{1}{8}$

Hence, option *(d)* is correct.

84. *(d)* Let $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\Rightarrow |A|=4-6=-2$

$\therefore A^{-1}=-\frac{1}{2}\begin{bmatrix}4 & -2\\ -3 & 1\end{bmatrix}$

$\Rightarrow [b_{ij}]=-\frac{1}{2}\begin{bmatrix}4 & -2\\ -3 & 1\end{bmatrix}=\begin{bmatrix}-2 & 1\\ \frac{3}{2} & -\frac{1}{2}\end{bmatrix}$

$\Rightarrow [b_{ij}]=\frac{1}{2}$

Hence, option *(d)* is correct.

85. *(c)* Given set $S=\{1,2,3,\ldots\}$

For $xRy, \log_a x > \log_a y \Rightarrow x > y$

As xRx, $\log_a x > \log_a x$ is not valid.

Hence, relation is not reflexive.

For $xRy, \log_a x > \log_a y \Rightarrow x > y$

For $yRx, \log_a y > \log_a x \Rightarrow y > x$

This is also not valid. Hence, relation is not symmetric also.

For $xRy, \log_a x > \log_a y \Rightarrow x > y$

For $yRz, \log_a y > \log_a z \Rightarrow y > z$

So, $xRz, \log_a x > \log_a z \Rightarrow x > z$

This is a valid relation. Hence, relation is only transitive.

Hence, option *(c)* is correct.

86. *(b)* Let $I = \int_0^a f(x) g(x) dx$

$$I = \int_0^a f(a-x) g(a-x) dx$$

$$I = \int_0^a f(x) \cdot [2 - g(x)] dx \qquad \{\because f(a-x) = f(x)\}$$

$$I = \int_0^a 2f(x) dx - \int_0^a f(x) \cdot g(x) dx$$

$$I = 2\int_0^a f(x) dx - I$$

$$\Rightarrow 2I = 2\int_0^a f(x) dx$$

$$\therefore\ I = \int_0^a f(x) dx$$

Hence, option *(b)* is correct.

87. *(a)* For 2×2 matrix,

$$|A| = |\text{adj}A| = (ab - 0) = ab$$

$$\therefore\ A^{-1} = \frac{\text{adj}\, A}{|A|} = \frac{1}{ab}\begin{pmatrix} a & 0 \\ -1 & b \end{pmatrix}$$

$$\left|A^{-1}\right| = \frac{1}{ab}(ab) = 1$$

Hence, option *(a)* is correct.

88. *(a)* Let $I = \int_0^\pi e^x \sin x\, dx$

$$I = \left(\sin x \cdot e^x\right)_0^\pi - \int_0^\pi \cos x \cdot e^x dx$$

$$I = \left(e^\pi \sin \pi - e^0 \sin 0\right) - \left\{\left[\cos x \cdot e^x\right]_0^\pi - \int_0^\pi \sin x \cdot e^x dx\right\}$$

$$I = 0 - \left\{\left[e^\pi \cos \pi - e^0 \cos 0\right] - I\right\}$$

$$I = -\left[-e^\pi - 1\right] - I$$

$$\Rightarrow 2I = e^\pi + 1 \Rightarrow I = \frac{e^\pi + 1}{2}$$

Hence, option *(a)* is correct.

89. *(b)* Given direction ratios are $(2, -1, 2)$ and $(x, 3, 5)$

We know that the angle between the lines whose direction ratios are (a_1, b_1, c_1) and (a_2, b_2, c_2) is

$$\cos\theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$\Rightarrow \cos\frac{\pi}{4} = \frac{2x - 3 + 10}{\sqrt{4+1+4}\sqrt{x^2 + 9 + 25}} = \frac{2x + 7}{\sqrt{9} \cdot \sqrt{x^2 + 34}}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{2x + 7}{3\sqrt{x^2 + 34}} \Rightarrow 2x + 7 = 3\sqrt{\frac{x^2 + 34}{2}}$$

$$\Rightarrow 4x^2 + 49 + 28x = \frac{9\left(x^2 + 34\right)}{2} \qquad (\text{squaring on both sides})$$

$$\Rightarrow 2\left(4x^2 + 49 + 28x\right) = 9x^2 + 306$$

$$\Rightarrow 8x^2 + 98 + 56x = 9x^2 + 306$$

$$\Rightarrow x^2 - 56x + 208 = 0$$

$$\therefore\ x = \frac{56 \pm \sqrt{3136 - 812}}{2} = \frac{56 \pm 48}{2} = 28 \pm 24 = 4, 52$$

Therefore, the smaller value of $x = 4$

Hence, option *(b)* is correct.

