**HINTS & SOLUTION**

**1.** *(d)* Let $A$ denotes the collection of all complex numbers whose square is a negative real number, then square of a complex number is a negative real number only if it has no real part and has only imaginary part.
Hence, $A=\{iy \mid y\in R\}$.

**2.** *(b)* Since, the roots of $x^2-2mx+m^2-1=0$ lies between $-2$ and 4 i.e., $b^2-4ac\geq 0$ and $-2<\frac{-b}{2a}<4$

$\therefore (2m)^2-4(m^2-1)\geq 0$ ....(i)

and $-2<\frac{2m}{2}<4$

$\Rightarrow -2<m<4$

From equation (i),

$4m^2-4m^2+4\geq 0$

$\Rightarrow m\in R$

and $f(-2)>0$, also $f(4)>0$

$4+4m+m^2-1>0,\ 16-8m+m^2-1>0$

$\Rightarrow m^2+4m+3>0,\ m^2-8m+15>0$

$\Rightarrow (m+1)(m+3)>0,\ (m-3)(m-5)>0$

$\Rightarrow -3<m<-1$ and $3<m<5$

Thus, the interval in which it lies is $-1\leq m\leq 5$.

**3.** *(a)* Let first term and common difference of an AP are $a$ and $d$ respectively.

Its $p^{\text{th}}$ term $=a+(p-1)d=q$ ....(i)

and $q^{\text{th}}$ term $=a+(q-1)d=p$ ....(ii)

Solving Eqs. (i) and (ii), we find
$a=p+q-1,\ d=-1$

**4.** *(a)* Let $\sqrt{\frac{1}{2}-i\frac{\sqrt{3}}{2}}=x+iy$

Squaring both sides,

$\Rightarrow \frac{1}{2}-i\frac{\sqrt{3}}{2}=x^2-y^2-2ixy$

Comparing real and imaginary parts, we get

$x^2-y^2=\frac{1}{2}$ ...(i)

and $2xy=\frac{\sqrt{3}}{2}$ ...(ii)

$(x^2+y^2)^2=(x^2-y^2)^2+4x^2y^2$

$\Rightarrow (x^2+y^2)^2=\frac{1}{4}+\frac{3}{4}=1$

$\Rightarrow x^2+y^2=1$ ...(iii)

On solving (i) and (ii), we get

$x^2=\frac{3}{4},\ y^2=\frac{1}{4}$

$\Rightarrow x=\pm\frac{\sqrt{3}}{2},\ y=\pm\frac{1}{2}$

Thus, $\sqrt{\frac{1}{2}-i\frac{\sqrt{3}}{2}}=\pm\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)$

**5.** *(d)* There are total 8 letters in the word BASEBALL, in which we have 2B's, 2A's and 2L's.

$\therefore$ Required no. of permutations $=\frac{8!}{2!\times 2!\times 2!}$

$=\frac{8\times 7\times 6\times 5\times 4\times 3\times 2\times 1}{8}=5040$.

**6.** *(a)* Given equation is $px^2+qx+r=p(x-\alpha)(x-\beta)$

$\Rightarrow px^2+qx+r=px^2-p(\alpha+\beta)x+\alpha\beta p$

$\Rightarrow \alpha\beta p=r$ and $q=-(\alpha+\beta)p$ ....(i)

Also given that $p^3+pq+r=0$

Putting the value of $q$ and $r$ from (i), we get

$\Rightarrow p^3 - p^2(\alpha+\beta) + \alpha\beta p = 0$

$\Rightarrow p^2 - p(\alpha+\beta) + \alpha\beta = 0$

$\Rightarrow (p-\alpha)(p-\beta) = 0$

$\Rightarrow \alpha = \beta = p$

**7.** *(d)* Given that $F(n) =$ set of all divisors of $n$ except 1.

$\therefore F(20) = \{2,4,5,10,20\}$ and $F(16) = \{2,4,8,16\}$

$\therefore F(20) \cap F(16) = \{2,4,5,10,20\} \cap \{2,4,8,16\} = \{2,4\}$

Also, $\{F(20) \cap F(16)\} \subseteq F(y)$

So, least value of $y = 2$

**8.** *(a)* We have $(1.02)^8 = (1+0.02)^8$

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots$

Here, $n = 8$ and $x = 0.02$

$(1+0.02)^8 = 1 + 8\times 0.02 + \frac{8\times 7}{2!}(0.02)^2 + \frac{8\times 7\times 6}{3!}(0.02)^3 +$

Neglecting higher terms,

$= 1 + 0.16 + 28\times 0.0004 + 56\times 0.000008$

$\approx 1 + 0.16 + 0.0112 = 1.171$

**9.** *(c)* Given that $a^x = b,\ b^y = c,\ c^z = a$

$\Rightarrow c^z = b^{yz} = a \Rightarrow a^{xyz} = a$

$\Rightarrow xyz = 1$

Now, $\frac{1}{(xy+yz+zx)}\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$

$= \frac{1}{(xy+yz+zx)}\left(\frac{xy+yz+zx}{xyz}\right) = \frac{1}{xyz} = 1$

**10.** *(b)* We have $0.\bar{2} + 0.2\bar{3}$

$= \frac{2}{9} + \frac{23}{99} = \frac{22+23}{99} = \frac{45}{99} = 0.\overline{45}$

**11.** *(d)* Given the extremities of a diameter of a circle as $(0,0)$ and $\left(a^3, \frac{1}{a^3}\right)$, equation of circle is

$(x-0)(x-a^3) + (y-0)\left(y - \frac{1}{a^3}\right) = 0$

$\Rightarrow x^2 - xa^3 + y^2 - \frac{y}{a^3} = 0$

$\Rightarrow x^2 + y^2 - xa^3 - \frac{y}{a^3} = 0$

Putting $x = \frac{1}{a}$ and $y = a$, the equation is satisfied.

Thus, the circle passes through the point $\left(\frac{1}{a}, a\right)$

**12.** *(c)* The equation of ellipse is given as:

$\frac{x^2}{a^2} + \frac{y^2}{7} = 1$

Eccentricity is given by $e = \sqrt{1 - \frac{7}{a^2}}$

$\therefore$ foci of ellipse are $(\pm ae, 0)$ i.e. $\left(\pm a\sqrt{1-\frac{7}{a^2}}, 0\right)$

Now, the equation of given hyperbola is

$\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25} \Rightarrow \frac{x^2}{\frac{144}{25}} - \frac{y^2}{\frac{81}{25}} = 1$

So, $a = \frac{12}{5}$ and $b = \frac{9}{5}$

$\therefore e' = \sqrt{1 + \frac{81/25}{144/25}} = \sqrt{\frac{144+81}{144}} = \sqrt{\frac{225}{144}} = \frac{15}{12}$

$\therefore$ Foci of hyperbola is $\left(\pm \frac{12}{5}\cdot\frac{15}{12}, 0\right)$ i.e. $(\pm 3, 0)$

Since, these foci coincides

$3 = a\sqrt{1-\frac{7}{a^2}} \Rightarrow \frac{3}{a} = \sqrt{1-\frac{7}{a^2}}$

$\Rightarrow \frac{9}{a^2} = 1 - \frac{7}{a^2}$

$\Rightarrow \frac{16}{a^2} = 1 \Rightarrow a = 4$

**13.** *(c)* Given that $\sin(\pi\cos x)=\cos(\pi\sin x)$

So, $\cos\left(\frac{\pi}{2}-\pi\cos x\right)=\cos(\pi\sin x)$

$\Rightarrow \frac{\pi}{2}-\pi\cos x=\pi\sin x$

$\Rightarrow \sin x+\cos x=\frac{1}{2}$

Squaring both sides, we get

$\sin^2 x+\cos^2 x+2\sin x\cos x=\frac{1}{4}$

$\Rightarrow \sin 2x=\frac{1}{4}-1=-\frac{3}{4}$

**14.** *(c)* Let E be the event of total of 12.

$$E=\begin{Bmatrix}(2,2,2,3,3),(2,2,3,3,2),(2,3,3,2,2)\\(3,3,2,2,2),(3,2,3,2,2),(3,2,2,3,2)\\(3,2,2,2,3),(2,3,2,3,2),(2,3,2,2,3)\\(2,2,3,2,3)\end{Bmatrix}$$

So, $n(E)=10$

Sample space contains a total of $2^5=32$ elements.

Hence, $n(S)=32$

So, $P(E)=\frac{n(E)}{n(S)}=\frac{10}{32}=\frac{5}{16}$

**15.** *(c)* Here, $f\left(x^2\right)+2=x^2+\frac{1}{x^2}+2$

$=\left(x+\frac{1}{x}\right)^2=\{f(x)\}^2$

And $f\left(x^3\right)+3f(x)=x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)$

$=\left(x+\frac{1}{x}\right)^3=\{f(x)\}^3$

Thus, both 1 and 2 are correct.

**16.** *(a)* Given equations are $y=\frac{8}{x^2+4}$ and $x+y=2$

Putting value of y from 1st equation into 2nd equation

$x+\frac{8}{x^2+4}=2$

$\Rightarrow x^3+4x+8=2x^2+8$

$\Rightarrow x^3-2x^2+4x=0$

$\Rightarrow x\left(x^2-2x+4\right)=0$

$\Rightarrow x=0$ [The other value of $x$ is not real]

**17.** *(d)* Since, Latus-rectum of an ellipse $=\frac{2b^2}{a}$

And minor axis $=2b$

$\therefore\ b=\frac{2b^2}{a}\Rightarrow a=2b$

Also, $e=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{b^2}{4b^2}}=\sqrt{\frac{3}{4}}=\frac{\sqrt{3}}{2}$

**18.** *(c)* We have $P(n,r)=2520$ and $C(n,r)=21$

${}^nP_r=\frac{n!}{(n-r)!}=2520$ ....(i)

${}^nC_r=\frac{n!}{r!(n-r)!}=21$ ....(ii)

Divide Eqs. (i) and (ii) we get

$\Rightarrow r!=\frac{2520}{21}=120$

$\Rightarrow r!=5!\Rightarrow r=5$

$\because {}^nC_r=21\Rightarrow {}^nC_5=21\Rightarrow n=7$

$\therefore C(n+1,r+1)=C(8,6)={}^8C_6=28$

**19.** *(a)* Let $z=2\omega^2+3i$

Since, $\omega$ is the cube root of unity

$\therefore\ \omega=\frac{-1+\sqrt{3}i}{2}$ and $\omega^2=\frac{-1-\sqrt{3}i}{2}$

$\therefore z = 2\omega^2 + 3i$

$= 2\left[\frac{-1-\sqrt{3}i}{2}\right] + 3i$

$= -1 - \sqrt{3}i + 3i = -1 + \left(3 - \sqrt{3}\right)i$

$\therefore \bar{z} = -1\left(3-\sqrt{3}\right)i = -1 + \sqrt{3}i - 3i = 2\left(\frac{-1+\sqrt{3}i}{2}\right) - 3i$

$= 2\omega - 3i$

**20.** *(c)* Given equation is $x = 2^{1/3} - 2^{-1/3}$

On cubing both sides, we get

$\Rightarrow x^3 = \left(2^{1/3} - 2^{-1/3}\right)^3$

$\Rightarrow x^3 = 2 - 2^{-1} - 3 \cdot 2^{1/3} \cdot 2^{-1/3}\left(2^{1/3} - 2^{-1/3}\right)$

$\Rightarrow x^3 = -3x + 2 - \frac{1}{2} \Rightarrow x^3 = -3x + \frac{3}{2}$

$\Rightarrow 2x^3 + 6x = 3$

**21.** *(c)* No. of people who are illiterates

$= 15000 \times \frac{34.5}{100} = 5175$

No. of people who have education up to primary school

$= 15000 \times \frac{27}{100} = 4050$

Similarly, no. of people who have education upto middle school $= 2790$

Let the no. of people who have education upto high school $= x$

$\therefore$ According to the questions, no. of people who have education upto pre-university $= \frac{x}{2}$

So, total no. of people who are not graduates

$= 5175 + 4050 + 2790 + x + \frac{x}{2} = 12150 + x + \frac{x}{2}$

Since, 660 are graduates, therefore

$15000 - \left(12150 + x + \frac{x}{2}\right) = 660$

$\Rightarrow 15000 - 660 = 12150 + \frac{3x}{2}$

$\Rightarrow 2\left(15000 - 660\right) = 24300 + 3x$

$\Rightarrow 28680 - 24300 = 3x$

$\Rightarrow \frac{4380}{3} = x \Rightarrow x = 1460$

Thus, 1460 students have education upto high school.

