HINTS & SOLUTION

1. *(a)* Since the first term $= p$ and common difference $= q$.

Sum of first 10 terms $= \frac{10}{2}\left[2p+(10-1)q\right]$ and

Sum of first 5 terms $= \frac{5}{2}\left[2p+(5-1)q\right]$

According to the question,

$\frac{10}{2}[2p+9q] = 4\times\frac{5}{2}[2p+4q]$

$\Rightarrow 2p+9p = 4p+8q$

$\Rightarrow 2p = q$

$\Rightarrow p:q = 1:2$

Hence, option *(a)* is correct.

2. *(b)* Let $\frac{k}{k+1}$ and $\frac{k+1}{k+2}$ are the roots of the equation $4\beta^2+\lambda\beta-2=0$, then

Sum of roots $= \frac{k}{k+1}+\frac{k+1}{k+2} = -\frac{\lambda}{4}$ ....(i)

Product of roots $= \frac{k}{k+1}\times\frac{k+1}{k+2} = -\frac{2}{4}$

$\Rightarrow \frac{k}{k+2} = -\frac{1}{2} \Rightarrow 2k = -k-2 \Rightarrow k = -\frac{2}{3}$

Putting the value of k is (i), we get

$\frac{-\frac{2}{3}}{-\frac{2}{3}+1}+\frac{-\frac{2}{3}+1}{-\frac{2}{3}+2} = -\frac{\lambda}{4}$

$\Rightarrow \frac{-\frac{2}{3}}{\frac{1}{3}}+\frac{\frac{1}{3}}{\frac{4}{3}} = -\frac{\lambda}{4} \Rightarrow -2+\frac{1}{4} = -\frac{\lambda}{4}$

$\Rightarrow \lambda = 7$

Hence, option *(b)* is correct.

3. *(c)* We have $U = \{x \mid x^5-6x^4+11x^3-6x^2=0\}$

Solving for the values of x, we get

$x = \{0,1,2,3\}$

For $A = \{x \mid x^2-5x+6=0\}$

Solving for the values of x, we get

$x = \{2,3\}$

For $B = \{x \mid x^2-3x+2=0\}$

Solving for the values of x, we get

$x = \{2,1\}$

This gives $A\cap B = 2$

$\therefore (A\cap B)' = U-(A\cap B)$

$= \{0,1,2,3\}-\{2\}$

$= \{0,1,3\}$

Hence, option *(c)* is correct.

4. *(d)* Given that $S_m = n$ and $S_n = m$

Then the sum of the $(m+n)$ terms is given by the direct formula $S_{m+n} = -(m+n)$

Hence, option *(d)* is correct.

5. *(c)* Let the angles are α and β, then $\alpha-\beta = 1°$

$\Rightarrow \alpha-\beta = \frac{\pi}{180°}$ is circular measure ....(i)

As given $\alpha+\beta = 1$ ....(ii)

On solving equations (i) and (ii), we get

$\alpha = \frac{1}{2}\left[1+\frac{\pi}{180}\right]$ and $\beta = \frac{1}{2}\left[1-\frac{\pi}{180}\right]$

β is the smaller angle.

Hence, smaller angle $= \frac{1}{2}\left[1-\frac{\pi}{180}\right]$

Hence, option *(c)* is correct.

6. *(d)* In the expansion of $\left(\frac{x\sqrt{y}}{3}-\frac{3}{y\sqrt{x}}\right)^{12}$, then middle term is $\frac{12}{2}+1=7^{\text{th}}$ term

$$T_{r+1}={}^{12}C_r\left[\frac{x\sqrt{y}}{3}\right]^{12-r}\cdot\left(-\frac{3}{y\sqrt{x}}\right)^r$$

$$\therefore T_7=T_{6+1}={}^{12}C_6\left(\frac{x\sqrt{y}}{3}\right)^6\left(-\frac{3}{y\sqrt{x}}\right)^6$$

$$={}^{12}C_6\frac{x^6y^3}{y^6x^3}={}^{12}C_6x^3y^{-3}=C(12,6)x^3y^{-3}$$

Hence, option *(d)* is correct.

7. *(c)* Let $A=(1,0),B=(0,-6)$ and $C=(3,4)$

Equation of *AB* is L: $\frac{y-0}{-6-0}=\frac{x-1}{0-1}$

$$\Rightarrow\frac{y}{-6}=\frac{x-1}{-1}\Rightarrow y=6x-6\Rightarrow 6x-y-6=0$$

Equation of circle C with *AB* as diameter is

$$(x-1)(x-0)+(y-0)(y+6)=0$$

$$\Rightarrow x^2-x+y^2+6y=0$$

The system of circle passing through the intersection of the circle C and line L is given by $C+kL=0$

$$\Rightarrow x^2-x+y^2+6y+k(6x-y-6)=0$$

This circle is passing through $(3,4)$

$$\therefore\ (3)^2-3+(4)^2+6(4)+k(6(3)-4-6)=0$$

$$\Rightarrow 9-3+16+24+k(18-10)=0$$

$$\Rightarrow 46+8k=0\Rightarrow 8k=-46\Rightarrow k=\frac{-23}{4}$$

$\therefore$ Equation of circle is

$$x^2-x+y^2+6y+\left(\frac{-23}{4}\right)(6x-y-6)=0$$

$$\Rightarrow 4x^2-4x+4y^2+24y-138x+23y+138=0$$

$$\Rightarrow 4x^2+4y^2-142x+47y+138=0$$

Hence, option *(c)* is correct.

8. *(a)* Consider, $\tan\left(\tan^{-1}x+\tan^{-1}y+\tan^{-1}z\right)-\cot\left(\cot^{-1}x+\cot^{-1}y+\cot^{-1}z\right)$

$$=\tan\left(\tan^{-1}x+\tan^{-1}y+\tan^{-1}z\right)-\cot\left(\frac{\pi}{2}-\tan^{-1}x+\frac{\pi}{2}-\tan^{-1}y+\frac{\pi}{2}-\tan^{-1}z\right)$$

$$\left[\because\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}\right]$$

$$=\tan\left(\tan^{-1}x+\tan^{-1}y+\tan^{-1}z\right)-\cot\left\{\frac{3\pi}{2}-\left(\tan^{-1}x+\tan^{-1}y+\tan^{-1}z\right)\right\}$$

$$=\tan\left(\tan^{-1}x+\tan^{-1}y+\tan^{-1}z\right)-\tan\left(\tan^{-1}x+\tan^{-1}y+\tan^{-1}z\right)$$

$$=0$$

Hence, option *(a)* is correct.

9. *(c)* Let α and β be roots of $x^2-(p-2)x-(p+1)=0$

Then, $\alpha+\beta=(p-2)$ and $\alpha\beta=-(p+1)$

Since, $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=5$

$$\Rightarrow(\alpha+\beta)^2-2\alpha\beta=5$$

$$\Rightarrow(p-2)^2+2(p+1)=5$$

$$\Rightarrow p^2-4p+4+2p+2=5$$

$$\Rightarrow p^2-2p+1=0$$

$$\Rightarrow(p-1)^2=0$$

$$\Rightarrow p=1$$

Hence, option *(c)* is correct.

10. *(a)* R is a relation defined on the set Z of integers as follows:

$mRn\Leftrightarrow m+n$ is odd

(1) Then, $mRn=2m$ and $mRn=2n$ are not odd multiples of 2 are not good. Thus, it is not reflexive.

(2) If m and n are numbers such that $mRn\Leftrightarrow m+n$ is odd.

Thus, $nRm\Leftrightarrow n+m$ is odd.

$\therefore$ This relation is symmetric.

(3) $mRn = m+n$, if there is third number p and $nRp = n+p$ is odd. (for ex: $2+3=5$ is odd, $3+4=7$ is odd, but $2+4=6$ is not odd). Then $mRp = m+p$ may not be odd. So, this relation is not transitive.

Hence, option *(a)* is correct.

11. *(a)* Given that a, b, c are in GP.

Let r be common ratio of GP.

So, $a = a, b = ar$ and $c = ar^2$ ....(i)

Also, given that $a, 2b, 3c$ are in AP.

$\Rightarrow 2b = \frac{a+3c}{2}$

$\Rightarrow 4b = a+3c$

From equation (i), we get

$\Rightarrow 4ar = a + 3ar^2$

$\Rightarrow 3ar^2 - 4ar + a = 0$

$\Rightarrow 3r^2 - 4r + 1 = 0$

$\Rightarrow (r-1)(3r-1) = 0$

$\Rightarrow r = 1$ or $r = \frac{1}{3}$

Hence, option *(a)* is correct.

12. *(a)* Given that $\sqrt{3}\cos ec 20° - \sec 20°$

$= \frac{\sqrt{3}}{\sin 20°} - \frac{1}{\cos 20°} = \frac{\sqrt{3}\cos 20° - \sin 20°}{\sin 20° \cos 20°}$

$= \frac{4 \cdot \frac{1}{2}\left(\sqrt{3}\cos 20° - \sin 20°\right)}{2\sin 20° \cos 20°}$

$= \left(\frac{\sqrt{3}}{2}\cos 20° - \frac{1}{2}\sin 20°\right)\left(\frac{4}{2\sin 20° \cos 20°}\right)$

$= \left(\sin 60° \cos 20° - \cos 60° \sin 20°\right)\left(\frac{4}{\sin 40°}\right)$

$= \sin\left(60° - 20°\right)\left(\frac{4}{\sin 40°}\right)$

$= \sin\left(40°\right)\left(\frac{4}{\sin 40°}\right)$

$= 4$

Hence, option *(a)* is correct.

13. *(c)* Given that $P(5, r) = P(6, r-1)$

$\Rightarrow {}^5P_r = {}^6P_{r-1}$

$\Rightarrow \frac{5!}{(5-r)!} = \frac{6!}{(6-r+1)!}$

$\Rightarrow \frac{5!}{(5-r)!} = \frac{6 \times 5!}{(7-r)(6-r)(5-r)!}$

$\Rightarrow (7-r)(6-r) = 6$

$\Rightarrow 42 - 13r + r^2 = 6$

$\Rightarrow r^2 - 13r + 36 = 0$

$\Rightarrow (r-9)(r-4) = 0$

$\Rightarrow r = 4 \quad (\because r \neq 9)$

Hence, option *(c)* is correct.