90. *(d)* Let $\vec{a} = \hat{i} - \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + 3\hat{j} + 2\hat{k}$, $\vec{c} = \hat{i} - m\hat{j} + n\hat{k}$

Given that $\vec{a}, \vec{b}, \vec{c}$ are coplanar.

So, $\begin{vmatrix} 1 & -1 & 1 \\ 2 & 3 & 2 \\ 1 & m & n \end{vmatrix} = 0$

Applying $c_1 \to c_1 + c_2,\ c_3 \to c_3 + c_2$

$\Rightarrow \begin{vmatrix} 0 & 1 & 0 \\ 5 & 3 & 5 \\ 1+m & m & m+n \end{vmatrix} = 0$

$\Rightarrow 0 + 1(5m+5n) - (5+5m) = 0$

$\Rightarrow 5m + 5n - 5 - 5m = 0$

$\Rightarrow 5n = 5 \Rightarrow n = 1$

$\therefore |\vec{c}| = 6 \Rightarrow \sqrt{1+m^2+n^2} = \sqrt{6}$

$\Rightarrow 1+m^2+n^2 = 6 \Rightarrow 2+m^2 = 6 \Rightarrow m^2 = 4 \Rightarrow m = \pm 2$

Hence, option *(d)* is correct.

91. *(c)* We have $A = \{3,6,9,\ldots 750\}$ and $n(A) = 250$

Also $B = \{2,4,6,\ldots 400\}$ and $n(B) = 200$

$\therefore A \cap B = \{6,12,18,\ldots 396\}$ and $n(A \cap B) = 66$

$\therefore n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 250 + 200 - 66 = 384$

Hence, option *(c)* is correct.

92. *(a)* Given, $\dfrac{\sin(x+y)}{\sin(x-y)} = \dfrac{a+b}{a-b}$

Applying componendo and dividend, we get

$\dfrac{\sin(x+y)+\sin(x-y)}{\sin(x+y)-\sin(x-y)} = \dfrac{a+b+a-b}{a+b-a+b}$

$\Rightarrow \dfrac{2\sin\left(\dfrac{x+y+x-y}{2}\right)\cos\left(\dfrac{x+y-x+y}{2}\right)}{2\cos\left(\dfrac{x+y+x-y}{2}\right)\sin\left(\dfrac{x+y-x+y}{2}\right)} = \dfrac{2a}{2b}$

$\Rightarrow \dfrac{\sin x \cos y}{\cos x \sin y} = \dfrac{a}{b}$

$\Rightarrow \dfrac{\tan x}{\tan y} = \dfrac{a}{b}$

Hence, option *(a)* is correct.

93. *(b)* Let $f(x) = x^3 + x^2 + kx$

$\Rightarrow f'(x) = 3x^2 + 2x + k$

For no local extremum, $D < 0$

$\Rightarrow (2)^2 - 4(3)(k) < 0$

$\Rightarrow 4 - 12k < 0$

$\Rightarrow 12k > 4$

$\Rightarrow 3k > 1$

Hence, option (b) is correct.

94. *(d)* Let $\Delta = \begin{vmatrix} x+1 & x+2 & x+4 \\ x+3 & x+5 & x+8 \\ x+7 & x+10 & x+14 \end{vmatrix}$

By applying $C_2 \to C_2 - C_1,\ C_3 \to C_3 - C_1$

$= \begin{vmatrix} x+1 & 1 & 3 \\ x+3 & 2 & 5 \\ x+7 & 3 & 7 \end{vmatrix}$

By applying $R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_2$ and, we get

$= \begin{vmatrix} x+1 & 1 & 3 \\ 2 & 1 & 2 \\ 4 & 1 & 2 \end{vmatrix}$

$= (x+1)(0) - 1(4-8) + 3(2-4)$

$= 4 - 6$

$= -2$

Hence, option *(d)* is correct.