**22.** *(c)* Let $f : R \to R$ be defined as $f(x) = \frac{|x|}{x}, x \neq 0$

Also, $f(0) = 2$

i.e. value of function at $x = 0$ is 2.

Consider, $f(x) = \begin{cases} \frac{x}{x} = 1 \text{ if } x > 0 \\ \frac{-x}{x} = -1 \text{ if } x < 0 \end{cases}$

$\because$ we have $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

Thus, Range of $f(x) = \{1, -1, 2\}$

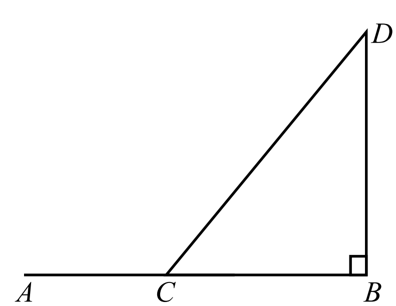
**23.** *(b)* Given that mid-point of $A(1,2)$ and $B(x,y)$ is $C(2,4)$,

$\therefore \frac{1+x}{2} = 2$ and $\frac{2+y}{2} = 4$

$\Rightarrow x = 3$ and $y = 6$

So, coordinates of $B$ are $(3,6)$

Given that $BD \perp AB$ and $CD = 3$ unit



$$BC=\sqrt{(2-3)^2+(4-6)^2}=\sqrt{1+4}=\sqrt{5}$$

In right angled $\Delta BCD$, $CD^2=BC^2+BD^2$

$\Rightarrow 9=5+BD^2 \Rightarrow BD^2=4 \Rightarrow BD=2$ unit

**24.** *(b)* Since, order of a set $A$ is 3 and order of set $B$ is 2.

Therefore, $n(A)=3$ and $n(B)=2$

$\therefore$ Number of relations from $A$ to $B$

$=n(A)\times n(B)=3\times 2=6$

**25.** *(d)* Since, $r$ and $s$ are the roots of $x^2+px+q=0$

Then, $r+s=-p$ and $rs=q$

Now, $\frac{1}{r^2}+\frac{1}{s^2}=\frac{r^2+s^2}{(rs)^2}=\frac{(r+s)^2-2rs}{(rs)^2}$

$$=\frac{(-p)^2-2q}{q^2}=\frac{p^2-2q}{q^2}$$

**26.** *(c)* As given $1, x, y, z, 16$ are in geometric progression.

Let common ratio be $r$,

$x=1\cdot r=r$

$y=1\cdot r^2=r^2$

$z=1\cdot r^3=r^3$

and $16=1\cdot r^4 \Rightarrow 16=r^4 \Rightarrow r=2$

$\therefore x=1\cdot r=2,\ y=1\cdot r^2=4,\ z=1\cdot r^3=8$

$\therefore x+y+z=2+4+8=14$

**27.** *(d)* Length of perpendicular from the origin on the straight line $x+2by+2p=0$ is

$$\left|\frac{0+2b\times 0-2p}{\sqrt{1^2+(2b)^2}}\right|=p$$

or $p=\left|\frac{-2p}{\sqrt{1^2+4b^2}}\right|$

or $p^2=\frac{4p^2}{1+4b^2}$

$\Rightarrow \frac{4}{1+4b^2}=1$

$\Rightarrow 1+4b^2=4$ or $4b^2=3$

$\Rightarrow b^2=\frac{3}{4} \Rightarrow b=\pm\frac{\sqrt{3}}{2} \Rightarrow b=\frac{\sqrt{3}}{2}$

**28.** *(a)* Given equation is $\tan^2 B=\frac{1-\sin A}{1+\sin A}$

Applying componendo and dividend

$$\frac{1+\tan^2 B}{1-\tan^2 B}=\frac{2}{2\sin A}$$

$\Rightarrow \sin A=\frac{1-\tan^2 B}{1+\tan^2 B} \Rightarrow \sin A=\cos 2B$

$\Rightarrow \sin A=\sin\left(\frac{\pi}{2}-2B\right)$

$\Rightarrow A=\frac{\pi}{2}-2B \Rightarrow A+2B=\frac{\pi}{2}$

**29.** *(b)* Given that $\sin^{-1}x+\sin^{-1}y=\frac{2\pi}{3}$ ....(i)

and $\cos^{-1}x+\cos^{-1}y=\frac{\pi}{3}$

$\Rightarrow \frac{\pi}{2}-\sin^{-1}x-\frac{\pi}{2}+\sin^{-1}y=\frac{\pi}{3}$

$\left[\text{since, } \cos^{-1}x+\sin^{-1}x=\frac{\pi}{2}\right]$

$\Rightarrow -\sin^{-1}x+\sin^{-1}y=\frac{\pi}{3}$ ....(ii)

On solving Eqs. (i) and (ii), we get

$2\sin^{-1}y=\pi$ and $2\sin^{-1}y=\frac{\pi}{3}$

$\Rightarrow \sin^{-1}y=\frac{\pi}{2}$ and $\sin^{-1}y=\frac{\pi}{6}$

Hence, $y=\sin\frac{\pi}{2}$ and $x=\sin\frac{\pi}{6}$

$\Rightarrow x=\frac{1}{2}$ and $y=1$

**30.** *(d)* The given function is $f(x)=\dfrac{1}{\sqrt{18-x^2}}$.

So, $f(3)=\dfrac{1}{\sqrt{18-9}}=\dfrac{1}{\sqrt{9}}=\dfrac{1}{3}$

$$\Rightarrow \lim_{x\to 3}\frac{f(x)-f(3)}{x-3}=\lim_{x\to 3}\frac{\frac{1}{\sqrt{18-x^2}}-\frac{1}{2}}{x-3}$$

Putting $x = 3$ gives $\frac{0}{0}$ form, so apply L'Hospital's Rule

$$\lim_{x\to 3}\frac{f(x)-f(3)}{x-3}=\lim_{x\to 3}-\frac{1}{2}\left(18-x^2\right)^{-3/2}(-2x)$$
$$=-\frac{1}{2}(18-9)^{-3/2}(-2\times 3)$$
$$=\frac{1}{27}(3)=\frac{1}{9}$$

**31.** *(a)* Let $\alpha$ and $\beta$ are the roots of $ax^2+bx+b=0$

$\alpha+\beta=-\dfrac{b}{a}$ and $\alpha\beta=\dfrac{b}{a}$

Consider, $\dfrac{\sqrt{\alpha}}{\sqrt{\beta}}+\dfrac{\sqrt{\beta}}{\sqrt{\alpha}}+\dfrac{\sqrt{b}}{\sqrt{a}}=\dfrac{\alpha+\beta}{\alpha\beta}+\dfrac{\sqrt{b}}{\sqrt{a}}$

$$=\frac{-b/a}{\sqrt{b/a}}+\frac{\sqrt{b}}{\sqrt{a}}=-\frac{\sqrt{b}}{\sqrt{a}}+\frac{\sqrt{b}}{\sqrt{a}}\quad(\text{by rationalization})$$
$$=0$$

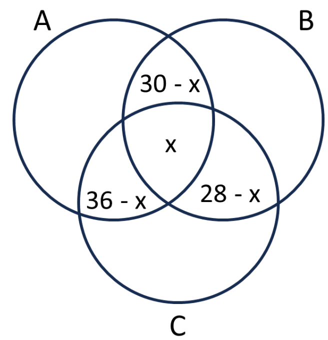
**32.** *(a)* Let the people who read all three papers A, B, and C be $x$%.

So, people who read only A and B but not C = $(30-x)\%$

So, people who read only B and C but not A = $(28-x)\%$

So, people who read only A and C but not B = $(36-x)\%$

The Venn diagram is shown below:

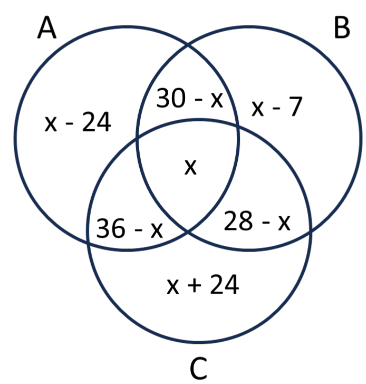



Remaining numbers in circles can be filled as shown below:

People who read only A =
$42-(30-x+x+36-x)=x-24$

People who read only B =
$51-(30-x+x+28-x)=x-7$

People who read only C =
$68-(36-x+x+28-x)=x+4$




Let $x$ % people read all the three newspaper. Since 8% people do not read any newspapers.

$$(x-24)+(x-7)+(x+4)+(30-x)+(28-x)+(36-x)+x=92$$
$$\Rightarrow x+98-31=92$$
$$\Rightarrow x+67=92$$
$$\Rightarrow x=92-67$$
$$\Rightarrow x=25$$

So, people who read only two newspapers is given by

$$(30-x)+(28-x)+(36-x)=94-3x$$
$$=94-75$$
$$=19$$

**33.** *(a)* Let $\alpha=\tan^{-1}x \Rightarrow \tan\alpha=x$

Then $\cos\alpha=\dfrac{1}{\sqrt{1+\tan^2\alpha}}=\dfrac{1}{\sqrt{1+x^2}}$

$\Rightarrow\cos\left(\tan^{-1}x\right)=\left\{\dfrac{1}{\sqrt{1+x^2}}\right\}$

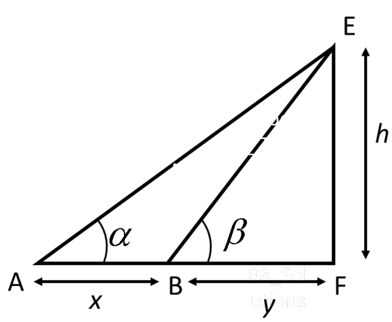
So, $\cot^{-1}\left\{\cos\left(\tan^{-1}x\right)\right\}=\cot^{-1}\left\{\dfrac{1}{\sqrt{1+x^2}}\right\}$