14. *(d)* The given integral is $\int \frac{dx}{\left(x^2+a^2\right)\left(x^2+b^2\right)}$

Breaking the expression under integral into partial fraction

$$\frac{1}{\left(x^2+a^2\right)\left(x^2+b^2\right)} = \left(\frac{1}{\left(x^2+a^2\right)} - \frac{1}{\left(x^2+b^2\right)}\right) \times \frac{1}{b^2-a^2}$$

The given integral is

$$\frac{1}{b^2-a^2}\int\left(\frac{1}{\left(x^2+a^2\right)} - \frac{1}{\left(x^2+b^2\right)}\right)dx$$

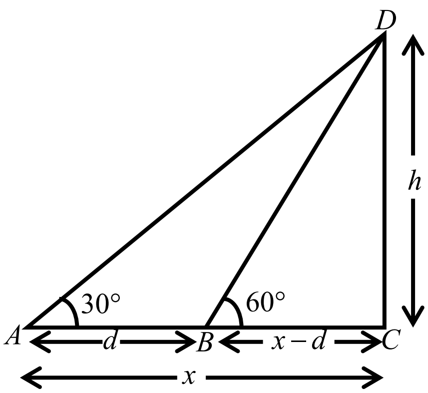
$$= \frac{1}{b^2-a^2}\left[\int \frac{1}{\left(x^2+a^2\right)}dx - \int \frac{1}{\left(x^2+b^2\right)}dx\right]$$

$$= \frac{1}{b^2-a^2}\left[\frac{\tan^{-1}\left(\frac{x}{a}\right)}{a} - \frac{\tan^{-1}\left(\frac{x}{b}\right)}{b}\right]$$

Hence, option *(d)* is correct.

15. *(b)* Let DC be the pillar of height h and A be the point at distance x from the pillar such that $\angle CAD = 30°$.

On walking a distance d towards pillar (point B) $\angle CBD = 60^\circ$.



So, in ΔBCD

$$\tan 60^\circ = \frac{CD}{BC}$$

$$\Rightarrow \sqrt{3} = \frac{h}{x-d}$$

$$\Rightarrow h = \sqrt{3}(x-d) \quad \text{....(i)}$$

and in ΔACD,

$$\tan 30^\circ = \frac{CD}{AC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$\Rightarrow x = h\sqrt{3}$$

$$\Rightarrow x = 3(x-d) \quad \{\text{using (i)}\}$$

$$\Rightarrow x = 3x - 3d$$

$$\Rightarrow 2x = 3d$$

$$\Rightarrow x = \frac{3d}{2}$$

Hence, option *(b)* is correct.

16. *(c)* We have $f(x) = \begin{vmatrix} 1+\sin^2 x & \cos^2 x & 4\sin 2x \\ \sin^2 x & 1+\cos^2 x & 4\sin 2x \\ \sin^2 x & \cos^2 x & 1+4\sin 2x \end{vmatrix}$

Applying $C_1 \to C_1 + C_2$

$$f(x) = \begin{vmatrix} 2 & \cos^2 x & 4\sin 2x \\ 2 & 1+\cos^2 x & 4\sin 2x \\ 1 & \cos^2 x & 1+4\sin 2x \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$f(x) = \begin{vmatrix} 2 & \cos^2 x & 4\sin 2x \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$$

$$f(x) = 2 + 4\sin 2x$$

$\therefore -1 \le \sin 2x \le 1$, maximum value of $\sin 2x = 1$

Thus, maximum value of $f(x) = 2 + 4 = 6$

Hence, option *(c)* is correct.

17. *(b)* Given that $u = \sin^{-1}(x-y)$ and $x = 3t, y = 4t^3$

So, $u = \sin^{-1}(3t - 4t^3)$

Let $t = \sin\theta \Rightarrow \theta = \sin^{-1} t$

So, $u = \sin^{-1}(3\sin\theta - 4\sin^3\theta) = \sin^{-1}(\sin 3\theta) = 3\theta$

Hence, $u = 3\sin^{-1} t$

$$\therefore \frac{du}{dt} = 3 \cdot \frac{1}{\sqrt{1-t^2}} = 3(1-t^2)^{-\frac{1}{2}}$$

Hence, option *(b)* is correct.

18. *(d)* Sum and product of roots of $Ax^2 - 4x + 1 = 0$ will be $\alpha + \gamma = \frac{4}{A}$ and $\alpha\gamma = \frac{1}{A}$ respectively.

Sum and product of roots of $Bx^2 - 6x + 1 = 0$ will be $\beta + \delta = \frac{6}{B}$ and $\beta\delta = \frac{1}{B}$ respectively.

Then, $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$ and $\frac{1}{\delta}$ will be in AP.

$$\Rightarrow \frac{1}{\beta} - \frac{1}{\alpha} = \frac{1}{\delta} - \frac{1}{\gamma} \Rightarrow \frac{1}{\beta} - \frac{1}{\delta} = \frac{1}{\alpha} - \frac{1}{\gamma}$$

$$\Rightarrow \frac{\delta - \beta}{\beta\delta} = \frac{\gamma - \alpha}{\alpha\gamma}$$

$$\Rightarrow \frac{\sqrt{(\delta+\beta)^2 - 4\beta\delta}}{\beta\delta} = \frac{\sqrt{(\gamma+\alpha)^2 - 4\alpha\gamma}}{\alpha\gamma}$$

$$\Rightarrow 36 - 4B = 16 - 4A$$

$$\Rightarrow 4A + 4B = 20$$

$$\Rightarrow A + B = 5$$

It is possible only, if $A = -3$ and $B = 8$

Hence, option *(d)* is correct.

19. *(d)* The given relation is $aRb \Leftrightarrow a+2b$ is an integral multiple of 3.

In this relation

$aRa \Leftrightarrow a+2a=3a$, an integral multiple of 3. So, it is reflexive.

$aRb \Leftrightarrow a+2b$ and

$bRa = b+2a+4b-4b = 2(a+2b)-3b$ is also an integral multiple of 3.

So, it is symmetric.

Let there be another value c, $bRc = b+2c$ be an integral multiple of 3.

Then $aRc = a+2c$

So, $aRb + bRc = a+2b+b+2c = a+2c+3b$ is integral multiple of 3. Hence, $a+2c$ is also integral multiple of 3.

So, aRb and $bRc \Rightarrow aRc$. So, it is transitive.

Therefore, relation is reflexive, symmetric as well as transitive.

Hence, R is an equivalence relation.

Hence, option *(d)* is correct.

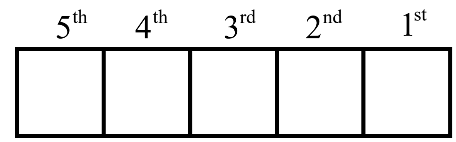
20. *(b)* Given that $\tan 9° - \tan 27° - \tan 63° + \tan 81°$

$$= \tan 9° - \tan 27° - \tan(90°-27°) + \tan(90°-9°)$$
$$= \tan 9° - \tan 27° - \cot 27° + \cot 9°$$
$$= (\tan 9° + \cot 9°) - (\tan 27° + \cot 27°)$$
$$= \frac{1}{\sin 9° \cos 9°} - \frac{1}{\sin 27° \cos 27°}$$
$$= \frac{2}{\sin 18°} - \frac{2}{\sin 54°}$$
$$= \frac{2}{\sin 18°} - \frac{2}{\sin(90°-36°)}$$
$$= \frac{2}{\sin 18°} - \frac{2}{\cos 36°}$$
$$= 2\left[\frac{4}{\sqrt{5}-1} - \frac{4}{\sqrt{5}+1}\right] = 8 \times \frac{2}{4} = 4$$

Hence, option *(b)* is correct.

21. *(b)* We have to construct 5 digit numbers which are greater than 41000.

So, we have only 2 ways to choose 5^{th} digit.



[$\because$ only 4 or 5 can come at 5^{th} place]

Thus, for 4^{th} place we have 4 ways to choose digits.

For 3^{rd} place we have 3 ways.

For 2^{nd} place we have 2 ways.

And for the unity place we have only 1 way.

$\therefore$ Required number of ways $2 \times 4 \times 3 \times 2 \times 1 = 48$

Hence, option *(b)* is correct.

22. *(b)* The length of latus rectum of an ellipse is $\frac{2b^2}{a}$ where b is semi-minor axis and a is semi-major axis.

As given,

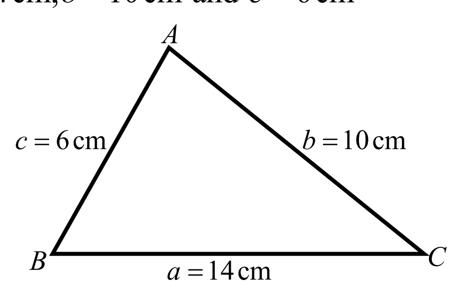
$$\frac{2b^2}{a} = b \Rightarrow 2b = a \Rightarrow \frac{b}{a} = \frac{1}{2}$$

We know that eccentricity $e = \sqrt{1-\frac{b^2}{a^2}} = \sqrt{1-\frac{1}{4}} = \frac{\sqrt{3}}{2}$

Hence, option *(b)* is correct.

23. *(b)* We know that the largest side has greatest angle opposite it.

$\therefore a = 14\text{cm}, b = 10\text{cm}$ and $c = 6\text{cm}$



$$\therefore \cos A = \frac{c^2+b^2-a^2}{2bc} = \frac{36+100-196}{2 \times 6 \times 10}$$
$$\cos A = -\frac{1}{2} = \cos 120°$$
$$\Rightarrow \angle A = 120°$$

Hence, option *(b)* is correct.