95. *(a)* Consider $\int (\ln x)^{-1}\,dx - \int (\ln x)^{-2}\,dx$

$= \int \left[\dfrac{1}{\ln x} - \dfrac{1}{(\ln x)^2}\right] dx$

Put $\ln x = t \Rightarrow x = e^t \Rightarrow dx = e^t\,dt$

$$\int\left[\frac{1}{\ln x}-\frac{1}{(\ln x)^2}\right]dx=\int\left[\frac{1}{t}-\frac{1}{t^2}\right]e^t\,dt$$

$$\therefore \quad =\int e^t\left(\frac{1}{t}-\frac{1}{t^2}\right)dt$$

$$=\frac{e^t}{t}+c=\frac{x}{\ln x}+c$$

$$=x(\ln x)^{-1}+c$$

Hence, option *(a)* is correct.

96. *(b)* Given $\vec{a}=\hat{i}-2\hat{j}+5\hat{k}$ and $\vec{b}=2\hat{i}+\hat{j}-3\hat{k}$

$$\therefore \vec{b}-\vec{a}=\left(2\hat{i}+\hat{j}-3\hat{k}\right)-\left(\hat{i}-2\hat{j}+5\hat{k}\right)=\hat{i}+3\hat{j}-8\hat{k}$$

And $3\vec{a}+\vec{b}=\left(3\hat{i}-6\hat{j}+15\hat{k}\right)+\left(2\hat{i}+\hat{j}-3\hat{k}\right)$

$$=5\hat{i}-5\hat{j}+12\hat{k}$$

Therefore,

$$\left(\vec{b}-\vec{a}\right)\cdot\left(3\vec{a}+\vec{b}\right)=\left(\hat{i}+3\hat{j}-8\hat{k}\right)\cdot\left(5\hat{i}-5\hat{j}+12\hat{k}\right)$$

$$=5-15-96$$

$$=-106$$

Hence, option *(b)* is correct.

97. *(d)* Let $I=\int_0^a \frac{f(a-x)}{f(x)+f(a-x)}dx$

$$=\int_0^a \frac{f(a+0-a+x)}{f(a+0-x)+f(a+0-a+x)}dx$$

$$=\int_0^a \frac{f(x)}{f(a-x)+f(x)}dx$$

$$2I=\int_0^a \frac{f(a-x)+f(x)}{f(a-x)+f(x)}dx=[x]_0^a$$

$$\therefore I=\frac{a}{2}$$

Hence, option (d) is correct.

98. *(a)* (i) Let $\cos\theta+\sec\theta=\frac{3}{2}$

$$\Rightarrow \frac{1+\cos^2\theta}{\cos\theta}=\frac{3}{2}$$

$$\Rightarrow 2\cos^2\theta-3\cos\theta+2=0$$

Here, discriminant is negative.

$\therefore$ No real roots.

$\therefore$ No solution.

(ii) In second quadrant both $\tan\theta$ and $\cot\theta$ are negative.

$\therefore$ In second quadrant value is less than 2.

So, only statement (i) is true.

Hence, option *(a)* is correct.

99. *(b)* We know $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$$\Rightarrow 0.8+0.9-p\le 1$$

$$\Rightarrow \quad 1.7-p\le 1$$

$$\Rightarrow \quad 0.7\le p$$

Now, $P(A)<P(B)$

$\therefore\ P(A\cap B)\le P(A)\ \Rightarrow\ p\le 0.8$

Thus, $0.7\le p\le 0.8$

Hence, option *(b)* is correct.

100. *(c)* Given, matrix A and x rows and $x+5$ columns.

Matrix B has y rows and $11-y$ columns.

Also, given *AB* and *BA* exist.

If *AB* exists, then the number of rows In A must be equal to number of columns in *B*.

i.e., $x=11-y$ ....(i)

If *BA* exists, then the number of rows in B must be equal to number of rows in A .

i.e., $x+5=y$

$\Rightarrow 11-y+5=y$ $\left[\text{from}(\text{i})\right]$

$\Rightarrow 2y=16$

$\Rightarrow y=8$

$\therefore$ from (i), we get

$x=11-8=3$

So, $x=3$ and $y=8$

101. *(c)* Slope of line of regression of Y and X, is $30°$.

So, $b_{yx} = \tan 30° = \frac{1}{\sqrt{3}}$ and for X and Y it is $60°$.