Let $\cot^{-1}\left\{\dfrac{1}{\sqrt{1+x^2}}\right\}=\beta\Rightarrow\cot\beta=\dfrac{1}{\sqrt{1+x^2}}$

and $\sin\beta=\dfrac{1}{\sqrt{1+\cot^2\beta}}=\dfrac{\sqrt{1+x^2}}{\sqrt{x^2+1+1}}=\sqrt{\dfrac{x^2+1}{x^2+2}}$

$\Rightarrow\sin\left[\cot^{-1}\left\{\cos\left(\tan^{-1}x\right)\right\}\right]=\sqrt{\dfrac{x^2+1}{x^2+2}}$

**34.** *(c)* Let EF be the height of the tower, and EB = $y$, AB = $x$, and EF = $h$.




In ΔBEF,

$\tan\beta=\dfrac{EF}{BF}$

$\tan\beta=\dfrac{h}{y}$ ...(i)

and in ΔAFE,

$\tan\alpha=\dfrac{EF}{AF}$

$\tan\alpha=\dfrac{h}{x+y}$

$\tan\left(\dfrac{\pi}{2}-\beta\right)=\dfrac{h}{x+y}\quad\left(\because\alpha+\beta=\dfrac{\pi}{2}\right)$

$\Rightarrow\quad\cot\beta=\dfrac{h}{x+y}$ ...(ii)

From (i) and (ii),

$\tan\beta\cdot\cot\beta=\dfrac{h}{y}\times\dfrac{h}{x+y}$

$\Rightarrow xy+y^2=h^2$

**35.** *(b)* As given $\alpha+\beta=\dfrac{\pi}{2}$ and $\beta+\gamma=\alpha$

$\Rightarrow\tan(\beta+\gamma)=\tan\alpha$

$\Rightarrow\dfrac{\tan\beta+\tan\gamma}{1-\tan\beta\tan\gamma}=\tan\alpha$

$\Rightarrow\tan\beta+\tan\gamma=\tan\alpha-\tan\alpha\tan\beta\tan\gamma$

$\Rightarrow\tan\beta+\tan\gamma=\tan\alpha-\tan\alpha\cot\alpha\tan\gamma$

$\left(\because\beta+\alpha=\dfrac{\pi}{2}\Rightarrow\beta=\dfrac{\pi}{2}-\alpha\Rightarrow\tan\left(\dfrac{\pi}{2}-\alpha\right)=\cot\alpha\right)$

$\Rightarrow\tan\beta+\tan\gamma=\tan\alpha-\tan\gamma$

$\Rightarrow\tan\beta+2\tan\gamma=\tan\alpha$

**36.** *(a)* $x$ and $y$ are positive numbers.

$x\le y^2$

**Reflexive**

$x\le x^2$ $\forall$ positive numbers.

Hence, relation is reflexive.

**Transitive**

$x\le y^2\qquad y\le z^2$

Let $x=5, y=3, z=2$

$5\le(3)^2\qquad(3)\le(2)^2$

but $5\not\le(2)^2$

Hence, $x\le y^2\qquad y\le z^2$

but $x\not\le z^2$

Thus, relation is not transitive.

**Symmetric**

$1 \le (2)^2$ while $2 \not\le (1)^2$

Hence relation is not symmetric.

Thus $x \le y^2$ $\forall$ positive numbers is reflexive, but not transitive and symmetric.

**37.** *(a)* Let the point of intersection divide the line segment joining points $(3,-1)$ and $(8,9)$ in $k:1$ ration, then

The point is $\left(\frac{8k+3}{k+1}, \frac{9k-1}{k+1}\right)$

Since, this point lies of the line $y-x+2=0$

We have, $\frac{9k-1}{k+1} - \frac{8k+3}{k+1} + 2 = 0$

$\Rightarrow \frac{9k-1-8k-3}{k+1} + 2 = 0 \Rightarrow \frac{k-4}{k+1} + 2 = 0$

$\Rightarrow k-4+2k+2=0 \Rightarrow 3k-2=0$

$\Rightarrow k = \frac{2}{3}$ i.e. $2:3$

**38.** *(d)* As given, $\tan\alpha = 2\tan\beta$

$\Rightarrow \frac{\tan\alpha}{\tan\beta} = 2 \Rightarrow \frac{\sin\alpha/\cos\alpha}{\sin\beta/\cos\beta} = 2$

$\Rightarrow \frac{\sin\alpha\cos\beta}{\cos\alpha\sin\beta} = 2$

Using componendo and dividend, we get

$\frac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\sin\alpha\cos\beta-\cos\alpha\sin\beta} = \frac{2+1}{2-1} = 3$

$\Rightarrow \frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)} = 3$

$\Rightarrow \sin(\alpha+\beta) = 3\sin(\alpha-\beta)$

**39.** *(b)* If any equation has $p-\sqrt{q}$ as a root, then another root will be $p+\sqrt{q}$.

So, sum of roots $= p-\sqrt{q}+p+\sqrt{q} = 2p$

and product of roots $= (p-\sqrt{q})(p+\sqrt{q}) = p^2 - q$

Now, required equation is

$x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0$

$\Rightarrow x^2 - 2px + (p^2-q) = 0$

**40.** *(c)* Two joining points are $(p,q)$ and $(q,-p)$

Mid-point of $(p,q)$ and $(q,-p)$ is $\left(\frac{p+q}{2}, \frac{q-p}{2}\right)$

But it is given that the mid-point is $\left(\frac{r}{2}, \frac{s}{2}\right)$

$\therefore \frac{p+q}{2} = \frac{r}{2}$ and $\frac{qs-p}{2} = \frac{s}{2}$

$\Rightarrow p+q=r$ and $q-p=s$

Now, length of segment $= \sqrt{(p-q)^2+(q+p)^2}$

$= \sqrt{s^2+r^2} = (s^2+r^2)^{1/2}$

**41.** *(c)* As given, $\sin 2A = \frac{4}{5}$

$\Rightarrow \frac{2\tan A}{1+\tan^2 A} = \frac{4}{5}$ $\left[\because \sin 2A = \frac{2\tan A}{1+\tan^2 A}\right]$

$\Rightarrow 10\tan A = 4 + 4\tan^2 A$

$\Rightarrow 5\tan A = 2 + 2\tan^2 A$

$\Rightarrow 2\tan^2 A - 5\tan A + 2 = 0$

$\Rightarrow 2\tan^2 A - 4\tan A - \tan A + 2 = 0$

$\Rightarrow 2\tan A(\tan A - 2) - 1(\tan A - 2) = 0$

$\Rightarrow (2\tan A - 1)(\tan A - 2) = 0$

$\Rightarrow \tan A = \frac{1}{2}$ $\left(\text{since } A \le \frac{\pi}{4} \Rightarrow \tan A \ne 2\right)$

**42.** *(a)* We have $(1p101)_2 + (10q1)_2 = (100r00)_2$

$\Rightarrow (1\times 2^4 + p\times 2^3 + 1\times 2^2 + 0\times 2^1 + 1\times 2^0)$

$+ (1\times 2^3 + 0\times 2^2 + q\times 2^1 + 1\times 2^0)$

$= (1\times 2^5 + 0 + 0 + r\times 2^2 + 0 + 0)$

$\Rightarrow 16+8p+4+1+8+2q+1=32+4r$

$\Rightarrow 30+8p+2q=32+4r$

$\Rightarrow 8p+2q=2+4r$

From options, substitute $p=0, q=1, r=0$, we get

$0+2(1)=2+0 \Rightarrow 2=2$

**43.** *(b)* We have $\sin\left(\frac{5\pi}{12}\right)=\sin 75°$

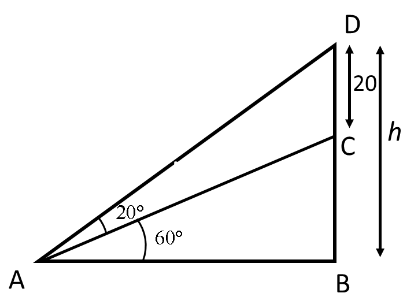
$=\sin(45°+30°)$

$=\sin 45° \cos 30° + \cos 45° \sin 30°$

$=\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}\cdot\frac{1}{2}=\frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}+1}{2}\right)$

$=\frac{\sqrt{3}+1}{2\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{6}+\sqrt{2}}{4}$

**44.** *(b)* The figure is given below:




In $\Delta$ABD,

$\tan 60° = \frac{BD}{AB}$

$\sqrt{3}=\frac{h}{AB}$

$AB=\frac{h}{\sqrt{3}}$

$AB=\frac{h\sqrt{3}}{3}$

Now in $\Delta$ABC,

$AC^2 = AB^2 + BC^2$

$\Rightarrow (20)^2=\left(\frac{h}{\sqrt{3}}\right)^2+(h-20)^2$

$\Rightarrow h^2+3h^2-120h=0$

$\Rightarrow \quad 4h^2-120h=0$

$\Rightarrow \quad 4h(h-30)=0$

$\Rightarrow \quad h=0, 30$

As $h$ = 0 is not possible, we have $h$ = 30 ft

**45.** *(c)* Given $\lim\limits_{x\to 0}\frac{x\sin 5x}{\sin^2 4x}$

Multiple numerator and denominator by $x$

$\lim\limits_{x\to 0}\frac{x^2\sin 5x}{x\sin^2 4x}=\lim\limits_{x\to 0}\frac{\sin 5x}{x}\cdot\frac{x^2}{\sin^2 4x}$

Rearranging to bring it in standard form,

$\lim\limits_{x\to 0}\frac{5\sin 5x}{5x}\cdot\frac{(4x)^2}{16\sin^2 4x}$

$=\frac{5}{16}\lim\limits_{x\to 0}\frac{5\sin 5x}{5x}\cdot\frac{1}{\lim\limits_{x\to 0}\left(\frac{\sin 4x}{4x}\right)^2}=\frac{5}{16}$

**46.** *(b)* Since the roots of the equation

$(a^2+b^2)x^2-2b(a+c)x+(b^2+c^2)=0$ are equal, so discriminant is equal to 0.