24. *(c)* Given that $x+y=t-\frac{1}{t}$ and $x^2+y^2=t^2+\frac{1}{t^2}$

$\therefore (x+y)^2=x^2+y^2+2xy$

$\Rightarrow\left(t-\frac{1}{t}\right)^2=\left(t^2+\frac{1}{t^2}\right)+2xy$

$\Rightarrow t^2+\frac{1}{t^2}-2=t^2+\frac{1}{t^2}+2xy$

$\Rightarrow -2=2xy$

$\Rightarrow xy=-1$

Also,

$(x-y)^2=(x+y)^2-4xy$

$\Rightarrow (x-y)^2=\left(t-\frac{1}{t}\right)^2-4(-1)$

$\Rightarrow (x-y)^2=t^2+\frac{1}{t^2}-2+4$

$\Rightarrow (x-y)^2=\left(t+\frac{1}{t}\right)^2$

$\Rightarrow x-y=t+\frac{1}{t}$

$\Rightarrow x=t,\ y=-\frac{1}{t}\Rightarrow xy=-1$

$\therefore x\frac{dy}{dx}+y=0$

$\Rightarrow \frac{dy}{dx}=-\frac{y}{x}=\frac{1}{t^2}=\frac{1}{x^2}$

Hence, option *(c)* is correct.

25. *(a)* Required area $=\int_{-2}^{2}\left(1-\frac{x^2}{4}\right)dx$

Since, $\left(1-\frac{x^2}{4}\right)$ is an even function, therefore

$$\int_{-2}^{2}\left(1-\frac{x^2}{4}\right)dx=2\int_{0}^{2}\left(1-\frac{x^2}{4}\right)dx$$

$$=2\left[x-\frac{x^3}{13}\right]_0^2=2\left[2-\frac{2^3}{12}\right]$$

$$=2\left(2-\frac{2}{3}\right)=\frac{8}{3}\text{sq unit}$$

Hence, option *(a)* is correct.

26. *(b)* The equation is $\left|x^2-x-6\right|=x+2$

For $x\ge 0$

$\therefore\ x^2-x-6=x+2$

$\Rightarrow x^2-2x-8=0$

$\Rightarrow (x-4)(x+2)=0$

$\Rightarrow x=4,-2$

For $x<0$

$-x^2+x+6=x+2$

$\Rightarrow x^2=4\Rightarrow x=\pm 2$

Thus, the number of real solutions are $-2, 2$ and 4.

So, the numbers of real solutions of the equation $\left|x^2-x-6\right|=x+2$ are 3.

Hence, option *(b)* is correct.

27. *(d)* Let $A=\{(n,2n):n\in N\}$ and $B=\{(2n,3n):n\in N\}$

Lisitng few members of each set:

$A=\{(1,2),(2,4),(3,6),....\}$

$B=\{(2,3),(4,6),(6,9),....\}$

There is no member common to both these sets.

Therefore, $A\cap B=\phi$

Hence, option *(d)* is correct.

28. *(c)* Given that the n^{th} term, $T_n=3n+7$

$\therefore$ Sum of n terms, $S_n=\sum T_n$

$=\sum(3n+7)=3\sum n+7\sum 1$

$=\frac{3n(n+1)}{2}+7n=n\left[\frac{3n+3+14}{2}\right]=n\left[\frac{3n+17}{2}\right]$

$\therefore$ Sum of 50 terms, $S_{50} = 50\left[\frac{3\times50+17}{2}\right]$

$= 50\left[\frac{167}{2}\right] = 4175$

Hence, option *(c)* is correct.

29. *(a)* We have $\sin(A+B)\sin(A-B)$

$= \frac{1}{2}\{2\sin(A+B)\sin(A-B)\}$

$= \frac{1}{2}\{\cos(A-B-A-B)-\cos(A-B+A+B)\}$

$[\because 2\sin X\sin Y = \cos(X-Y)-\cos(X+Y)]$

$= \frac{1}{2}\{\cos 2B - \cos 2A\}$

Similarly, $\sin(B+C)\sin(B-C) = \frac{1}{2}\{\cos 2C - \cos 2B\}$

And, $\sin(C+A)\sin(C-A) = \frac{1}{2}\{\cos 2A - \cos 2C\}$

$\therefore \sin(A+B)\sin(A-B)+\sin(B+C)\sin(B-C)$
$+\sin(C+A)\sin(C-A)$

$= \frac{1}{2}\{\cos 2C - \cos 2B + \cos 2A - \cos 2C + \cos 2B - \cos 2A\}$

$= 0$

Hence, option *(a)* is correct.

30. *(d)* As given $(p+2q)x+(p-3q)y = p-q$

$\Rightarrow px+2qx+py-3qy = p-q$

$\Rightarrow p(x+y)-(3y-2x) = p-q$

Equating co-efficient of p and q, we get

$x+y=1$ and $3y-2x=1$

Solving these, we get

$x=\frac{2}{5}, y=\frac{3}{5}$

So, line passes through $\left(\frac{2}{5},\frac{3}{5}\right)$

Hence, option *(d)* is correct.

31. *(c)* Since, the probabilities of failure for engines *A, B* and *C* are $P(A)=0.03, P(B)=0.02$ and $P(C)=0.05$ respectively.

The aircraft will crash only when all three engine fail.

So, probability that it crashes $= P(A)P(B)P(C)$

$= 0.03\times0.02\times0.05$

$= 0.00003$

Hence, the probability that the aircraft will not crash is

$= 1-0.00003$

$= 0.99997$

Hence, option *(c)* is correct.

32. *(c)* Let $\begin{bmatrix}1 & 2\\0 & 3\end{bmatrix} = B$, then

$BA = \begin{bmatrix}-1 & 0\\6 & 3\end{bmatrix} \Rightarrow A = B^{-1}\begin{bmatrix}-1 & 0\\6 & 3\end{bmatrix}$ and $|B| = 3$

$\therefore \text{adj}\,B = \begin{bmatrix}3 & -2\\0 & 1\end{bmatrix}$ and $B = \frac{1}{3}\begin{bmatrix}3 & -2\\0 & 1\end{bmatrix}$

$\Rightarrow A = \frac{1}{3}\begin{bmatrix}3 & -2\\0 & 1\end{bmatrix}\begin{bmatrix}-1 & 0\\6 & 3\end{bmatrix}$

$= \frac{1}{3}\begin{bmatrix}-3-12 & -6\\6 & 3\end{bmatrix} = \begin{bmatrix}-5 & -2\\2 & 1\end{bmatrix}$

Hence, option *(c)* is correct.

33. *(c)* Let $I = \int(x^2+1)^{\frac{5}{2}}x\,dx$

Put $x^2+1=t \Rightarrow 2xdx = dt \Rightarrow xdx = \frac{1}{2}dt$

$\therefore I = \frac{1}{2}\int t^{\frac{5}{2}}dt = \frac{1}{2}\left(\frac{t^{7/2}}{7/2}\right)+c = \frac{1}{7}t^{7/2}+c$

$\Rightarrow I = \frac{1}{7}(x^2+1)^{7/2}+c$

Hence, option *(c)* is correct.

34. *(b)* Here $\frac{d\left(\sqrt{1-x^2}\right)}{d\left(\sin^{-1}x\right)} = \frac{\frac{d}{dx}\left(\sqrt{1-x^2}\right)}{\frac{d}{dx}\left(\sin^{-1}x\right)}$ is to be found.

We have $f(x)=\sqrt{1-x^2}$

So, $f'(x)=\frac{d}{dx}\left(\sqrt{1-x^2}\right)=\frac{1}{2\sqrt{1-x^2}}(-2x)=\frac{-x}{\sqrt{1-x^2}}$

Also, $g(x)=\sin^{-1}x$

So, $g'(x)=\frac{d}{dx}\left(\sin^{-1}x\right)=\frac{1}{\sqrt{1-x^2}}$

$\therefore \frac{f'(x)}{g'(x)}=\frac{-x/\sqrt{1-x^2}}{1/\sqrt{1-x^2}}=-x$

Hence, option *(b)* is correct.

35. *(a)* Let $(1,3,4),(-1,6,10),(-7,4,7)$ and $(-5,1,1)$ be the coordinates of points A,B,C and D respectively. Therefore,

$AB=\sqrt{(-1-1)^2+(6-3)^2+(10-4)^2}=\sqrt{4+9+36}=7$

$BC=\sqrt{(-7+1)^2+(4-6)^2+(7-10)^2}=\sqrt{36+4+9}=7$

$CD=\sqrt{(-5+7)^2+(1-4)^2+(1-7)^2}=\sqrt{4+9+36}=7$

$DA=\sqrt{(1+5)^2+(3-1)^2+(4-1)^2}=\sqrt{36+4+9}=7$

Also, $AC=\sqrt{(-7-1)^2+(4-3)^2+(7-4)^2}$

$=\sqrt{64+1+9}=\sqrt{74}$

And, $BD=\sqrt{(-5+1)^2+(1-6)^2+(1-10)^2}$

$=\sqrt{16+25+81}=\sqrt{122}$

$\because AB=BC=CD=DA$ but $BD\neq AC$

$\therefore A,B,C$ and D are the vertices of a rhombus.

Hence, option *(a)* is correct.

36. *(b)* If $\vec{p}$ and $\vec{q}$ are unit vectors which makes an angle $\frac{\pi}{3}$ with each other.

Then, $\vec{p}\cdot\vec{q}=|\vec{p}||\vec{q}|\cos\frac{\pi}{3}=\frac{1}{2}$

Now, $\left|\vec{p}-\frac{1}{2}\vec{q}\right|^2=|\vec{p}|^2+\frac{1}{4}|\vec{q}|^2-\frac{2}{2}\vec{p}\cdot\vec{q}$

$=1+\frac{1}{4}-\frac{1}{2}\quad\left[\text{since}\,|\vec{p}|=|\vec{q}|=1\right]$

$=\frac{5}{4}-\frac{1}{2}=\frac{5-2}{4}=\frac{3}{4}$

So, $\left|\vec{p}-\frac{1}{2}\vec{q}\right|=\frac{\sqrt{3}}{2}$

Hence, option (b) is correct.

37. *(b)* Given equation is $(\log_3 x)^2+\log_3 x<2$

$\Rightarrow (\log_3 x)^2+\log_3 x-2<0$

$\Rightarrow (\log_3 x+2)(\log_3 x-1)<0$

$\Rightarrow -2<\log_3 x<1$

$\Rightarrow \log_3 3^{-2}<\log_3 x<\log_3 3$

$\Rightarrow \frac{1}{9}<x<3$

Hence, option *(b)* is correct.