Hence, $\frac{1}{b_{xy}} = \tan 60° = \sqrt{3}$

$b_{yx} = \frac{1}{\sqrt{3}}$ and $b_{xy} = \frac{1}{\sqrt{3}}$

$r(x, y) = r^2 = b_{yx} \cdot b_{xy} = \frac{1}{3}$

So, $r = \pm\frac{1}{\sqrt{3}}$

Since, b_{yx} and b_{xy} are both positive, $r = \frac{1}{\sqrt{3}}$

Hence, option *(c)* is correct.

102. *(b)* Mean of the numbers $= \frac{\frac{n(n+1)}{2}}{n} = \frac{n+1}{2}$

Therefore,

$$\text{Variance} = \frac{\left(1-\frac{n+1}{2}\right)^2 + \left(2-\frac{n+1}{2}\right)^2 + \left(3-\frac{n+1}{2}\right)^2 + \cdots}{n}$$

$$2 = \frac{(1^2+2^2+3^2+\cdots) + n\left(\frac{n+1}{2}\right)^2 + 2\left(\frac{n+1}{2}\right)[1+2+3+\ldots]}{n}$$

$$2n = \frac{1}{6}n(n+1)(2n+1) + \frac{n(n+1)^2}{4} - 2\left(\frac{n+1}{2}\right)\left\{\frac{n(n+1)}{2}\right\}$$

$$2n = n(n+1)\left[\frac{2n+1}{6} + \frac{n+1}{4} - \frac{n+1}{2}\right]$$

$$2 = (n+1)\left[\frac{4n+2-3n-3}{12}\right]$$

$\Rightarrow 24 = (n+1)(n-1)$ $\Rightarrow n^2 - 1 = 24$

$\Rightarrow n^2 = 25$ $\Rightarrow n = 5$

Hence, option *(b)* is correct.

103. *(c)* Let $p = \text{Magnitude of } 3\hat{i} - 2\hat{j} = \sqrt{9+4} = \sqrt{13}$

$q = \text{Magnitude of } 2\hat{i} + 2\hat{j} + \hat{k} = \sqrt{4+4+1} = 3$

$r = \text{Magnitude of } 4\hat{i} - \hat{j} + \hat{k} = \sqrt{16+1+1} = 3\sqrt{2}$

$s = \text{Magnitude of } 2\hat{i} + 2\hat{j} + 3\hat{k} = \sqrt{4+4+9} = \sqrt{17}$

$\therefore r > s > p > q$

Hence, option *(c)* is correct.

104. *(b)* Let $I = \int_{-2}^{-1} \frac{x}{|x|}dx$

$= \int_{-2}^{-1} \frac{x}{-x}dx \qquad [\because |x| = -x, x < 0]$

$= -\int_{-2}^{-1} 1dx = -[x]_{-2}^{-1}$

$= -[(-1)-(-2)]$

$= -1$

Hence, option *(b)* is correct.

105. *(d)* Given, $7\sin\theta + 24\cos\theta = 25$

$\Rightarrow \frac{7}{25}\sin\theta + \frac{24}{25}\cos\theta = 1$

Again, $(7)^2 + (24)^2 = (25)^2$

$\therefore$ Let $\frac{7}{25} = \cos\alpha$ and $\frac{24}{25} = \sin\alpha$

Then,

$\sin\theta + \cos\theta = \sin\left(\frac{\pi}{2} - \alpha\right) + \cos\left(\frac{\pi}{2} - \alpha\right)$

$= \cos\alpha + \sin\alpha$

$= \frac{7}{25} + \frac{24}{25} = \frac{31}{25}$

Hence, option *(d)* is correct.

106. *(c)* Prime number between 1 to 10 are 2, 3, 5, 7

Now, number of ways of selecting 2 prime number out of 4 prime number $= {}^4C_2 = 6$

Number of ways of selecting 2 numbers out of 10 numbers $= {}^{10}C_2 = 45$

$\therefore$ Required probability $= \frac{6}{45} = \frac{2}{15}$

Hence, option *(c)* is correct.

107. *(d)* Given lines of regression are $3X+Y-12=0$ and $X+2Y-14=0$

Since, lines of regression pass through $(\overline{X},\overline{Y})$

Therefore, $(\overline{X},\overline{Y})$ satisfies the given equations.