$[-2b(a+c)]^2-4(a^2+b^2)(b^2+c^2)=0$

$\Rightarrow \quad 4b^2(a+c)^2=4(a^2+b^2)(b^2+c^2)$

$\Rightarrow \quad b^2(a^2+c^2+2ac)=a^2b^2+b^2c^2+a^2c^2+b^4$

$\Rightarrow a^2b^2+b^2c^2+2acb^2=a^2b^2+b^2c^2+a^2c^2+b^4$

$\Rightarrow b^4+a^2c^2-2acb^2=0$

$\Rightarrow \quad (b^2-ac)^2=0$

$\Rightarrow \quad b^2=ac$

**47.** *(c)* Let the locus of a point be $(h,k)$

Let the given points be $P(m+n,n-m)$ and $Q(m-n,n+m)$

$\therefore$ By distance formula, we have

$$\sqrt{[h-(m+n)]^2+[k-(n-m)]^2}=\sqrt{[h-(m-n)]^2+[k-(n+m)]^2}$$

$$\Rightarrow h^2+(m+n)^2-2h(m+n)+k^2+(n-m)^2-2k(n-m)$$

$$=h^2+(m-n)^2-2h(m-n)+k^2+(n+m)^2-2k(n+m)$$

$$\Rightarrow 2h(m-n-m-n)+2k(n+m-n+m)=0$$

$$\Rightarrow -4hn+4km=0 \Rightarrow mk=nh$$

Thus, locus of a point is $nx=my$

**48.** *(c)* Given, $\sin x+\sin y=a$ and $\cos x+\cos y=b$

$$\therefore a^2+b^2=(\sin x+\sin y)^2+(\cos x+\cos y)^2$$

$$=\sin^2 x+\sin^2 y+2\sin x\sin y+\cos^2 x+\cos^2 y+2\cos x\cos y$$

$$=\left(\sin^2 x+\cos^2 x\right)+\left(\sin^2 y+\cos^2 y\right)+2(\cos x\cos y+\sin x\sin y)$$

$$=1+1+2\cos(x-y)$$

$$\Rightarrow 2\cos(x-y)=a^2+b^2-2$$

$$\Rightarrow \cos(x-y)=\frac{1}{2}\left(a^2+b^2-2\right)$$

**49.** *(c)* Let $A(a,0)$ and $B(0,b)$ be two points on $x$-axis and $y$-axis respectively.

Given $(-5,4)$ divides line $AB$ in the ratio 1:2.

By section formula, we have

$$-5=\frac{1\times 0+2\times a}{3}\Rightarrow a=\frac{-15}{2} \text{ and}$$

$$4=\frac{1\times b+2\times 0}{3}\Rightarrow b=12$$

Thus, $A\left(\frac{-15}{2},0\right)$ and $B=(0,12)$

Hence, equation of line joining $\left(\frac{-15}{2},0\right)$ and $(0,12)$ is

$$(y-0)=\frac{12-0}{0+\frac{15}{2}}\left(x+\frac{15}{2}\right)$$

$$\Rightarrow y=\frac{4}{5}(2x+15)$$

$$\Rightarrow 5y=(8x+60)\Rightarrow 8x-5y+60=0$$

**50.** *(d)* Given equation is $x^2-4x-\log_3 N=0$

Since, roots are real

$$\therefore b^2-4ac=0\Rightarrow (4)^2-4(-\log_3 N)\ge 0$$

$$\Rightarrow 16\ge -4\log_3 N$$

$$\Rightarrow 4\ge -\log_3 N$$

$$\Rightarrow 4\ge \log_3 N^{-1}$$

$$\Rightarrow N^{-1}\ge 3^4\ge 81$$

$$\Rightarrow N\ge\frac{1}{81}$$

So, minimum value of $N$ is $\frac{1}{81}$

**51.** *(d)* Let $f(x)=\log_{10}(1+x)$

$$\therefore 4f(4)+5f(1)-\log_{10}2$$

$$=4\log_{10}(1+4)+5\log_{10}(1+1)-\log_{10}2$$

$$=4\log_{10}5+5\log_{10}2-\log_{10}2$$

$$=4\log_{10}5+4\log_{10}2$$

$$=4(\log_{10}5+\log_{10}2)$$

$$=4(\log_{10}10)$$

$$=4 \qquad \{\therefore \log_a a=1\}$$

**52.** *(a)* Given that, $\sec A+\tan A=p$

$\Rightarrow \frac{1}{\cos A}+\frac{\sin A}{\cos A}=p$

$\Rightarrow \frac{1+\sin A}{\cos A}=p$

$\Rightarrow \frac{(1+\sin A)^2}{\cos^2 A}=p^2$

$\Rightarrow \frac{(1+\sin A)^2}{1-\sin^2 A}=p^2$

$\Rightarrow \frac{(1+\sin A)^2}{(1-\sin A)(1+\sin A)}=p^2$

$\Rightarrow \frac{1+\sin A}{1-\sin A}=p^2$

Using componendo and dividend, we get

$\Rightarrow \frac{(1+\sin A)+(1-\sin A)}{(1+\sin A)-(1-\sin A)}=\frac{p^2+1}{p^2-1}$

$\Rightarrow \frac{2}{2\sin A}=\frac{p^2+1}{p^2-1}$

$\Rightarrow \sin A=\frac{p^2-1}{p^2+1}$

**53.** *(b)* Let $p$ and $q$ are the roots of the equation

$x^2-30x+221=0$

$\Rightarrow x^2-13x-17x+17\times 13=0$

$\Rightarrow (x-13)(x-17)=0$

$\therefore\ p=13$ and $Q=17$

$\therefore\ p^3+q^3=2197+4913=7110$

**54.** *(c)* Since, the straight lines $x-2y=0$ and $kx+y=1$ intersect at the point $\left(1,\frac{1}{2}\right)$

$\therefore$ The point $\left(1,\frac{1}{2}\right)$ satisfies the equation $kx+y=1$

$\therefore$ Put $x=1$ and $y=\frac{1}{2}$ in eqation $kx+y=1$

we get $k\cdot 1+\frac{1}{2}=1\Rightarrow k=\frac{1}{2}$

**55.** *(c)* Given that, $\sin^2 x+\sin^2 y=1$

$\Rightarrow \sin^2 x=1-\sin^2 y$

$\Rightarrow \sin^2 x=\cos^2 y$

$\Rightarrow \sin x=\cos y$

Similarly by considering $\sin^2 y=1-\sin^2 x$, we have $\cos x=\sin y$

Now, Consider $\cot(x+y)=\frac{\cos(x+y)}{\sin(x+y)}$

$=\frac{\cos x\cos y-\sin x\sin y}{\sin(x+y)}$

$=\frac{\cos x\cos y-\cos x\cos y}{\sin(x+y)}=0$

**56.** *(b)* $f(x)=\sqrt{x}(7x-6)=7x^{3/2}-6x^{1/2}$

Differentiate both sides wrt $x$

$f'(x)=7\times\frac{3}{2}x^{1/2}-6\times\frac{1}{2}x^{-1/2}$

When the tangent is parallel to $x$-axis, $f'(x)=0$

$7\times\frac{3}{2}x^{1/2}-6\times\frac{1}{2}x^{-1/2}=0$

$\Rightarrow \frac{21}{2}x^{1/2}-3x^{-1/2}=0$

$\Rightarrow \frac{21}{2}\sqrt{x}=\frac{3}{\sqrt{x}}$

$\Rightarrow 7x=2$

$\Rightarrow x=\frac{2}{7}$

**57.** *(c)* Let $I=\int \sec x\,\mathrm{cosec}\,x\,dx=\int\frac{2}{2\sin x\cos x}dx$

$$I=\int\frac{2}{\sin 2x}dx$$
$$=2\int\frac{1}{\frac{2\tan x}{1+\tan^2 x}}dx$$
$$=\int\frac{\sec^2 x}{\tan x}dx$$

Let $\tan x=t\Rightarrow\sec^2 x\,dx=dt$

$\therefore\ I=\int\frac{dt}{t}=\log|t|+c$

Comparing with $\int\sec x\operatorname{cosec}x\,dx=\log|g(x)|+c$, we get $g(x)=\tan x$

**58.** *(c)* Given $f(x)=\frac{1}{1-|1-x|}$

$$\text{LHL}=\lim_{h\to 0}\frac{1}{1-|1-(1-h)|}$$
$$=\lim_{h\to 0}\frac{1}{1-|h|}$$
$$=\lim_{h\to 0}\frac{1}{1-h}=1$$
$$\text{RHL}=\lim_{h\to 0}\frac{1}{1-|1-(1+h)|}$$
$$=\lim_{h\to 0}\frac{1}{1-|-h|}$$
$$=\lim_{h\to 0}\frac{1}{1-h}=1$$
$$\therefore\lim_{x\to 0}f(x)=1$$

**59.** *(a)* Given $\sin x\cos y=\frac{1}{2}$

Differentiating both the sides w.r.t. $x$

$$\sin x(-\sin y)\frac{dy}{dx}+\cos y\cos x=0$$
$$\Rightarrow\frac{dy}{dx}=\frac{\cos x\cos y}{\sin x\sin y}=\cot x\cot y$$

$$\frac{d^2y}{dx^2}=-\operatorname{cosec}^2 x\cot y+\cot x\left(-\operatorname{cosec}^2 y\right)\frac{dy}{dx}$$

For the given point, $\left(\frac{\pi}{4},\frac{\pi}{4}\right)$

$$\frac{dy}{dx}=\cot\frac{\pi}{4}\cot\frac{\pi}{4}=1$$

This gives,

$$\frac{d^2y}{dx^2}=-\operatorname{cosec}^2\frac{\pi}{4}\cot\frac{\pi}{4}+\cot\frac{\pi}{4}\left(-\operatorname{cosec}^2\frac{\pi}{4}\right)$$
$$=-(2)(1)+(1)(-1)$$
$$=-4$$

**60.** *(d)* Given equation of the line $\frac{x}{4}+\frac{y}{3}=1$ can be written as

$$3x+4y=12\Rightarrow 4y=-3x+12$$
$$\Rightarrow y=-\frac{3}{4}x+\frac{12}{4}$$

The slope of the line $\frac{x}{4}+\frac{y}{3}=1$ is $-\frac{3}{4}$

$\therefore$ Slope of the line perpendicular to this line

$$=-\left(\frac{-1}{3/4}\right)=\frac{4}{3}$$

**61.** *(c)* Given function is $f(x)=A\sin\left(\frac{\pi x}{2}\right)+B$.

Differentiating it w.r.t $x$

$$f'(x)=A\cos\left(\frac{\pi x}{2}\right)\cdot\frac{\pi}{2}$$
$$f'\left(\frac{1}{2}\right)=\sqrt{2}\Rightarrow A\cos\left(\frac{\pi}{4}\right)\cdot\frac{\pi}{2}=\sqrt{2}$$
$$\Rightarrow A\frac{1}{\sqrt{2}}\cdot\frac{\pi}{2}=\sqrt{2}$$
$$\Rightarrow\quad A=\frac{\sqrt{2}\times\sqrt{2}\times 2}{\pi}$$
$$\Rightarrow\quad A=\frac{4}{\pi}$$

Given, $\int_0^1 f(x)\,dx = \frac{2A}{\pi}$

$\Rightarrow \int_0^1 \left\{A\sin\left(\frac{\pi x}{2}\right)+B\right\}dx = \frac{2}{\pi}\times\frac{4}{\pi}$

$\Rightarrow \left[-A\cos\left(\frac{\pi x}{2}\right)\cdot\frac{2}{\pi}+Bx\right]_0^1 = \frac{8}{\pi^2}$

$\Rightarrow \left[-\frac{4}{\pi}\cos\left(\frac{\pi x}{2}\right)\cdot\frac{2}{\pi}+Bx\right] = \frac{8}{\pi^2}$

$\Rightarrow -\frac{8}{\pi^2}\left(\cos\frac{\pi}{2}-\cos 0\right)+B = \frac{8}{\pi^2}$

$\Rightarrow \quad \frac{8}{\pi^2}+B = \frac{8}{\pi^2}$

$\Rightarrow \quad B = 0$

**62.** *(a)* Let $f(x) = \begin{cases} \frac{x^3-3x+2}{(x-1)^2} & \text{for } x \neq 1 \\ k & \text{for } x = 1 \end{cases}$.

and $f(x)$ is continuous

$\therefore \lim_{x\to 1} f(x) = f(1) = k$

$\Rightarrow \lim_{x\to 1} \frac{x^3-3x+2}{(x-1)^2} = k$

$\Rightarrow \lim_{x\to 1} \frac{3x^2-3}{2(x-1)} = k$ [By L'Hospital's Rule]

$\Rightarrow \lim_{x\to 1} \frac{6x}{2} = k$ [By L'Hospital's Rule]

$\Rightarrow \quad \frac{6}{2} = k$

$\Rightarrow \quad 3 = k$

**63.** *(c)* Given $f(x) = \frac{x-1}{x+1} \Rightarrow x = \frac{1+f(x)}{1-f(x)}$

$\Rightarrow f(2x) = \frac{2x-1}{2x+1}$

$= \frac{2\left[\frac{1+f(x)}{1-f(x)}\right]-1}{2\left[\frac{1+f(x)}{1-f(x)}\right]+1}$

$= \frac{2+2f(x)-1+f(x)}{2+2f(x)+1-f(x)}$

$= \frac{3f(x)+1}{f(x)+3}$

**64.** *(d)* Given integral is $I = \int_0^1 (x-1)e^{-x}dx$

Integrating by parts taking $x-1$ as the first function.