38. *(c)* Given that $A=\{(x,y)\,|\,x+y\le 4\}$ and $B=\{(x,y)\,|\,2x+y\le 0\}$

Set A contains all the pairs in the interval $(-\infty,2)$ and set B contains all the pairs in the interval $(-\infty,0)$. So, $A\cap B$ shows a set containing all the pairs in the interval $[-\infty,0]$.

So, $A\cap B=\{(x,y)\,|\,x+y\le 0\}$

Hence, option *(c)* is correct.

39. *(d)* The given series is $3,7,13,21,31,43,\ldots.$

Let,

$$\begin{aligned} S&=3+7+13+21+31+\cdots\cdots+a_n\\ -S&=-(3+7+13+21+31+\cdots+a_{n-1}+a_n)\\ \hline 0&=3+4+6+8+10+12+\ldots-a_n \end{aligned}$$

$\Rightarrow a_n = 3+\left[4+6+8+10+12+\ldots(n-1)\right]$

$\Rightarrow a_n = 3+\frac{(n-1)}{2}\left[8+\{(n-1)-1\}\times 2\right]$

$\Rightarrow a_n = 3+\frac{(n-1)}{2}\left[8+2n-4\right]$

$\Rightarrow a_n = 3+\frac{(n-1)}{2}(2n+4)$

$\Rightarrow a_n = 3+(n-1)(n+2)$

$\therefore$ The 15^{th} term $= a_{15} = 3+(15-1)(15+2) = 241$

Hence, option *(d)* is correct.

40. *(b)* From the relation between minute 6 seconds measure $60'' = 1'$

$\Rightarrow 30'' = \frac{1'}{2} \Rightarrow 35'30'' = \left(35+\frac{1}{2}\right)' = \left(\frac{71}{2}\right)'$

Also, $60' = 1°$

$\Rightarrow \left(\frac{71}{2}\right)' = \left(\frac{71}{2}\times\frac{1}{60}\right)^\circ = \left(\frac{71}{120}\right)^\circ$

$\therefore 114°35'30'' = \left(114+\frac{71}{120}\right)^\circ = \left(\frac{13751}{120}\right)^\circ$

We know that $2\pi \text{ rad} = 360° \Rightarrow 1° = \frac{2\pi}{360}\text{ rad}$

$\therefore \left(\frac{13751}{120}\right)^\circ = \frac{2\pi}{360°}\times\frac{13751}{120}\text{ rad}$

$= \frac{2\times 22\times 13751}{7\times 360\times 120}\text{ rad} \approx 2.0008069\text{ rad}$

$\Rightarrow 114°35'30'' = 2\pi\text{ rad}(\text{approx})$

Hence, option *(b)* is correct.

41. *(b)* Let point $P(x_1, y_1)$ be equidistant from point $A(1,2)$ and $B(3,4)$

$\therefore PA = PB$

$\Rightarrow PA^2 = PB^2$

$\Rightarrow (1-x_1)^2+(2-y_1)^2 = (3-x_1)^2+(4-y_1)^2$

$\Rightarrow 1+x_1^2-2x_1+4+y_1^2-4y_1 = 9+x_1^2-6x_1+16+y_1^2-8y_1$

$\Rightarrow 4x_1+4y_1 = 20$

$\Rightarrow x_1+y_1 = 5$ ....(i)

As $P(x_1, y_1)$ lies on $2x-3y=5$

$\therefore 2x_1-3y_1 = 5$ ....(ii)

On solving equations (i) and (ii), we get $x_1 = 4$ and $y_1 = 1$

$\therefore$ Coordinates of p are $(4,1)$.

Hence, option *(b)* is correct.

42. *(b)* Given hyperbola is $4x^2-9y^2-1=0$

$\Rightarrow \frac{x^2}{\left(\frac{1}{2}\right)^2}-\frac{y^2}{\left(\frac{1}{3}\right)^2} = 1$

$\therefore$ Eccentricity, $e = \sqrt{\frac{1+\left(\frac{1}{3}\right)^2}{\left(\frac{1}{2}\right)^2}} = \frac{\sqrt{13}}{3}$

So, foci $= \left(\pm\frac{1}{2}\times\frac{\sqrt{13}}{3}, 0\right) = \left(\pm\frac{\sqrt{13}}{6}, 0\right)$

Hence, option *(b)* is correct.

43. *(b)* Consider $\sin^{-1}\frac{4}{5}+2\tan^{-1}\frac{1}{3}$

$$= \tan^{-1}\left(\frac{4/5}{\sqrt{1-16/25}}\right) + \tan^{-1}\left(\frac{2/3}{1-1/9}\right)$$

$$\left[\because \sin^{-1} x = \tan^{-1}\frac{x}{\sqrt{1-x^2}} \text{ and } 2\tan^{-1} x = \tan^{-1}\left(\frac{2x}{1=x^2}\right)\right]$$

$$= \tan^{-1}\left(\frac{4/5}{3/5}\right) + \tan^{-1}\left(\frac{2/3}{8/9}\right)$$

$$= \tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1}\left(\frac{3}{4}\right)$$

$$= \tan^{-1}\left(\frac{4}{3}\right) + \cot^{-1}\left(\frac{4}{3}\right)$$

$$= \frac{\pi}{2} \qquad \left[\because \tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}\right]$$

Hence, option *(b)* is correct.

44. *(c)* Let $I = \int \frac{\log x}{(1+\log x)^2}dx$

Put $\log x = t \Rightarrow \frac{1}{x}dx = dt \Rightarrow dx = xdt \Rightarrow dx = e^t dt$

Therefore,

$$I = \int \frac{e^t t}{(1+t)^2}dt = \int \frac{e^t(t+1-1)}{(1+t)^2}dt$$

$$= \int \frac{e^t(1+t)}{(1+t)^2}dt - \int \frac{e^t}{(1+t)^2}dt$$

$$= \int \frac{e^t}{(1+t)}dt - \int \frac{e^t}{(1+t)^2}dt$$

$$= \frac{e^t}{(1+t)} - \int -e^t \frac{1}{(1+t)^2}dt - \int \frac{e^t}{(1+t)^2}dt$$

$$= \frac{e^t}{(1+t)} + \int \frac{e^t}{(1+t)^2}dt - \int \frac{e^t}{(1+t)^2}dt$$

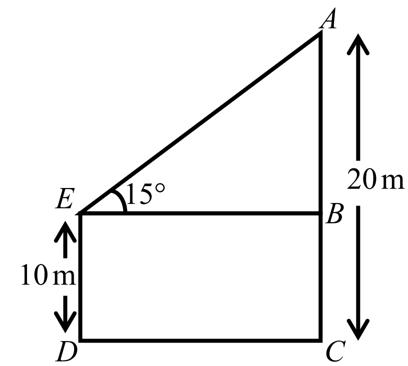
$$= \frac{e^t}{(1+t)} + c$$

$$\therefore I = \frac{x}{(1+\log x)} + c$$

Hence, option (c) is correct.

45. *(b)* Let AC and ED be two poles of height 20m and 10m respectively.

Let $\angle AEB = 15°$



Now, $AB = AC - BC = AC - ED = 20 - 10 = 10\text{m}$

Now, in ΔABE, $(\because BC = ED)$

$$\tan 15° = \frac{AB}{BE}$$

$$\Rightarrow \tan(45° - 30°) = \frac{10}{BE}$$

$$\Rightarrow \frac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°} = \frac{10}{BE}$$

$$\left(\because \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}\right)$$

$$\Rightarrow \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{10}{BE}$$

$$\Rightarrow BE = 10\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right) = \frac{10\times(\sqrt{3}+1)^2}{2}$$

$$\Rightarrow BE = 5(3+1+2\sqrt{3}) = 37.3$$

Thus $CD = BE = 37.3\,\text{m}$

Hence, option *(b)* is correct.

46. *(d)* Given that $f(x) = \frac{x^2}{1+x^2}$

Since, numerator is less than denominator.

$f(x) < 1$ for all values of x (negative or positive) and $f(x) = 0$ for $x = 0$.

So, range of f is $[0,1)$

Hence, option *(d)* is correct.

47. *(a)* Let $y=\tan^{-1}\left(\frac{\sqrt{x}-x}{1+x^{3/2}}\right)=\tan^{-1}\left(\frac{\sqrt{x}-x}{1+x\sqrt{x}}\right)$

$=\tan^{-1}\sqrt{x}-\tan^{-1}x$

On differentiating w.r.t. x, we get

$\frac{dy}{dx}=\frac{1}{1+x}\cdot\frac{1}{2\sqrt{x}}-\frac{1}{1+x^2}$

Now, $\left(\frac{dy}{dx}\right)_{x=1}=\frac{1}{1+1}\cdot\frac{1}{2}-\frac{1}{1+1}=\frac{1}{4}-\frac{1}{2}=-\frac{1}{4}$

Hence, option *(a)* is correct.

48. *(b)* Given that $AB=\begin{bmatrix}4 & 11\\ 4 & 5\end{bmatrix}$ and $A=\begin{bmatrix}3 & 2\\ 1 & 2\end{bmatrix}$

Let $B=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$

$\Rightarrow \begin{bmatrix}3 & 2\\ 1 & 2\end{bmatrix}\begin{bmatrix}a & b\\ c & d\end{bmatrix}=\begin{bmatrix}4 & 11\\ 4 & 5\end{bmatrix}$

$\Rightarrow \begin{bmatrix}3a+2c & 3b+2d\\ a+2c & b+2d\end{bmatrix}=\begin{bmatrix}4 & 11\\ 4 & 5\end{bmatrix}$

$\Rightarrow 3a+2c=4$ and $a+2c=4$ ....(i)

$3b+2d=11$ and $b+2d=5$ ....(ii)

From equation set (i) $a=0$ and $c=2$ and from equation set (ii), $b=3$ and $d=1$

$\Rightarrow B=\begin{bmatrix}0 & 3\\ 2 & 1\end{bmatrix}$

Thus, $|B|=\begin{vmatrix}0 & 3\\ 2 & 1\end{vmatrix}=0-6=-6$

Hence, option *(b)* is correct.