$\therefore 3\overline{X}+\overline{Y}-12=0 \quad ....(\text{i})$

And $\overline{X}+2\overline{Y}-14=0 \quad ....(\text{ii})$

Multiply equation (ii) by 3 and subtract from (i), we get

$(3\overline{X}+\overline{Y}-12)-(3\overline{X}+6\overline{Y}-42)=0$

$\Rightarrow 5\overline{Y}+30=0 \Rightarrow \overline{Y}=6$

Thus, $\overline{X}=14-2\overline{Y}=14-12=2$

Hence, mean, $X=2$

Hence, option *(d)* is correct.

108. *(a)* Let $A=\begin{bmatrix}1 & a\\ 0 & 1\end{bmatrix}$, then

$\Rightarrow A^2=\begin{bmatrix}1 & 2a\\ 0 & 1\end{bmatrix} \Rightarrow A^3=\begin{bmatrix}1 & 3a\\ 0 & 1\end{bmatrix}$

$\Rightarrow A^n=\begin{bmatrix}1 & na\\ 0 & 1\end{bmatrix}$

Now, $A^{100}-A^{50}-2A^{25}$

$=\begin{bmatrix}1 & 100a\\ 0 & 1\end{bmatrix}-\begin{bmatrix}1 & 50a\\ 0 & 1\end{bmatrix}-2\begin{bmatrix}1 & 25a\\ 0 & 1\end{bmatrix}$

$=\begin{bmatrix}-2 & 0\\ 0 & -2\end{bmatrix}=-2\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$=-2I$

Hence, option *(a)* is correct.

109. *(a)* Given table can be rewritten as

x	y	x^2	y^2	xy
5	2	25	4	10
7	4	49	16	28
8	3	64	9	24
4	2	16	4	8
6	4	36	16	24
$\sum x=30$	$\sum y=15$	$\sum x^2=190$	$\sum y^2=49$	$\sum xy=94$

Here, $\bar{x}=\frac{30}{5}=6$ and $\bar{y}=\frac{15}{5}=3$

$\therefore b_{yx}=\frac{n\sum xy-(\sum x)(\sum y)}{n\sum x^2-(\sum x)^2}$

$=\frac{5\times 94-30\times 15}{5\times 190-(30)^2}=\frac{2}{5}=0.4$

Hence, line of regression is

$y-3=0.4(x-6) \Rightarrow y=0.4x+0.6$

Hence, option *(a)* is correct.

110. *(a)* Given natural numbers are 1, 2, 3, 4,….,16

This is an A.P. with first term $=1$

And common difference $=1$ and $n=16$

$\therefore$ By using sum of first 16 natural numbers

i.e., $S_n=\frac{n}{2}[2a+(n-1)d]$

we have

$S_{16}=\frac{16}{2}[2(1)+(16-1)(1)]=8(17)=136$

$\therefore \text{AM}=\frac{136}{16}=\frac{17}{2}$

Hence, option *(a)* is correct.

111. *(c)* Given $P(A\cup B)=\frac{5}{6}, P(A\cap B)=\frac{1}{3}, P(A')=\frac{1}{2}$

Since, $P(A)=1-P(A')=1-\frac{1}{2}=\frac{1}{2}$

$\therefore P(A\cup B)=P(A)+P(B)-P5(A\cap B)$

$\frac{5}{6}=\frac{1}{2}+P(B)-\frac{1}{3}$

(a) $P(B)=\frac{5}{6}-\frac{1}{2}+\frac{1}{3}=\frac{4}{6}=\frac{2}{3}$

(b) $P(A\cap B)=\frac{1}{3}=\frac{1}{2}\cdot\frac{2}{3}=P(A)\cdot P(B)$

$\therefore P(A\cap B)=P(A)\cdot P(B)$

(c) $P(A\cup B)=\frac{5}{6}$

$P(A)+P(B)=\frac{1}{2}+\frac{2}{3}=\frac{7}{6}$

$\therefore P(A\cup B)<P(A)+P(B)$

Hence, option *(c)* is not correct.

(d) $P(A'\cap B')=P(A')\cdot P(B')$

$1-P(A\cup B)=1-\frac{5}{6}=\frac{1}{6}$

And, $P(A')\cdot P(B')=(1-P(A))\cdot(1-P(B))$

$=\left(1-\frac{1}{2}\right)\left(1-\frac{2}{3}\right)=\frac{1}{2}\cdot\frac{1}{3}=\frac{1}{6}$

Hence, $P(A'\cap B')=P(A')\cdot P(B')$

Hence, option *(c)* is correct.