$I = \left[-(x-1)e^{-x}\right]_0^1 - \int_0^1 1\cdot\left(-e^{-x}\right)dx$

$= -(1-1)\frac{1}{e}+(-1)e^0+\left[-e^{-x}\right]_0^1$

$= -1-\left(\frac{1}{e}-1\right) = -\frac{1}{e}$

**65.** *(a)* A matrix is singular if value of its determinant is zero.

Given that matrix $\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is singular.

$\Rightarrow \begin{vmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = 0$

$\Rightarrow \quad \begin{vmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{vmatrix} = 0$

$\Rightarrow \quad \cos^2\theta - \sin^2\theta = 0 = \cos\frac{\pi}{2}$

$\Rightarrow \quad \text{co}\,2\theta = \cos\frac{\pi}{2}$

$\Rightarrow \quad \theta = \frac{\pi}{4}$

**66.** *(a)* We know that if

$A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $\text{Adj}\,A=\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

and $|A| = ad - bc$

$A^{-1}=\dfrac{\text{Adj}\,A}{|A|}, |A| \neq 0$

So, $A^{-1}=\dfrac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Given that $A=\begin{bmatrix} 1+i & 1+i \\ -1+i & 1-i \end{bmatrix}$

Then,

$\text{Adj}\,A=\begin{bmatrix} 1-i & -1-i \\ 1-i & 1+i \end{bmatrix}$

$|A|=\begin{vmatrix} 1+i & 1+i \\ -1+i & 1-i \end{vmatrix}$

$=\left(1-i^2\right)-\left(i^2-1\right)$

$=1-i^2-i^2+1$

$=1+1+1+1$

$=4$

Thus, $A^{-1}=\dfrac{1}{4}\begin{bmatrix} 1-i & -1-i \\ 1-i & 1+i \end{bmatrix}$

**67.** *(d)* Let $y = 2x^2 - 3x + 5$

$\Rightarrow \dfrac{dy}{dx} = 4x - 3$ and $\dfrac{d^2y}{dx^2} = 4$

For maximum or minimum value of $y$,

$\dfrac{dy}{dx} = 0$

$\Rightarrow 4x - 3 = 0$

$\Rightarrow \quad x = \dfrac{3}{4}$

For every value of $x$, $\dfrac{d^2y}{dx^2} > 0$

So, minimum value of $y = 2x^2 - 3x + 5$ occurs at $x = \dfrac{3}{4}$

Minimum value $= 2\left(\dfrac{3}{4}\right)^2 - 3\left(\dfrac{3}{4}\right) + 5 = \dfrac{9}{8} - \dfrac{9}{4} + 5 = \dfrac{31}{8}$

**68.** *(a)* Let the given integral be $I = \displaystyle\int_a^b \frac{\log x}{x}\,dx$

Put $\log x = t \Rightarrow \dfrac{dx}{x} = dt$

When $x = a$, $t = \log a$ and for $x = b$, $t = \log b$

$\therefore I = \displaystyle\int_{\log a}^{\log b} t\,dt$

$= \left[\dfrac{t^2}{2}\right]_{\log a}^{\log b}$ \

Substitute the limits,

$\therefore I = \dfrac{1}{2}\left[(\log b)^2 - (\log a)^2\right]$

$= \dfrac{1}{2}(\log b + \log a)(\log b - \log a)$

$= \dfrac{1}{2}(\log ba)\left(\log \dfrac{b}{a}\right)$

**69.** *(b)* Given $f(x) = \begin{cases} \dfrac{\tan kx}{x} & \text{for } x < 0 \\ 3x + 2k^2 & \text{for } x \geq 0 \end{cases}$

When the given function is continuous at $x = 0$, then

$\displaystyle\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$

Now,

$\displaystyle\lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h)$

$= \displaystyle\lim_{h\to 0}\left[\frac{\tan k(0-h)}{0-h}\right]$

$= \displaystyle\lim_{h\to 0} \frac{\tan kh}{h}$

$$\lim_{x\to0^+} f(x) = \lim_{h\to0} f(0+h)$$
$$= \lim_{h\to0}\left[3(0+h)+2k^2\right]$$
$$= \lim_{h\to0}\left[3h+2k^2\right] = 2k^2$$

This gives,

$$\lim_{h\to0}\frac{\tan kh}{h} = 2k^2$$
$$\Rightarrow k\lim_{h\to0}\frac{\tan kh}{kh} = 2k^2$$
$$\Rightarrow \quad k = 2k^2 \quad \left[\because \lim_{x\to0}\frac{\tan x}{x} = 0\right]$$
$$\Rightarrow \quad k = \frac{1}{2}$$

**70.** *(c)* Given that $y^2 = p(x)$.

Differentiating, we get

$$\Rightarrow 2yy_1 = p'(x)$$
$$\Rightarrow \quad 2y_1 = \frac{p'(x)}{y}$$

Differentiating again,

$$2y_2 = \frac{yp''(x) - p'(x)y_1}{y^2}$$
$$\Rightarrow 2y_2 = \frac{yp''(x) - p'(x)\frac{p'(x)}{2y}}{y^2}$$
$$\Rightarrow 2y_2 = \frac{2y^2p''(x) - (p'(x))^2}{2y^3}$$
$$\Rightarrow 2y^3y_2 = \frac{1}{2}\left[2p(x)p''(x) - (p'(x))^2\right]$$
$$\Rightarrow 2\frac{d}{dx}\left(y^3y_2\right)$$
$$= \frac{1}{2}\left[2p'(x)p''(x) + 2p(x)p'''(x) - 2p'(x)p''(x)\right]$$
$$= p(x)p'''(x)$$

**71.** *(b)* We know that $A^{-1} = \dfrac{\text{Adj}\,A}{|A|}, |A| \neq 0$

$$\text{or, } AA^{-1} = \frac{A\,\text{Adj}\,A}{|A|}$$
$$\text{or, } I_n = \frac{A\,\text{Adj}\,A}{|A|}$$
$$\Rightarrow A(\text{Adj}\,A) = |A|I_n$$
$$= |A|\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$
$$= \begin{bmatrix}4 & 0 & 0\\0 & 4 & 0\\0 & 0 & 4\end{bmatrix}$$

**72.** *(b)* Consider the line $x\cos\theta + y\sin\theta = 2$

$$\Rightarrow y\sin\theta = -x\cos\theta + 2$$
$$\Rightarrow \quad y = -x\frac{\cos\theta}{\sin\theta} + \frac{2}{\sin\theta}$$
$$\Rightarrow \quad y = -x\cot\theta + \frac{2}{\sin\theta}$$

On comparing this equation with $y = mx + c$,

Slope of the line $x\cos\theta + y\sin\theta = 2$ is $-\cot\theta$

Also, we have line $x - y = 3$

$$\Rightarrow y = x - 3$$

Slope of the line $x - y = 3$ is 1.

Since both the lines are perpendicular to each other,

Product of their slopes = $-1$

$$\Rightarrow (-\cot\theta)\times1 = -1$$
$$\Rightarrow \quad \cot\theta = 1$$
$$\Rightarrow \quad \cot\theta = \cot\frac{\pi}{4}$$
$$\Rightarrow \quad \theta = \frac{\pi}{4}$$

**73.** *(c)* Probability of getting a spade = $\dfrac{13}{52}$

Probability of getting a ace = $\dfrac{4}{52}$

and probability of getting a spade ace $= \frac{1}{52}$

So, required probability $= \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$

Odd against his winning $= \frac{1-\frac{4}{13}}{\frac{4}{13}} = \frac{9}{4}$

**74.** *(c)* The given vector is $\vec{V} = 2\hat{i} - \hat{j} + 2\hat{k}$ and for $z$-axis,

$x = 0,\ y = 0$.

So, the vector equation is,

$\vec{A} = 0\hat{i} + 0\hat{j} + \hat{k}$

$\cos\alpha = \frac{\vec{V}\cdot\vec{A}}{|\vec{V}||\vec{A}|}$

$\cos\alpha = \frac{2\cdot 0 + (-1)\cdot 0 + 2\cdot 1}{\sqrt{4+1+4}\sqrt{0+0+1}} = \frac{2}{3}$

Hence,

$\sin\alpha = \sqrt{1-\cos^2\alpha}$

$= \sqrt{1-\frac{4}{9}} = \sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{3}$

**75.** *(c)* For $-1 \le x \le 2$

$f(x) = 3x^2 + 12x - 1$

$f'(x) = 6x + 12$

If we take any point in the interval $[-1, 2]$, then

$f'(1) = 6\times 1 + 12 = 18 > 0$

$\Rightarrow f(x)$ is increasing in the interval $[-1, 2]$

For $2 < x \le 3$

$f(x) = 37 - x$

$f'(x) = -1 < 0$

$\Rightarrow f(x)$ is decreasing in the interval $(2, 3]$

**76.** *(c)* Given $x = \cos t,\ y = \sin t$

$\Rightarrow \frac{dx}{dt} = -\sin t$ and $\frac{dy}{dt} = \cos t$

$\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\cos t}{-\sin t} = -\cot t$

$\Rightarrow \frac{d^2y}{dx^2} = \text{cosec}^2 t\,\frac{dt}{dx} = \text{cosec}^2 t \cdot \frac{1}{-\sin t} = -\frac{1}{\sin^3 t}$

$\Rightarrow \frac{d^2y}{dx^2} = -\frac{1}{y^3} = -y^{-3}$

**77.** *(c)* Let the volume of the box be $V$.

$V(x) = (24-2x)(9-2x)\cdot x$

$= \left(216 - 48x - 18x + 4x^2\right)\cdot x$

$= 4x^3 - 66x^2 + 216x$

$\Rightarrow V'(x) = 12x^2 - 132x + 216$

For maximum value, $V'(x) = 0$

$12x^2 - 132x + 216 = 0$

$\Rightarrow\ 12\left(x^2 - 11x + 18\right) = 0$

$\Rightarrow\ 12(x-2)(x-9) = 0$

$\Rightarrow\ x = 2, 9$

Here $x = 9$ is not possible. So, $x = 2$.