49. *(c)* Let $I=\int_{\ln 2}^{x}\left(e^x-1\right)^{-1}dx=\int_{\ln 2}^{x}\frac{1}{\left(e^x-1\right)}dx$

Put $e^x-1=t \Rightarrow e^x=t+1$

$\therefore e^x dx=dt \Rightarrow dx=\frac{dt}{e^x} \Rightarrow dx=\frac{dt}{t+1}$

when $x=\ln 2$, $t=e^{\ln 2}-1=2-1=1$ and $I=\int_1^t\frac{1}{t(t+1)}dt$

Breaking into partial fractions, we get

$\frac{1}{t(t+1)}=\frac{1}{t}-\frac{1}{t+1}$

And,

$I=\int_1^t\left(\frac{1}{t}-\frac{1}{t+1}\right)dt=\left[\log_e t-\log_e(t+1)\right]_i^t$

$=\left[\log_e\frac{t}{t+1}\right]_i^t=\log_e\frac{t}{t+1}-\log_e\frac{1}{2}$

$=\log_e\frac{2t}{t+1}=\log_e\frac{3}{2}$ $\left[\text{since, }\int_{\ln 2}^{x}\left(e^x-1\right)^{-1}dx=\log_e\frac{3}{2}\right]$

So, $\frac{2t}{t+1}=\frac{3}{2}\Rightarrow 4t=3t+3\Rightarrow t=3$

Thus, $e^x-1=3\Rightarrow e^x=4\Rightarrow x=\ln 4$

Hence, option *(c)* is correct.

50. *(b)* For a function to be continuous at a point, the limit should exist and should be equal to the value of the function at that point.

Here, point is $x=0$

And $\lim_{x\to 0}f(x)=\lim_{x\to 0}(x+1)^{\cot x}$

$=\lim_{x\to 0}(x+1)^{\frac{1}{x}\cdot x\cot x}=\lim_{x\to 0}(x+1)^{\frac{1}{x}\cdot\lim_{x\to 0}\frac{x}{\tan x}}$

$=e^1=e$

Since, limiting value of $f(x)=e$, when $x\to 0$, $f(0)$ should also be equal to e.

Hence, option *(b)* is correct.

51. *(b)* Given equation $\cos 2x+a\sin x=2a-7$ can be written as

$\cos^2 x-\sin^2 x+a\sin x=2a-7$

$\Rightarrow 1-\sin^2 x-\sin^2 x+a\sin x=2a-7$

$\Rightarrow 2\sin^2 x-a\sin x+(2a-7-1)=0$

$\Rightarrow 2\sin^2 x-a\sin x+2a-8=0$

This is a quadratic equation in $\sin x$ and its discriminant ≥ 0

Here, $a=2, b=-a, c=2a-8$

$\Rightarrow a^2 - 4\times 2\times(2a-8)\geq 0$

$\Rightarrow a^2 - 16a + 64 \geq 0$

$\Rightarrow (a-8)^2 \geq 0 \Rightarrow a \geq 8$

Hence, option *(b)* is correct.

52. *(b)* The total number of students $= 500$

Let H be the set showing number of students who can speak Hindi $= 475$ and B be the set showing number of students who can speak Bengali $= 200$

So, $n(H) = 475, n(B) = 200$ and $n(B\cup H) = 500$.

We have

$$n(B\cup H) = n(B) + n(H) - n(B\cap H)$$

$\Rightarrow 500 = 200 + 475 - n(B\cap H)$

So, $n(B\cap H) = 175$

Therefore, the persons who speak Hindi only is given by $= n(H) - n(B\cap H) = 475 - 175 = 300$.

Hence, option *(b)* is correct.

53. *(c)* Let $z_1 = \alpha + i\beta$ and $z_2 = \beta + i\alpha$

Since, $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$

$\therefore \left|\frac{\alpha+i\beta}{\beta+i\alpha}\right| = \frac{|\alpha+i\beta|}{|\beta+i\alpha|} = \frac{\sqrt{\alpha^2+\beta^2}}{\sqrt{\alpha^2+\beta^2}} = 1$

Hence, option *(c)* is correct.

54. *(c)* Given that $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 4A}}}$

$= \sqrt{2+\sqrt{2+\sqrt{2(1+\cos 4A)}}}$

$= \sqrt{2+\sqrt{2+2\cos 2A}} = \sqrt{2+\sqrt{2(1+\cos 2A)}}$

$(\because 1+\cos 4A = 2\cos^2 2A)$

$= \sqrt{2+2\cos A} = \sqrt{2(1+\cos A)}$

$(\because 1+\cos 2A = 2\cos^2 A)$

$= 2\cos\left(\frac{A}{2}\right) \qquad (\because 1+\cos 2A = 2\cos^2 A)$

Hence, option *(c)* is correct.

55. *(a)* Equation of a straight line in intercept form is

$\frac{x}{a} + \frac{y}{b} = 1 \Rightarrow \frac{3}{a} + \frac{4}{b} = 1 \qquad$ ....(i)

Given $a + b = 14 \qquad$ ....(ii)

On solving (i) and (ii), we get

$\frac{3}{a} + \frac{4}{14-a} = 1 \Rightarrow a^2 - 13a + 42 = 0$

$\Rightarrow (a-7)(a-6) = 0$

$\Rightarrow a = 6$ and $b = 8$

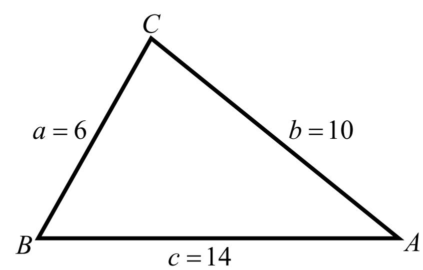
or $a = 7$ and $b = 7$

$\therefore$ Required equations are $4x + 3y = 24$ or $x + y = 1$

Hence, option *(a)* is correct.

56. *(c)* Since, $c = 14$ is the largest side

$\therefore$ Angle C will be obtuse



$\cos C = \frac{a^2+b^2-c^2}{2ab} = \frac{(6)^2+(10)^2-(14)^2}{2(6)(10)}$

$= \frac{36+100-196}{2\times 6\times 10} = -\frac{1}{2}$

$\Rightarrow C = \cos^{-1}\left(-\frac{1}{2}\right) = \pi - \frac{\pi}{3} = \frac{2\pi}{3} = \frac{2\times 180}{3} = 120°$

Hence, option *(c)* is correct.

57. *(c)* As given $\vec{p} = \frac{\vec{b}\times\vec{c}}{[\vec{a}\vec{b}\vec{c}]}, \vec{q} = \frac{\vec{c}\times\vec{a}}{[\vec{a}\vec{b}\vec{c}]}, \vec{r} = \frac{\vec{a}\times\vec{b}}{[\vec{a}\vec{b}\vec{c}]}$

$\therefore (\vec{a}-\vec{b}-\vec{c})\cdot\vec{p} + (\vec{b}-\vec{c}-\vec{a})\cdot\vec{q} + (\vec{c}-\vec{a}-\vec{b})\cdot\vec{r}$

$= \frac{\vec{a}\cdot(\vec{b}\times\vec{c})}{[\vec{a}\vec{b}\vec{c}]} + \frac{\vec{b}\cdot(\vec{c}\times\vec{a})}{[\vec{a}\vec{b}\vec{c}]} + \frac{\vec{c}\cdot(\vec{a}\times\vec{b})}{[\vec{a}\vec{b}\vec{c}]}$

[Since, $\vec{b}\cdot(\vec{b}\times\vec{c}) = 0, \vec{c}\cdot(\vec{b}\times\vec{c}) = 0, \vec{c}\cdot(\vec{c}\times\vec{a}) = 0$

$\vec{a}\cdot(\vec{c}\times\vec{a})=0\ \vec{a}\cdot(\vec{a}\times\vec{b})=0$ and $\vec{b}\cdot(\vec{a}\times\vec{b})=0]$

$=\frac{[\vec{a}\vec{b}\vec{c}]}{[\vec{a}\vec{b}\vec{c}]}+\frac{[\vec{a}\vec{b}\vec{c}]}{[\vec{a}\vec{b}\vec{c}]}+\frac{[\vec{a}\vec{b}\vec{c}]}{[\vec{a}\vec{b}\vec{c}]}$

$=3$

Hence, option *(c)* is correct.

58. *(a)* Let $\frac{dy}{dx}=1+x+y+xy$

$\Rightarrow \frac{dy}{dx}=(1+x)(1+y)$

$\Rightarrow \frac{dy}{1+y}=(1+x)dx$

$\Rightarrow \int\frac{1}{1+y}dy=\int(1+x)dx$

$\Rightarrow \log(1+y)=x+\frac{x^2}{2}+c$

Given, at $x=-1, y=0$

$\Rightarrow \log(1+0)=-1+\frac{1}{2}+c$

$\Rightarrow c=\frac{1}{2}$

$\therefore \log(1+y)=x+\frac{x^2}{2}+\frac{1}{2}=\frac{(1+x)^2}{2}$

$\Rightarrow 1+y=e^{\frac{(1+x)^2}{2}}$

$\Rightarrow y=e^{\frac{(1+x)^2}{2}}-1$

Hence, option *(a)* is correct.

59. *(b)* We know that sum of square of direction cosines $=1$

i.e. $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$

$\Rightarrow \cos^2 45°+\cos^2\beta+\cos^2\beta=1$

(As given $\alpha=45°$ and $\beta=\gamma$)

$\Rightarrow \frac{1}{2}+2\cos^2\beta=1$

$\Rightarrow \cos^2\beta=\frac{1}{4}$

$\Rightarrow \cos\beta=\pm\frac{1}{2}$

Negative value is discarded, since the line makes angle with positive axes.

Thus, $\cos\beta=\frac{1}{2}$

$\Rightarrow \cos\beta=\cos 60°$

$\Rightarrow \beta=60°$

$\therefore$ Required sum $\alpha+\beta+\gamma=45°+60°+60°=165°$

Hence, option *(b)* is correct.