112. *(d)* Given that mean of 10 observations is odd.

$\Rightarrow \frac{\sum_{i=1}^{10} x_i}{10}=5$

According to the question,

$\frac{\sum_{i=1}^{10} 3(x_i+2)}{10}=$ New mean

$\Rightarrow \frac{3\sum_{i=1}^{10} x_i}{10}+\frac{3\times 2\times 10}{10}=$ New mean

$\Rightarrow 3\times 5+6=$ New mean

$\Rightarrow$ New mean $=21$

Hence, option *(d)* is correct.

113. *(a)* Probability for Husband's selection $P(H)=\frac{1}{7}$

Probability when Husband is not selected

$P(H')=1-\frac{1}{7}=\frac{6}{7}$

Probability for wife's selection $P(W)=\frac{1}{5}$

Probability when wife is not selected

$P(W')=1-\frac{1}{5}=\frac{4}{5}$

As both are independent event.

So, probability for at least one of them will be selected.

$=P(H)P(W')+P(H')P(W)$

$=\frac{1}{7}\times\frac{4}{5}+\frac{6}{7}\times\frac{1}{5}=\frac{11}{35}$

Hence, option *(a)* is correct.

114. *(c)* The series is 1, 2, 3,……,20

$\text{Variance}(\sigma)=\frac{\sum x^2}{n}-\sum(\bar{x})^2$

$=\frac{n(n+1)(2n+1)}{6}-\left(\frac{n(n+1)}{2n}\right)^2=\frac{(n+1)}{12}(n-1)$

$=\frac{n^2-1}{12}=\frac{(20)^2-1}{12}=\frac{399}{12}=33.25$

$\because$ Numbers are multiplied by 3,

$\text{Variance}(\sigma)=33.25\times 9=299.25$

Hence, option *(c)* is correct.

115. *(b)* We know that three lines $a_1x+b_1y+c_1=0$, $a_2x+b_2y+c_2=0$ and $a_3x+b_3y+c_3=0$ are concurrence if

$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}=0$

$\therefore \begin{vmatrix} 2k^2 & 3 & -1 \\ 7 & -2 & 3 \\ 6k & 1 & 1 \end{vmatrix}=0$

$\Rightarrow 2k^2(-2-3)-7(3+1)+6k(9-2)=0$

$\Rightarrow 5k^2-21k+14=0$

$\therefore k=\frac{21\pm\sqrt{161}}{10}$

Hence, option *(b)* is correct.

116. *(c)* We know that $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$

$\Rightarrow \cos^2\alpha+\cos^2\beta=1-\cos^2\gamma$

$\Rightarrow \cos^2\alpha+\cos^2\beta=\sin^2\gamma$

$\therefore$ Statement 1 is correct.

Now, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow 1 - \sin^2\alpha + 1 - \sin^2\beta + 1 - \sin^2\gamma = 1$

$\Rightarrow 3 - (\sin^2\alpha + \sin^2\beta + \sin^2\gamma) = 1$

$\Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

$\therefore$ Statement 3 is correct.

117. *(d)* The probability that exactly 3 out of 6 workers suffer from a disease is

$$P(X = 3) = {}^6C_3\left(\frac{1}{3}\right)^3\left(\frac{1}{3}\right)^3$$

$$= \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} \times \frac{8}{3^6} = \frac{160}{729}$$

118. *(b)* The probability that no one out of 6 workers suffers from a disease is

$$P(X = 0) = {}^6C_0\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^6$$

$$= \frac{2^6}{3^6} = \frac{64}{729}$$

119. *(b)* The probability that at least one out of 6 workers suffer from a disease is

$$P(X \geq 1) = 1 - P(X = 0)$$

$$= 1 - \frac{64}{729} = \frac{665}{729}$$

120. *(d)* Coefficient of Variation (C.V.)

$$= \frac{\text{standard deviation}(\sigma)}{\text{mean}(\mu)} \times 100$$

Now, standard deviation $(\sigma) = \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}$

$$= \sqrt{\frac{200}{10} - \left(\frac{20}{10}\right)^2} = 4$$

And, Mean $(\mu) = \frac{\sum x_i}{n} = \frac{20}{10} = 2$

$\therefore$ Co-efficient of variation (C.V.) $= \frac{4}{2} \times 100 = 200$