Also, $V''(x) = 24x - 132$

$V''(2) = 48 - 132 = -84 < 0$

This implies that the volume is maximum when $x = 2$.

**78.** *(d)* Given equations are:

$x - y + 2z = 0$ ...(i)

$kx - y + z = 0$ ...(ii)

$3x + y - 3z = 0$ ...(iii)

System of equations possess a unique solution if

$$|A|=\begin{vmatrix}1 & -1 & 2\\ k & -1 & 1\\ 3 & 1 & -3\end{vmatrix}\neq 0 \quad \dots \text{(i)}$$

Applying, $C_1 \to C_1 + C_2$ and $C_2 \to C_2 + \frac{1}{2}C_3$

$$|A|=\begin{vmatrix}0 & 0 & 2\\ k-1 & -\frac{1}{2} & 1\\ 4 & -\frac{1}{2} & -3\end{vmatrix}=\frac{1}{2}\begin{vmatrix}0 & 0 & 2\\ k-1 & -1 & 1\\ 4 & -1 & -3\end{vmatrix}$$

Expanding along Row 1,

$$|A|=\frac{1}{2}\times 2\left[(k-1)(-1)-4(-1)\right]$$

$$=-k+1+4$$

$$=-k+5$$

Now, from (i), $-k+5\neq 0 \Rightarrow k\neq 5$

**79.** *(b)* The given function

$$f(x)=(\log_{\tan x}\cot x)(\log_{\cot x}\tan x)^{-1}$$

$$=\left(\frac{\log\cot x}{\log\tan x}\right)\left(\frac{\log\tan x}{\log\cot x}\right)^{-1}$$

$$=\left(\frac{\log\cot x}{\log\tan x}\right)\left(\frac{\log\cot x}{\log\tan x}\right)$$

$$=\left(\frac{\log\cot x}{\log\tan x}\right)^2$$

$$=\left(\frac{\log\frac{1}{\tan x}}{\log\tan x}\right)^2=\left(-\frac{\log\tan x}{\log\tan x}\right)^2=1$$

$\Rightarrow f(x)=1$ (Constant function)

$\Rightarrow f'(x)=1$

This is true for $0<x<\frac{\pi}{2}$

$\Rightarrow f'\left(\frac{\pi}{4}\right)=0$

**80.** *(d)* Given differential equation is

$$\frac{dy}{dx}=1+x+y^2+xy^2$$

$$\Rightarrow \frac{dy}{dx}=(1+x)(1+y^2)$$

Separate the variables

$$\frac{dy}{1+y^2}=(1+x)dx$$

$$\Rightarrow \int\frac{dy}{1+y^2}=\int(1+x)dx$$

$$\Rightarrow \tan^{-1}y=\frac{x^2}{2}+x+C$$

Given that when $x=0$, $y=0$, then $c=0$

$$\Rightarrow y=\tan\left(\frac{x^2}{2}+x\right)$$

**81.** *(b)* Total number of possible arrangements, $n(S)=6!$

Since 2 and 3 occupy consecutive places, so, they are grouped together.
So, there will be 5! such arrangements. But 2 and 3 can be arranged in themselves in 2! Ways.

So, required probability $=\frac{5!\times 2!}{6!}=\frac{2}{6}=\frac{1}{3}$

**82.** *(c)* Given that $A=\begin{bmatrix}1 & 2\\ 2 & 2\end{bmatrix}$

$$\therefore\ A^2=\begin{bmatrix}1 & 2\\ 2 & 2\end{bmatrix}\begin{bmatrix}1 & 2\\ 2 & 2\end{bmatrix}=\begin{bmatrix}1+4 & 2+4\\ 2+4 & 4+4\end{bmatrix}=\begin{bmatrix}5 & 6\\ 6 & 8\end{bmatrix}$$

Let $A^2+xA+yI=0$, where $x$ and $y$ are constant

$$\Rightarrow \begin{bmatrix}5 & 6\\ 6 & 8\end{bmatrix}+x\begin{bmatrix}1 & 2\\ 2 & 2\end{bmatrix}+y\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}5+x+y & 6+2x\\ 6+2x & 8+2x+y\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

So, $6+2x=0 \Rightarrow x=-3$

Now, $5+x+y=0 \Rightarrow 5-3+y=0 \Rightarrow y=-2$

This means that $A^2-3A-2I=0$

**83.** *(b)* Given equation,

$x\sqrt{1+y}+y\sqrt{1+x}=0$

This can be re-written as,

$x\sqrt{1+y}=-y\sqrt{1+x}$

Squaring both sides, we get

$x^2(1+y)=y^2(1+x)$

$\Rightarrow x^2+x^2y=y^2+y^2x$

$\Rightarrow x^2-y^2=y^2x-x^2y$

$\Rightarrow (x+y)(x-y)=xy(x-y)$

$\Rightarrow \quad x+y=-xy$

$\Rightarrow \quad y(1+x)=-x$

$\Rightarrow \quad y=\frac{-x}{1+x}$, which is in explicit form

Differentiating both sides,

$$\frac{dy}{dx}=\frac{(1+x)(-1)+x(1)}{(1+x)^2}=\frac{-1}{(1+x)^2}$$

**84.** *(b)* There are 7 days in a week If 1st person's birth day falls on any day out of 7.

So, probability is $\frac{7}{7}$.

Since birthday of second person will fall on any of the remaining six days, then its probability is $\frac{6}{7}$

Birthday of 3rd person will fall on any of the remaining 5 days, so, its probability is $\frac{5}{7}$.

This means that the probability that all three persons will have different day as their birthday

$$=\frac{7}{7}\times\frac{6}{7}\times\frac{5}{7}=\frac{30}{49}=0.612\approx 0.60$$

**85.** *(a)* Given $x=\sin t-t\cos t$ and $y=t\sin t+\cos t$.

$\Rightarrow \frac{dx}{dt}=\cos t-\{\cos t+t(-\sin t)\}$

$=\cos t-\cos t+t\sin t$

$=t\sin t$

$\Rightarrow \frac{dy}{dt}=t\cos t+\sin t-\sin t$

$=t\cos t$

This gives,

$$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{t\cos t}{t\sin t}=\cot t$$

$=t\cos t$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{t=\frac{\pi}{2}}=\cot\frac{\pi}{2}=0$$

**86.** *(d)* Given $\vec{a}=2\hat{i}-3\hat{k}$, $\vec{b}=\hat{j}+3\hat{k}$ and $\vec{c}=-3\hat{i}+3\hat{j}+\hat{k}$.

Let $\vec{n}=x\hat{i}+y\hat{j}+z\hat{k}$.

Since $\vec{a}$ and $\vec{n}$ are perpendicular to each other,

$\vec{a}\cdot\vec{n}=0\Rightarrow(2\hat{i}-3\hat{k})\cdot(x\hat{i}+y\hat{j}+z\hat{k})=0$

$\Rightarrow 2x-3z=0 \quad ...(i)$

Also, $\vec{b}$ and $\vec{n}$ are perpendicular to each other,

$\vec{b}\cdot\vec{n}=0\Rightarrow(\hat{j}+3\hat{k})\cdot(x\hat{i}+y\hat{j}+z\hat{k})=0$

$\Rightarrow y+3z=0 \quad ...(ii)$

On solving equations (i) and (ii),

$y=z=0$

Since $\vec{n}=x\hat{i}+y\hat{j}+z\hat{k}$ is a unit vector,

$\sqrt{x^2+y^2+z^2}=1\Rightarrow x=1\ (\because y=z=0)$

Hence, $\hat{n}=\hat{i}$

This gives, $\vec{c}\cdot\vec{n}=(-3\hat{i}+3\hat{j}+\hat{k})\cdot(\hat{i})=-3$

**87.** *(a)* We know that the mean and variance of Binomial distribution are $np$ and $npq$ respectively.

So, $np=8$ and $npq=4$

On dividing, we get

$$q=\frac{npq}{np}=\frac{4}{8}=\frac{1}{2}$$

Since, $p+q=1 \Rightarrow p=1-\frac{1}{2}=\frac{1}{2}$

We know that, $P(X=r)={}^{n}C_{r}p^{r}q^{n-r}$

This gives,

$$P(X=1)={}^{16}C_{1}\left(\frac{1}{2}\right)^{1}\left(\frac{1}{2}\right)^{16-1}=\frac{16}{2^{15}\cdot 2}=\frac{1}{2^{12}}$$

In a binomial distribution, the value of $r$ for which $P(X=r)$ is maximum is the mod of binomial distribution. Hence,

$$(n+1)p-1\le r\le (n+1)p$$

$$\Rightarrow \quad \frac{17}{4}-1\le r\le \frac{17}{4}$$

$$\Rightarrow \quad \frac{13}{4}\le r\le \frac{17}{4}$$

$$\Rightarrow \quad 3.25\le r\le 4.25$$

$$\Rightarrow \quad r=4$$

**88.** *(b)* The equation of the sphere is $x^2+y^2+z^2-10z=0$.

So, its center is $(0,0,0)$.

Coordinates of one end point of a diameter of the sphere is given as $(-3,-4,5)$.

Let coordinates of another end point of this diameter is $(x_1,y_1,z_1)$.

$$\therefore \ \frac{-3+x_1}{2}=0\Rightarrow x_1=3$$

$$\frac{-4+y_1}{2}=0\Rightarrow y_1=4$$

$$\frac{5+z_1}{2}=5\Rightarrow z_1=5$$

So, required coordinates are $(3,4,5)$.

**89.** *(c)* Given mean and variance of binomial distribution are 4 and 3 respectively.

So, $np=4,\ npq=3$.

Here $p$ is the probability of success, $q$ is the probability of failure, and $n$ is the number of trials.

$$\Rightarrow \frac{npq}{np}=\frac{3}{4}$$

$$\Rightarrow q=\frac{3}{4}$$

Also, $p=1-q=1-\frac{3}{4}=\frac{1}{4}$

$\therefore n=16$

**90.** *(b)* Total no. of letters = 26

No. of selected letters = 3

So, no. of ways of selecting 3 letters out of 26 letters is given by ${}^{26}C_3$.

Since the letter 'A' will not be included in our choice, so, no. of letters becomes 25.

Now, no. of ways of selecting 3 letters out of 25 letters is given by ${}^{25}C_3$

So, required probability $=\frac{{}^{25}C_3}{{}^{26}C_3}=\frac{25!}{3!22!}\times\frac{3!23!}{26!}=\frac{23}{26}$

**91.** *(b)* It is given that $\vec{a}$ is perpendicular to both $\vec{b}$ and $\vec{c}$.

$\vec{a}\cdot\vec{b}=0$ and $\vec{a}\cdot\vec{c}=0$

and angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$.

$$\therefore \ \vec{b}\cdot\vec{c}=|\vec{b}||\vec{c}|\cos\frac{\pi}{3}=1\cdot1\cdot\frac{1}{2}=\frac{1}{2}$$

Now,

$$|\vec{a}+\vec{b}+\vec{c}|^2=|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{a}\cdot\vec{c})$$

$$=1+1+1+2\left(0+\frac{1}{2}+0\right)$$

$$=1+1+1+1$$

$$=4$$

$$\Rightarrow |\vec{a}+\vec{b}+\vec{c}|=2$$

**92.** *(d)* The equation of family of circles having centres at the origin is

$$x^2 + y^2 = r^2$$

Where $r$ is the radius

Differentiate both sides w.r.t. $x$, we get

$$2x + 2y\frac{dy}{dx} = 0$$

$$\Rightarrow xdx + ydy = 0$$

This is the required differential equation.