60. *(c)* Given function is:

$f(x)=\begin{cases}3x-4, & 0\le x\le 2\\ 2x+l, & 2<x\le 9\end{cases}$

And also given that $f(x)$ is continuous at $x=2$.

For a function to be continuous at a point, LHL=RHL=V.F at that point, $f(2)=2=$ V.F.

$\Rightarrow \text{RHL}=\lim_{x\to 2}(2x+l)=3(2)-4$

$\Rightarrow \lim_{h\to 0}\{2(2+h)+l\}=6-4$

$\Rightarrow 4+l=2$

$\Rightarrow l=-2$

Hence, option *(c)* is correct.

61. *(c)* The given system are

$kx+y+z=k-1$

$x+ky+z=k-1$

$x+y+kz=k-1$

$\therefore A=\begin{bmatrix}k&1&1\\1&k&1\\1&1&k\end{bmatrix}, B=\begin{bmatrix}k-1\\k-1\\k-1\end{bmatrix}$ and $x=\begin{bmatrix}x\\y\\z\end{bmatrix}$

Now, $|A|=\begin{vmatrix}k&1&1\\1&k&1\\1&1&k\end{vmatrix}$

$= k(k^2-1) - 1(k-1) + 1(k-1)$

$= k^3 - k - k + 1 + 1 - k$

$= k^3 - 3k + 2$

This given system of equation has no solution if $|A| = 0$

$\Rightarrow k^3 - 3k + 2 = 0 \Rightarrow (k-1)^2(k+2) = 0$

$\Rightarrow k = 1$ or $k = -2$

Hence, option *(c)* is correct.

62. *(c)* 16 tickets are sold and 4 prizes are awarded.

A person buys 4 tickets, then the required probability is

$= \frac{4}{16} = \frac{1}{4}$

Hence, option *(c)* is correct.

63. *(c)* Let $x = t^2$ and $y = t^3$

$\Rightarrow \frac{dx}{dt} = 2t$ and $\frac{dy}{dt} = 3t^2$

$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2}{2t} = \frac{3}{2}t$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{3}{2} \cdot \frac{dt}{dx} = \frac{3}{2} \cdot \frac{1}{2t} = \frac{3}{4t} \qquad \left(\because \frac{dx}{dt} = 2t\right)$

Hence, option *(c)* is correct.

64. *(d)* The equation of the line is $\frac{x-2}{2} = \frac{y-3}{3} = \frac{z-4}{5} = r$

where r is a constant. Any point on this line is given by $x = 2r+2,\ y = 3r+2,\ z = 5r+4$

Since, a plate that is parallel to z-axis will have no z-coordinates, $z = 0$

$z = 0 \Rightarrow 5r + 4 = 0 \Rightarrow r = -\frac{4}{5}$

Putting this value of r for x and y coordinates,

$x = 2r + 2 = 2\left(-\frac{4}{5}\right) + 2$

$\Rightarrow 5x = -8 + 10$

$\Rightarrow x = \frac{2}{5}$ or $\frac{2}{x} = 5 \qquad$ ....(i)

Similarly,

$y = 3r + 3 = 3\left(-\frac{4}{5} + 3\right)$

$\Rightarrow 5y = -12 + 15$

$\Rightarrow y = \frac{3}{5}$ or $\frac{3}{y} = 5 \qquad$ ....(ii)

From equation (i) and (ii), we get

$\frac{2}{x} = \frac{3}{y} \Rightarrow 3x - 2y = 0$

Hence, option (d) is correct.

65. *(c)* Let the numbers are x and y

So, $x + y = 20$, Let $P = x^2y^3 = x^2(20-x)^3 \quad$ (As given)

Differentiating w.r.t. x, we get

$\frac{dP}{dx} = x^2 \cdot 3(20-x)^2(-1) + (20-x)^3 \cdot 2x$

$= (20-x)^2\left[-3x^2 + 40x - 2x^2\right]$

$= (20-x)^2(40x - 5x^2)$

$\frac{d^2P}{dx^2} = (20-x)^2[40 - 10x] + (40x - 5x^2) \cdot 2(20-x)(-1)$

$\frac{dP}{dx} = 0$ for maxima or minima

So, $(20-x)^2(40x - 5x^2) = 0$

$\Rightarrow x(20-x)^2(40 - 5x) = 0$

$\Rightarrow x = 0, 8, 20$

We get $\left(\frac{d^2P}{dx^2}\right)_{x=8} < 0;\ \left(\frac{d^2P}{dx^2}\right)_{x=0} > 0$ and $\left(\frac{d^2P}{dx^2}\right)_{x=20} = 0$

Hence, P is maximum at $x = 8$ and numbers are 12 and 8.

Hence, option *(c)* is correct.

66. *(b)* Since, circle is touching y-axis at origin, its center lies on x-axis. Let the centre be $(a, 0)$ and its radius is a

$\therefore\ (x-a)^2+y^2=a^2$

$\Rightarrow x^2+a^2-2ax+y^2=a^2$

$\Rightarrow x^2+y^2-2ax=0$ ....(i)

$\Rightarrow a=\dfrac{x^2+y^2}{2x}$

Differentiating both sides, we get

$\Rightarrow 2x+2y\dfrac{dy}{dx}-2a=0$

$\Rightarrow 2x+2y\dfrac{dy}{dx}-\dfrac{x^2+y^2}{x}=0$

$\Rightarrow 2x^2+2xy\dfrac{dy}{dx}-x^2-y^2=0$

$\Rightarrow 2xy\dfrac{dy}{dx}=y^2-x^2$

Hence, option *(b)* is correct.

67. *(a)* The given integral is $I=\int\dfrac{dx}{\sqrt{x^2+a^2}}$

Let $x=a\tan u\Rightarrow dx=a\sec^2 u du$

Therefore, we have

$$I=\int\frac{a\sec^2 udu}{\sqrt{a^2\tan^2 u+a^2}}=a\int\frac{\sec^2 udu}{\sqrt{a^2(\tan^2 u+1)}}$$

$$=\frac{a}{a}\int\frac{\sec^2 udu}{\sqrt{\tan^2 u+1}}=\int\frac{\sec^2 u}{\sqrt{\sec^2 u}}du$$

$$=\int\sec udu$$

$$=\ln[\tan(u)+\sec(u)]+c$$

$$=\ln\left[\frac{x}{\sqrt{a^2}}+\sqrt{1+\frac{x^2}{a^2}}\right]+c$$

$$=\ln\left[\frac{x}{a}+\frac{\sqrt{a^2+x^2}}{a}\right]+c$$

$$=\ln\left[\frac{x+\sqrt{a^2+x^2}}{a}\right]+c$$

Hence, option *(a)* is correct.

68. *(b)* Let $A=\begin{bmatrix}3&2\\1&4\end{bmatrix}$

If A is a square matrix of order n, then

$A(\text{adj}A)=|A|\cdot I_n$

Here $n=2$

$$=\begin{bmatrix}1&0\\0&1\end{bmatrix}\begin{vmatrix}3&2\\1&4\end{vmatrix}=\begin{bmatrix}1&0\\0&1\end{bmatrix}(12-2)=10\begin{bmatrix}1&0\\0&1\end{bmatrix}$$

$$=\begin{bmatrix}10&0\\0&10\end{bmatrix}$$

Hence, option *(b)* is correct.

69. *(a)* Given that $x^{1/3}+y^{1/3}+z^{1/3}=0\Rightarrow x^{1/3}=-(y^{1/3}+z^{1/3})$

Raising both the sides to the power of cube, we get

$x=-\{y+z+3y^{1/3}z^{1/3}(y^{1/3}+z^{1/3})\}$

$\{\because (a+b)^3=a^3+b^3+3ab(a+b)\}$

$x=-\{y+z+3y^{1/3}z^{1/3}(-x^{1/3})\}$

$x=-\{y+z-3x^{1/3}y^{1/3}z^{1/3}\}$

$x=-y-z+3x^{1/3}y^{1/3}z^{1/3}$

$\Rightarrow x+y+z=3x^{1/3}y^{1/3}z^{1/3}$

$\Rightarrow x+y+z=3(xyz)^{1/3}$

$\Rightarrow (x+y+z)^3=27xyz$

Hence, option *(d)* is correct.

70. *(d)* Converting from binary to decimal, we have

$(1001)_2=1\times2^3+0\times2^2+0\times2^1+1\times2^0=8+1=9$

$(11)_2=1\times2^1+1\times2^0=2+1=3$

$(101)_2=1\times2^2+0\times2^1+1\times2^0=4+1=5$

$(10)_2=1\times2^1+0\times2^0=2$

$(01)_2=0\times2^1+1\times2^0=1$

$$\therefore \frac{(1001)_2^{(11)_2}-(1001)_2^{(11)_2}}{(1001)_2^{(10)_2}+(1001)_2^{(01)_2}(101)_2^{(01)_2}+(101)_2^{(10)_2}}$$

$$=\frac{9^3-5^3}{9^2+9\times5+5^2}$$

$$=\frac{(9-5)(9^2+9\times5+5^2)}{9^2+9\times5+5^2}$$

$$=4$$

$$=(100)_2 \qquad \{\text{converting from decimal to binary}\}$$

Hence, option *(d)* is correct.

71. *(c)* Let 1, ω and ω^2 are the three cube roots of unity.

$1+\omega+\omega^2=0$ and $\omega^3=1$

The given expression is

$$\frac{a\omega^6+b\omega^4+c\omega^2}{b+c\omega^{10}+a\omega^8}=\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2} \qquad [\omega^6=1,\omega^4=\omega]$$

$$=\frac{\omega(a+b\omega+c\omega^2)}{\omega(b+c\omega+a\omega^2)}$$

$$=\frac{\omega(a+b\omega+c\omega^2)}{(b\omega+c\omega^2+a\omega^3)}$$

$$=\frac{\omega(a+b\omega+c\omega^2)}{(a+b\omega+c\omega^2)}=\omega$$

Hence, option *(c)* is correct.

72. *(d)* Given that $1-\sin 10°\sin 50°\sin 70°$

$$=1-\frac{1}{2}[2\sin 70°\sin 10°\sin 50°]$$

$$=1-\frac{1}{2}[(\cos 60°-\cos 80°)\sin 50°]$$

$$(\because 2\sin A\sin B=\cos(A-B)-\cos(A+B))$$

$$=1-\frac{1}{2}\left[\frac{1}{2}\sin 50°-\frac{1}{2}\cos 80°\sin 50°\right]$$

$$=1-\frac{1}{4}[\sin 50°-\sin 130°+\sin 30°]$$

$$(\because 2\cos A\sin B=\sin(A+B)-\sin(A-B))$$

$$=1-\frac{1}{4}[\sin 50°-\sin 50°+\sin 30°]$$

$$(\because \sin 130°=\sin(180°-50°)=\sin 50°)$$

$$=1-\frac{1}{4}\left[\frac{1}{2}\right]$$

$$=1-\frac{1}{8}=\frac{7}{8}$$

Hence, option *(d)* is correct.

73. *(a)* Let the equation of hyperbola be $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$$\text{Latus rectum}=8=\frac{2b^2}{a}\Rightarrow b^2=4a \qquad \text{....(i)}$$

Also, $b^2=a^2(e^2-1)$

$$\Rightarrow 4a=a^2(e^2-1) \qquad [\text{from (i)}]$$

$$\Rightarrow 4a=a^2\left[\left(\frac{3}{\sqrt{5}}\right)^2-1\right]$$

$$\Rightarrow a=5 \text{ and } b^2=20$$

$\therefore$ Equation of hyperbola is $\frac{x^2}{25}-\frac{y^2}{20}=1$

Hence, option *(a)* is correct.

74. *(d)* We know that $\cos C=\frac{a^2+b^2-c^2}{2ab}$

$$\Rightarrow \cos C=\frac{(18)^2+(24)^2-(30)^2}{2\times18\times24}=\frac{9+16-25}{2\times3\times4}=0$$

Now, $\sin C=\sqrt{1-\cos^2 C}=\sqrt{1-0}=1$

Hence, $\sin C=1$

Hence, option *(d)* is correct.

75. *(a)* Given function is $f(x)=\sin(|x|)$

$$\Rightarrow f(x)=\begin{cases}\sin(x), & x\ge 0\\ \sin(-x), & x<0\end{cases}$$

$$\Rightarrow f(x)=\begin{cases}\sin x, & x\ge 0\\ -\sin x, & x<0\end{cases}$$

LHD at $x=0=\lim_{h\to 0}\frac{f(0-h)-f(0)}{0-h-0}$

$=\lim_{h\to 0}\frac{f(0-h)-f(0)}{-h}=\lim_{h\to 0}\frac{-\sin(-h)-0}{-h}=-1$

RHD at $x=0=\lim_{h\to 0}\frac{f(0+h)-f(0)}{0+h-0}$

$=\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}=\lim_{h\to 0}\frac{\sin(h)-0}{h}=1$

Since, LHD $\neq$ RHD

$f(x)$ is not differentiable at $x=0$.

Hence, option *(a)* is correct.

76. *(c)* Total number of selecting 3 components out of 10 $= {}^{10}C_3$

Out of 3 selected components two defective pieces can be selected in ${}^4C_2$ ways and one non-defective piece will be selected in ${}^6C_1$ ways.

Thus, required probability is

$=\frac{{}^6C_1\times {}^4C_2}{{}^{10}C_3}=\frac{6\times 6\times 6}{10\times 9\times 8}=\frac{3}{10}$

Hence, option *(c)* is correct.

77. *(a)* Since, α and β are the roots of the equation $ax^2+bx+c=0$ then,

Sum of the roots, $\alpha+\beta=-\frac{b}{a}$

Product of the roots, $\alpha\beta=\frac{c}{a}$

The expression $(a\alpha+b)^{-1}+(a\beta+b)^{-1}$

$=\frac{1}{a\alpha+b}+\frac{1}{a\beta+b}=\frac{a\beta+b+a\alpha+b}{(a\alpha+b)(a\beta+b)}$

$=\frac{a(\alpha+\beta)+2b}{a^2\alpha\beta+ab(\alpha+\beta)+b^2}$

$=\frac{a\left(-\frac{b}{a}\right)+2b}{a^2\left(\frac{c}{a}\right)+ab\left(-\frac{b}{a}\right)+b^2}$

$=\frac{-b+2b}{ac-b^2+b^2}=\frac{b}{ac}$

Hence, option *(b)* is correct.

78. *(c)* Given that $P=\{p_1,p_2,p_3,p_4\}$, $Q=\{q_1,q_2,q_3,q_4\}$ and $R=\{r_1,r_2,r_3,r_4\}$

$\therefore S_{10}=\{(p_2,q_4,r_4),(p_3,q_3,r_4),(p_3,q_4,r_3),(p_4,q_2,r_4)$
$(p_4,q_3,r_3),(p_4,q_4,r_2)\}$

$\therefore$ Total number of elements in S_{10} is 6.

Hence, option *(c)* is correct.

79. *(b)* From cube root of infinity, we have

$\omega=\frac{-1+\sqrt{3}i}{2}$ and $\omega=\frac{-1-\sqrt{3}i}{2}$

Now,

$\omega^3=\left(\frac{-1+\sqrt{3}i}{2}\right)\left(\frac{-1-\sqrt{3}i}{2}\right)=\frac{(-1)^2-(\sqrt{3}i)^2}{4}=\frac{4}{4}=1$

$\therefore \left(\frac{-1+i\sqrt{3}}{2}\right)^{3n}+\left(\frac{-1-i\sqrt{3}}{2}\right)^{3n}=(\omega)^{3n}+(\omega^2)^{3n}=1+1=2$

Hence, option *(b)* is correct.

80. *(b)* Given that $(1+\tan\theta)(1+\tan\phi)=2$

$\Rightarrow 1+\tan\theta+\tan\phi+\tan\theta\tan\phi=2$

$\Rightarrow \tan\theta+\tan\phi=1-\tan\theta\tan\phi$

$\Rightarrow \frac{\tan\theta+\tan\phi}{1-\tan\theta\tan\phi}=1$

$\Rightarrow \tan(\theta+\phi)=\tan 45^\circ$

$\Rightarrow \theta+\phi=45^\circ$

Hence, option *(b)* is correct.

81. *(b)* Given that $f(x)=\sqrt{9-x^2}$

$\therefore f'(x)=\frac{1}{2\sqrt{9-x^2}}\cdot(-2x)=-\frac{x}{\sqrt{9-x^2}}$

For function to be increasing $f'(x)>0$

$-\frac{x}{\sqrt{9-x^2}} > 0 \Rightarrow -x > 0 \text{ or } x < 0$

But $\sqrt{9-x^2}$ is defined only when

$9-x^2 > 0$ or $x^2-9<0$

$(x+3)(x-3)<0$

i.e. $-3<x<3$

$-3<x<3 \cap x<0$

$\Rightarrow -3<x<0$

Hence, option *(b)* is correct.

82. *(a)* The given differential equation can be written as

$\frac{dy}{dx} = y\tan x + \sec x$

or, $\frac{dy}{dx} - y\tan = \sec x$

which is of the form $\frac{dy}{dx} + P(x)y = Q(x)$

Here $P(x) = -\tan x$ and $Q(x) = \sec x$

Integrating factor $\text{IF} = e^{\int P(x)dx}$

$\text{IF} = e^{\int -\tan x dx} = e^{\int -\frac{\sin x}{\cos x}dx}$

Putting $\cos x = t \Rightarrow -\sin x dx = dt$

$\therefore IF = e^{\int \frac{dt}{t}} = e^{\log_e t} = t = \cos x$

Therefore, the solution is

$y \cdot Q(x) = \int \text{IF} \cdot Q(x)dx + c$

$y\sec x = \int \cos x \cdot \sec x dx + c$

$y\sec x = \int dx + c$

$y\sec x = x + c$

Since, the curve passes through the origin

$0 = 0 + c \Rightarrow c = 0$

And $y\sec x = x \Rightarrow y = x\cos x$

Hence, option *(a)* is correct.

83. *(d)* Consider first: $x^2-3x+2>0$

$\Rightarrow (x-1)(x-2)>0$

$\Rightarrow x<1$ or $x>0$ ....(1)

And $x^2-3x-4 \le 0$

$\Rightarrow (x-4)(x+2) \le 0$

$\Rightarrow -1 \le x \le 4$ ....(2)

Combining (1) and (2), we get

$-1 \le x < 1$ or $2 < x \le 4$

Hence, option *(d)* is correct.

84. *(c)* The maximum three digit integer in decimal system $=999$

We go on dividing till we get a dividend <2 and write remainders from last to first as shown below:

2	999	
2	499	1
2	249	1
2	124	1
2	62	0
2	31	0
2	15	1
2	7	1
2	3	1
	1	1

Hence, $(999)_{10} = (1111100111)_2$

Hence, option *(c)* is correct.

85. *(c)* Consider $\begin{vmatrix} \cos 15° & \sin 15° \\ \cos 45° & \sin 45° \end{vmatrix} \times \begin{vmatrix} \cos 45° & \sin 15° \\ \cos 45° & \sin 15° \end{vmatrix}$

$= (\sin 45° \cos 15° - \cos 45° \sin 15°)$

$\times(\cos 45° \sin 15° - \sin 45° \cos 15°)$

$= -\sin(45°-15°) \times \sin(45°-15°)$

$\{\because \sin(A-B) = -\cos A \sin B + \sin A \cos B\}$

$= -\sin(30°) \times \sin(30°)$

$= -\frac{1}{2} \times \frac{1}{2} = -\frac{1}{4}$

Hence, option *(c)* is correct.