**93.** *(a)* Let $n$ denote the number of workers and $x$, the pay.

$n_1 = 30, n_2 = 20,\ x_1 = 500,\ x_2 = 600$

$$\text{Combined average} = \frac{n_1x_1 + n_2x_2}{n_1 + n_2}$$

$$= \frac{30\times 500 + 20\times 600}{30+20}$$

$$= \frac{15000+12000}{50}$$

$$= \frac{27000}{50}$$

$$= 540$$

Combined average pay = ` 540

**94.** *(c)* Given $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = 0$.

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

$$\Rightarrow \begin{vmatrix} 1+a & -a & -a \\ 1 & b & 0 \\ 1 & 0 & c \end{vmatrix} = 0$$

Expanding along Row 3,

$$1(ab) + c(b + ab + a) = 0$$

$$\Rightarrow ab + bc + ca + abc = 0$$

$$\Rightarrow \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = -1$$

**95.** *(a)* Let $\vec{\alpha} = a\hat{i} + b\hat{j} + c\hat{k}$

Now, $\vec{\alpha}\cdot\hat{i} = \left(a\hat{i} + b\hat{j} + c\hat{k}\right)\cdot\hat{i} = a$

Similarly,

$$\vec{\alpha}\cdot\hat{j} = \left(a\hat{i} + b\hat{j} + c\hat{k}\right)\cdot\hat{j} = b$$

$$\vec{\alpha}\cdot\hat{k} = \left(a\hat{i} + b\hat{j} + c\hat{k}\right)\cdot\hat{k} = c$$

So, $\left(\vec{\alpha}\cdot\hat{i}\right)\hat{i} + \left(\vec{\alpha}\cdot\hat{j}\right)\hat{j} + \left(\vec{\alpha}\cdot\hat{k}\right)\hat{k}$

$$= a\hat{i} + b\hat{j} + c\hat{k}$$

$$= \vec{\alpha}$$

**96.** *(c)* For $\left\langle \frac{1}{\sqrt{2}}, \frac{1}{2}, k \right\rangle$ to be direction cosines,

$$\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + k^2 = 1$$

$$\Rightarrow \frac{1}{2} + \frac{1}{4} + k^2 = 1$$

$$\Rightarrow \frac{3}{4} + k^2 = 1$$

$$\Rightarrow k^2 = \frac{1}{4}$$

$$\Rightarrow k = \pm\frac{1}{2}$$

**97.** *(a)* Given table can be rewritten as

| $x$ | $y$ | $x^2$ | $y^2$ | $xy$ |
|---|---|---|---|---|
| 5 | 2 | 25 | 4 | 10 |
| 7 | 4 | 49 | 16 | 28 |
| 8 | 3 | 64 | 9 | 24 |
| 8 | 3 | 64 | 9 | 24 |
| 4 | 2 | 16 | 4 | 8 |
| 6 | 4 | 36 | 16 | 24 |
| $\sum x = 30$ | $\sum y = 15$ | $\sum x^2 = 190$ | $\sum y^2 = 49$ | $\sum xy = 94$ |

$$\bar{x} = \frac{30}{5} = 6 \quad \text{and} \quad \bar{y} = \frac{15}{5} = 3$$

$$\therefore byx = \frac{n\sum xy - \left(\sum x\right)\left(\sum y\right)}{n\sum x^2 - \left(\sum x\right)^2}$$

$$= \frac{5\times 94 - 30\times 15}{5\times 190 - (30)^2} = \frac{2}{5} = 0.4$$

Hence, line of regression is

$y - 3 = 0.4(x-6) \Rightarrow y = 0.4x + 0.6$

**98.** *(a)* Let the line segment joining the points $(2,4,5)$ and $(3,5,-4)$ is internally divided by the xy-plane in the ratio k:1.

For *x*y plane, $z = 0$.

$$\Rightarrow \quad 0 = \frac{-k\times 4 + 5}{k+1}$$

$$\Rightarrow 4k = 5$$

$$\Rightarrow \quad k = \frac{5}{4}$$

So, the ratio is 5:4.

**99.** *(b)* Since the rate of growth of bacteria is proportional to the number of bacteria present at that time,

$$\Rightarrow \frac{dx}{dt} \propto x \Rightarrow \frac{dx}{dt} = x$$

$(\because \text{ proportional constant} = 1)$

$$\Rightarrow \int \frac{1}{x}dx = \int dt$$

$$\Rightarrow \log x = t + \log c$$

$$\Rightarrow \log x - \log c = t$$

$$\Rightarrow \quad \log\frac{x}{c} = t$$

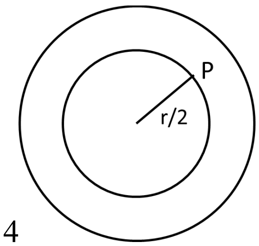
$$\Rightarrow \quad \frac{x}{c} = e^t$$

$$\Rightarrow \quad x = ce^t$$

**100.** *(a)* Let radius of the circle be *r*.

Total possible outcomes = Area of circle = $\pi r^2$

Observe the figure, we have to find the probability of point, P in the ring which will be closer to circumference.




4

Area of ring = Area6 of circle – Area of inner circle

$$= \pi r^2 - \pi\left(\frac{r}{2}\right)^2 = \pi r^2 - \frac{\pi r^2}{4} = \frac{3\pi r^2}{4}$$

So, favourable outcome $= \frac{3\pi r^2}{4}$

Required probability $= \frac{\frac{3\pi r^2}{4}}{\pi r^2} = \frac{3}{4}$

**101.** *(b)* Given differential equation is:

$$\left(\frac{d^2y}{dx^2}\right)^{5/6} = \left(\frac{dy}{dx}\right)^{1/3}$$

Raising both the side to power of 6, to make it a polynomial of derivatives.

$$\Rightarrow \left(\frac{d^2y}{dx^2}\right)^5 = \left(\frac{dy}{dx}\right)^{6/3}$$

$$\Rightarrow \left(\frac{d^2y}{dx^2}\right)^5 = \left(\frac{dy}{dx}\right)^2$$

Highest derivative has a power of 5. So, the order and degree of given differential equation are 2 and 5 respectively.

**102.** *(a)* Since, $\sin^3\theta + \cos^3\theta = 0$

$\Rightarrow (\sin\theta+\cos\theta)(\sin^2\theta-\sin\theta\cos\theta+\cos^2\theta)=0$

$\Rightarrow (\sin\theta+\cos\theta)(1-\sin\theta\cos\theta)=0$

$\Rightarrow (\sin\theta+\cos\theta)\left(1-\frac{\sin 2\theta}{2}\right)=0$

$\Rightarrow \sin\theta+\cos\theta=0$ or $\sin 2\theta=2$

(disccarded since $\sin 2\theta=2$ not possible)

$\Rightarrow \sin\theta+\cos\theta=0 \Rightarrow \sin\theta=-\cos\theta$

$\Rightarrow \tan\theta=-1 \Rightarrow \theta=-\frac{\pi}{4}$

**103.** *(d)* Given differential equation is

$-\text{cosec}^2(x+y)dy=dx$

$\Rightarrow \frac{dy}{dx}=-\sin^2(x+y)$

Put $x+y=t \Rightarrow 1+\frac{dy}{dx}=\frac{dt}{dx}$

$\Rightarrow \frac{dy}{dx}=\frac{dt}{dx}-1$

$\Rightarrow \frac{dt}{dx}-1=-\sin^2 t$

$\Rightarrow \frac{dt}{dx}=1-\sin^2 t$

$\Rightarrow \frac{dt}{dx}=\cos^2 t$

$\Rightarrow \frac{dt}{dx}=\frac{1}{\sec^2 t}$

Separate the variables and then integrate,

$\int \sec^2 t\,dt=\int dx \Rightarrow \tan t=x-c$

**104.** *(c)* Probability that machine stops working

$=P(A\cup B\cup C)$

$\Rightarrow P(A\cup B\cup C)=P(A)+P(A)+P(C)-P(A\cap B)$
$-P(B\cap C)-P(A\cap C)+P(A\cap B\cap C)$

$\Rightarrow P(A\cup B\cup C)=P(A)+P(A)+P(C)-P(A)P(B)$
$-P(B)P(C)-P(A)P(C)+P(A)P(B)P(C)$

[$\because A, B, C$ are independent events]

$\Rightarrow P(A\cup B\cup C)=0.02+0.1+0.05-(0.02\times 0.1)$
$-(0.02\times 0.05)-(0.1\times 0.05)+(0.2\times 0.05\times 0.1)$

$\Rightarrow P(A\cup B\cup C)=0.16$

So, the probability that the machine will not stop working is given by

$=1-P(A\cup B\cup C)=1-0.16=0.84$

**105.** *(a)* Force, $\vec{F}=m\hat{i}-3\hat{j}+\hat{k}$

Due to the force, point moves from $(20,3m,0)$ to $(0,0,7)$.

So, the displacement vector $\overrightarrow{AB}$ is given by,

$\overrightarrow{AB}=-20\hat{i}-3\hat{j}+7\hat{k}$

Work done $=\vec{F}\cdot\overrightarrow{AB}$

$=(m\hat{i}-3\hat{j}+\hat{k})\cdot(-20\hat{i}-3\hat{j}+7\hat{k})$

$=-20m+9m+7$

$=-11m+7$

But work done is given to be $-48$ unit . So, we have

$-11m+7=-48$

$\Rightarrow -11m=-55$

$\Rightarrow \quad m=5$

**106.** *(c)* **Case I**

When unit digit can be 1, 3, 5 or 7 & digit at thousand's place can be 1, 2, 3, 4, 5, 6, 7 or 8.

No. of ways digits can be filled are:

| | | | |
|---|---|---|---|
| 7 | 8 | 7 | 4 |

Total no's $=7\times 8\times 7\times 4=1568$

**Case II**

When unit digit can be 9 & digit at thousand's place can be 1, 2, 3, 4, 5, 6, 7 or 8.

No. of ways digits can be filled are:

| | | | |
|---|---|---|---|
| 8 | 8 | 7 | 1 |

Total no's $=8\times 8\times 7\times 1=448$

**Case III**

When unit digit can be 1, 3, 5 or 7 & digit at thousand's place can be 9.

No. of ways digits can be filled are:

| | | | |
|---|---|---|---|
| 1 | 8 | 7 | 4 |

Total no's $= 1\times 8\times 7\times 4 = 224$

$\therefore$ Number of odd digits between 1000 and 9999 with no digit repeated $= 1568 + 448 + 224 = 2240$

**107.** *(c)* We have $\tan\left(7\frac{1}{2}\right)^{\circ} = \dfrac{\sin\left(7\frac{1}{2}\right)^{\circ}}{\cos\left(7\frac{1}{2}\right)^{\circ}}$

Multiply and divide by $2\sin\left(7\frac{1}{2}\right)^{\circ}$, we get

$$\frac{2\sin^2\left(7\frac{1}{2}\right)^{\circ}}{2\sin\left(7\frac{1}{2}\right)^{\circ}\cos\left(7\frac{1}{2}\right)^{\circ}} = \frac{2\sin^2\left(\frac{15}{2}\right)^{\circ}}{2\sin\left(\frac{15}{2}\right)^{\circ}\cos\left(\frac{15}{2}\right)^{\circ}}$$

$$= \frac{1-\cos\left(2\times\frac{15}{2}\right)^{\circ}}{\sin\left(2\times\frac{15}{2}\right)^{\circ}}$$

$$\left(\because \cos 2\theta = 1-2\sin^2\theta \text{ and } \sin 2\theta = 2\sin\theta\cos\theta\right)$$

$$= \frac{1-\cos 15^{\circ}}{\sin 15^{\circ}} = \frac{1-\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)}{\frac{\sqrt{3}-1}{2\sqrt{2}}} = \sqrt{2}\left(\sqrt{3}+1\right)-\left(2+\sqrt{3}\right)$$

$$= \sqrt{6}+\sqrt{2}-2-\sqrt{3} = \sqrt{6}-\sqrt{3}+\sqrt{2}-2$$

**108.** *(d)* Any 5-digits number formed by the digits 1, 2, 3, 4 and 5 (without repetition) will be always divisible by number 3, because sum of digits of any number, thus formed $1+2+3+4+5=15$, is divisible by 3.

Hence, prime number of 5 digits cannot be obtained by using the digits 1, 2, 3, 4, 5

**109.** *(b)* Consider $\dfrac{\sin\theta+\cos\theta-\tan\theta}{\sec\theta+\operatorname{cosec}\theta-\cot\theta}$

Now, put value of $\theta = \dfrac{3\pi}{4}$, we get

$$= \frac{\sin\frac{3\pi}{4}+\cos\frac{3\pi}{4}-\tan\frac{3\pi}{4}}{\sec\frac{3\pi}{4}+\operatorname{cosec}\frac{3\pi}{4}-\cot\frac{3\pi}{4}}$$

$$= \frac{\sin\frac{\pi}{4}-\cos\frac{\pi}{4}+\tan\frac{\pi}{4}}{-\frac{1}{\cos\frac{\pi}{4}}+\frac{1}{\sin\frac{\pi}{4}}+\frac{1}{\tan\frac{\pi}{4}}} = \frac{\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}+1}{-\sqrt{2}+\sqrt{2}+1} = 1$$

**110.** *(c)* Given matrix is $A = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 4+4 & 4+4 \\ 4+4 & 4+4 \end{bmatrix} = \begin{bmatrix} 2^3 & 2^3 \\ 2^3 & 2^3 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix}\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 16+16 & 16+16 \\ 16+16 & 16+16 \end{bmatrix} = \begin{bmatrix} 2^5 & 2^5 \\ 2^5 & 2^5 \end{bmatrix}$$

Going this way, we get

$$A^4 = \begin{bmatrix} 2^7 & 2^7 \\ 2^7 & 2^7 \end{bmatrix} \Rightarrow A^n = \begin{bmatrix} 2^{2n-1} & 2^{2n-1} \\ 2^{2n-1} & 2^{2n-1} \end{bmatrix}$$

**111.** *(c)* The given word is 'NATION'.

Total number of words that can be formed from given word 'NATION' $= \dfrac{6!}{2!} = \dfrac{6\times5\times4\times3\times2\times1}{2\times1} = 360$

Now numbers of words that can be formed from given word 'NATION', so that all vowels never come together.

$$= 360-\left[4!\times\frac{3!}{2!}\right] = 360-\left[24\times3\right]$$

$$= 360-72 = 288$$

**112.** *(d)* Vectors are $2\hat{i}+m\hat{j}-3n\hat{k}$ and $5\hat{i}+3m\hat{j}+n\hat{k}$.

The magnitudes are given as $\sqrt{14}$ and $\sqrt{35}$ respectively. This gives,

$$\left|2\hat{i}+m\hat{j}-3n\hat{k}\right|=\sqrt{14}$$

$$\Rightarrow \sqrt{4+m^2+9n^2}=\sqrt{14}$$

$$\Rightarrow 4+m^2+9n^2=14$$

$$\Rightarrow m^2+9n^2=10 \quad ...(\text{i})$$

Also,

$$\left|5\hat{i}+3m\hat{j}+n\hat{k}\right|=\sqrt{35}$$

$$\Rightarrow \sqrt{25+9m^2+n^2}=\sqrt{35}$$

$$\Rightarrow 25+9m^2+n^2=35$$

$$\Rightarrow 9m^2+n^2=10 \quad ...(\text{ii})$$

From (i) and (ii),

$$m^2+9n^2=9m^2+n^2$$

$$\Rightarrow 8n^2=8m^2$$

$$\Rightarrow n^2=m^2$$

$$\Rightarrow n=\pm m$$

So, $n$ takes 2 values and $m$ takes 2 values.

**113.** *(b)* Given two lines are $8x-10y=66$ and $40x-18y=214$.

This gives,

$$8x-10y=66 \Rightarrow 10y=8x-66$$

$$\Rightarrow b_{yx}=\frac{8}{10}=\frac{4}{5}$$

$$40x-18y=214 \Rightarrow 40x=18x+214$$

$$\Rightarrow b_{xy}=\frac{18}{40}=\frac{9}{20}$$

Thus, $r=\pm\sqrt{\frac{4}{5}\times\frac{9}{20}}=\frac{3}{5}$

Also, $\sigma_x=\sqrt{9}=3$

$$\therefore \sigma_y=\frac{b_{yx}\times\sigma_x}{r}=\frac{\frac{4}{5}\times 3}{r}=\frac{12}{5r}=\frac{12}{5}\times\frac{5}{3}=4$$

**114.** *(c)* Given center of sphere is $(6,-1,2)$ and equation of plane is $2x-y+2z-2=0$.

Since, sphere touches the plane, therefore perpendicular distance from center to the plane is radius of the sphere.

$$\therefore \text{Radius}=\frac{2(6)-1(-1)+2(2)-2}{\sqrt{4+1+4}}=\frac{15}{3}=5$$

So, required equation of sphere is

$$(x-6)^2+(y+1)^2+(z-2)^2=5^2$$

$$\Rightarrow x^2+y^2+z^2-12x+2y-4z+16=0$$

**115.** *(c)* Here $\vec{\mathbf{F}}=3\hat{i}+2\hat{j}-\hat{k}$

and

$$\vec{r_1}-\vec{r_2}=(\hat{i}-\hat{j}+\hat{k})-(2\hat{i}-3\hat{j}-\hat{k})$$

$$=-\hat{i}+2\hat{j}+2\hat{k}$$

Moment of couple $=(\vec{r_1}-\vec{r_2})\times\vec{F}$

$$=(-\hat{i}+2\hat{j}+2\hat{k})\times(3\hat{i}+2\hat{j}-\hat{k})$$

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -1 & 2 & 2\\ 3 & 2 & -1\end{vmatrix}$$

$$=\hat{i}(-2-4)-\hat{j}(1-6)+\hat{k}(-2-6)$$

$$=-6\hat{i}+5\hat{j}-8\hat{k}$$

Magnitude of moment is

$$\left|-6\hat{i}+5\hat{j}-8\hat{k}\right|=\sqrt{26+25+64}$$

$$=5\sqrt{5}$$

**116.** *(d)* To find the consistent demand, we will calculate coefficient of variance.

We know that coefficient of variance $=\frac{\sqrt{\text{SD}}}{\text{mean}}$

Also, $\text{SD}=\sqrt{\text{variance}}$

Coefficient of variance of A $=\frac{\sqrt{12}}{60}=\frac{3.46}{60}=0.057$

Coefficient of variance of B $=\frac{\sqrt{25}}{90}=\frac{5}{90}=0.055$

Coefficient of variance of C $=\frac{\sqrt{36}}{80}=\frac{6}{80}=0.075$

Coefficient of variance of D $=\frac{\sqrt{16}}{120}=\frac{4}{120}=0.033$

We see that minimum coefficient of variance is of D, so product D is consistent.

**117.** *(d)* Here $A=\begin{bmatrix}-2 & 1 & 1\\ 1 & -2 & 1\\ 1 & 1 & -2\end{bmatrix}$, $B=\begin{bmatrix}l\\ m\\ n\end{bmatrix}$

and $X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$

$\therefore |A|=-2(4-1)-1(-2-1)+(1+2)$

$=-6+3+3=0$

Now, $\text{Adj}\,A=\begin{bmatrix}3 & 3 & 3\\ 3 & 3 & 3\\ 3 & 3 & 3\end{bmatrix}$

$\therefore (\text{Adj}\,A)B=0$

So, the given system of equations has an infinitely many solutions.

**118.** *(a)* Given $x+z=0,\ y=0$ and $20x=15y=12z$.

$\Rightarrow \frac{x}{1}=\frac{y}{0}=\frac{z}{-1}$ and $\frac{x}{3}=\frac{y}{4}=\frac{z}{5}$

Let $\theta$ be the angle between two lines.

$\therefore \cos\theta=\left|\frac{(1)(3)+(0)(4)+(-1)(5)}{\sqrt{1+0+1}\sqrt{9+16+25}}\right|$

$=\left|\frac{3+0-5}{\sqrt{2}\sqrt{50}}\right|$

$=\frac{2}{\sqrt{2}\,5\sqrt{2}}=\frac{1}{5}$

$\therefore \theta=\cos^{-1}\left(\frac{1}{5}\right)$

***119.*** *(b)* Let there be $x$ number of boys and $y$ number of girls.

Total students = $x+y$

Total weight of the students $=(x+y)60$

Total weight of boys $=x\times 70$

Total weight of girls $=y\times 55$

Hence, $(x+y)60=70x+55y$

$\Rightarrow 60x+60y=70x+55y$

$\Rightarrow 5y=10x$

$\Rightarrow \frac{x}{y}=\frac{5}{10}$

$\Rightarrow x:y=1:2$

***120.*** *(a)* Since $l^2+m^2+n^2=1$

$\left(\because\right.$ a line makes the same angle $\alpha$ with $x$ and $y$ axis and $\theta$ with $z$ axis$\left.\right)$

Also, $\sin^2\theta=1-\cos^2\theta$

$\Rightarrow \sin^2\theta=2\sin^2\alpha$

$\Rightarrow 1-\cos^2\theta=2\left(1-\cos^2\alpha\right)$

$\left(\because \sin^2 A+\cos^2 A=1\right)$

$\Rightarrow \cos^2\theta=2\cos^2\alpha-1 \quad ...(ii)$

From (i) and (ii),

$2\cos^2\alpha+2\cos^2\alpha-1=1$

$\Rightarrow 4\cos^2\alpha=1$

$\Rightarrow \cos^2\alpha=\frac{1}{4}$

$\Rightarrow \cos\alpha=\pm\frac{1}{2}$

$\Rightarrow \alpha=\frac{\pi}{4},\frac{3\pi}{4}$