86. *(c)* We have $P(A_1) = \frac{1}{1+1} = \frac{1}{2}$, $P(A_2) = \frac{1}{3}$ and

$P(A_3)=\frac{1}{4}$

$\therefore$ Probability that at least one of these events occur is $P(A_1\cup A_2\cup A_3)$. Also, A_1, A_2, A_3 are independent events.

$$P(A_1\cup A_2\cup A_3)=P(A_1)+P(A_2)+P(A_3)-P(A_1\cap A$$
$$-P(A_2\cap A_3)-P(A_1\cap A_3)+P(A_1\cap A_2\cap$$

$$P(A_1\cup A_2\cup A_3)=\frac{1}{2}+\frac{1}{3}+\frac{1}{4}-\left(\frac{1}{2}\times\frac{1}{3}\right)-\left(\frac{1}{2}\times\frac{1}{4}\right)$$
$$-\left(\frac{1}{3}\times\frac{1}{4}\right)+\left(\frac{1}{2}\times\frac{1}{3}\times\frac{1}{4}\right)$$

$$P(A_1\cup A_2\cup A_3)=\frac{3}{4}$$

Hence, option *(c)* is correct.

87. *(d)* Force, $\vec{F}$ is given by $\vec{F}=\hat{i}+\hat{j}+\hat{k}$

$\overrightarrow{OA}=\hat{i}+2\hat{j}+3\hat{k}$ and $\overrightarrow{OB}=-2\hat{i}+3\hat{j}+\hat{k}$

$$\therefore \vec{r}=\overrightarrow{AB}=\left(-2\hat{i}+3\hat{j}+\hat{k}\right)-\left(\hat{i}+2\hat{j}+3\hat{k}\right)=-3\hat{i}+\hat{j}-2\hat{k}$$

Moment $\vec{M}$ about the point $\hat{i}+2\hat{j}+3\hat{k}$

$$\vec{M}=\vec{r}\times\vec{F}=\left(-3\hat{i}+\hat{j}-2\hat{k}\right)\times\left(\hat{i}+\hat{j}+\hat{k}\right)$$

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -3 & 1 & -2\\ 1 & 1 & 1\end{vmatrix}=\hat{i}(1+2)-\hat{j}(-3+2)+\hat{k}(-3-1)$$

$$=3\hat{i}+\hat{j}-4\hat{k}$$

Hence, option *(d)* is correct.

88. *(d)* Given rule is: Distance $s=2-3t+4t^3$

$$\Rightarrow \text{Velocity}=\frac{ds}{dt}=-3+12t^2$$

$$\Rightarrow \text{Acceleration}=\frac{d^2s}{dt^2}=24t$$

Since, velocity is zero

$$\therefore \frac{ds}{dt}=0\Rightarrow -3+12t^2=0\Rightarrow t=\sqrt{\frac{3}{12}}=\frac{1}{2}$$

Acceleration (when velocity is zero)

$$\Rightarrow \frac{d^2s}{dt^2}=24t=24\times\frac{1}{2}=12\,\text{unit}$$

Hence, option *(d)* is correct.

89. *(b)* Let $2^x=3^y=12^z=k$

Taking log on both sides, we get

$x=\log_2 k,\ y=\log_3 k$ and $z=\log_{12} k$

$$\therefore \frac{x+2y}{xy}=\frac{\log_2 k+2\log_3 k}{\log_2 k\log_3 k}$$
$$=\frac{1}{\log_3 k}+\frac{2}{\log_2 k}$$
$$=\log_k 3+2\log_k 2$$
$$=\log_k 3+\log_k 4$$
$$=\log_k 12$$
$$=\frac{1}{\log_{12} k}=\frac{1}{z}$$

Hence, option *(b)* is correct.

90. *(b)* Given that $\sin 3A=1$

$$\Rightarrow 3\sin A-4\sin^3 A=1$$
$$\Rightarrow 4\sin^3 A-3\sin A+1=0$$
$$\Rightarrow(\sin A+1)\left(4\sin^2 A-4\sin A+1\right)=0$$
$$\Rightarrow(\sin A+1)(2\sin A-1)^2=0$$
$$\Rightarrow \sin A=-1 \text{ or } \frac{1}{2}$$

Thus, $\sin A$ can take two distinct values.

Hence, option (b) is correct.

91. *(b)* Here $m_1=2-\sqrt{3}$ and $m_1=2+\sqrt{3}$

Obtuse angle between them is given by:

$$\theta=\tan^{-1}\left|\frac{m_1-m_2}{1+m_1m_2}\right|$$
$$=\tan^{-1}\left(\frac{2-\sqrt{3}-2-\sqrt{3}}{1+\left(2-\sqrt{3}\right)\left(2+\sqrt{3}\right)}\right)$$
$$=\tan^{-1}\left(\frac{-2\sqrt{3}}{2}\right)=\tan^{-1}\left(-\sqrt{3}\right)=120^\circ$$

Hence, option *(b)* is correct.

92. *(c)* The equation of curve is $y=-x^3+3x^2+2x-27$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx}=-3x^2+6x+2$$

This represents slope of the curve at any point.

Let $A=\frac{dy}{dx}=-3x^2+6x+2$

$\Rightarrow \frac{dA}{dx}=-6x+6$ and

$$\frac{d^2A}{dx^2}=-6$$

Put $\frac{dA}{dx}=0$ for maxima or minima

$-6x+6=0 \Rightarrow x=1$

Now, $\left(\frac{d^2A}{dx^2}\right)_{x=1}=-6<0$

$\therefore$ A is maximum at $x=1$

$\therefore$ Maximum slope of curve $=-3+6+2=5$

Hence, option *(c)* is correct.

93. *(b)* We have $A=\begin{bmatrix}1 & 2\\ 1 & 1\end{bmatrix}$ and $B=\begin{bmatrix}0 & -1\\ 1 & 2\end{bmatrix}$

$\therefore |A|=-1$ and $|B|=1$

$$A^{-1}=\frac{1}{|A|}adjA=\frac{1}{-1}\begin{bmatrix}1 & -1\\ -2 & 1\end{bmatrix}^{T}=\begin{bmatrix}-1 & 2\\ 1 & -1\end{bmatrix}$$

$$B^{-1}=\frac{1}{|B|}adjB=\frac{1}{1}\begin{bmatrix}2 & -1\\ 1 & 0\end{bmatrix}^{T}=\begin{bmatrix}2 & 1\\ -1 & 0\end{bmatrix}$$

$$\therefore B^{-1}A^{-1}=\begin{bmatrix}2 & 1\\ -1 & 0\end{bmatrix}\begin{bmatrix}-1 & 2\\ 1 & -1\end{bmatrix}=\begin{bmatrix}-1 & 3\\ 1 & -2\end{bmatrix}$$

Hence, option *(b)* is correct.

94. *(c)* R is defined over the set of non-negative integers, $x^2+y^2=36$

$\Rightarrow y=\sqrt{36-x^2}=\sqrt{(6-x)(6+x)}, x=0 \text{ or } 6$

For $x=0 \Rightarrow y=6$ and for $x=6 \Rightarrow y=0$

So, y is 6 or 0

Thus, $R=\{(6,0),(0,6)\}$

Hence, option *(c)* is correct.

95. *(c)* Given that $3\tan^2 x=1$

$\Rightarrow \tan x=\pm\frac{1}{\sqrt{3}}$

$\Rightarrow \tan x=\tan\left(\pm\frac{\pi}{6}\right)$

$\Rightarrow x=n\pi\pm\frac{\pi}{6}$

Hence, option *(c)* is correct.

96. *(a)* We know that the angle between the planes $a_1x+b_1y+c_1z+d_1=0$ and $a_2x+b_2y+c_2z+d_2=0$ is given by

$$\cos\theta=\left|\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|$$

Given equation of planes are $px+2y+2z-3=0$ and $2x-y+z+2=0$

On comparing with standard equations, we get

$a_1=p, a_2=2, b_1=2, b_2=-1, c_1=2, c_2=1$

Also, $\theta=\frac{\pi}{4}$ (given)

$$\therefore \cos\frac{\pi}{4}=\left|\frac{p\times 2+2\times(-1)+2\times 1}{\sqrt{p^2+4+4}\sqrt{4+1+1}}\right|$$

$$\Rightarrow \frac{1}{\sqrt{2}}=\frac{2p}{\sqrt{6}\sqrt{p^2+4+4}} \Rightarrow \frac{1}{2}=\frac{4p^2}{6(p^2+8)}$$

$$\Rightarrow \frac{3}{4}=\frac{p^2}{p^2+8} \Rightarrow 3p^2+24=4p^2 \Rightarrow p^2=24$$

Hence, option (a) is correct.

97. *(b)* Let the vector be $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$. Since, $\vec{r}$ and $\hat{i}+\hat{j}$ are perpendicular to each other.

Hence, $\vec{r}\cdot(\hat{i}+\hat{j})=0 \Rightarrow x+y=0$ ....(i)

Also, $\vec{r}$ and $\hat{j}+\hat{k}$ are perpendicular to each other. So,

$\vec{r}\cdot\left(\hat{j}+\hat{k}\right)=0 \Rightarrow y+z=0$ ....(ii)

And $x^2+y^2+z^2=9$ ....(iii)

$\Rightarrow (-y)^2+y^2+(-y)^2=9$

$\Rightarrow 3y^2=9$

$\Rightarrow y=\pm\sqrt{3}$

$\therefore\ x=\mp\sqrt{3}$ and $z=\pm\sqrt{3}$ $\quad[\text{from}(i)\text{and}(ii)]$

So, vector is $\sqrt{3}\left(\hat{i}-\hat{j}+\hat{k}\right)$

Hence, option (b) is correct.

98. *(b)* Let r be the radius of the balloon.

Balloon is like a sphere and volume of sphere $=\frac{4}{3}\pi r^3$

$\therefore V=\frac{4}{3}\pi r^3$

Differentiate both sides w.r.t. t, we get

$\Rightarrow \frac{dV}{dt}=\frac{4}{3}\pi\cdot 3r^2\frac{dr}{dt}$

$\Rightarrow 4=\frac{4}{3}\pi\cdot 3(4)^2\frac{dr}{dt}$ $\quad\left(\because \frac{dV}{dt}=4\,\text{cm}^3/\text{s}\right)$

$\Rightarrow \frac{dr}{dt}=\frac{1}{16\pi}$ ....(i)

Now, surface area of the balloon, $S=4\pi r^2$

$\therefore \frac{dS}{st}=4\pi\cdot 2r\frac{dr}{dt}=4\pi\cdot 2(4)\times\frac{1}{16\pi}=2\,\text{cm}^2/\text{s}$

Hence, option *(b)* is correct.

99. *(d)* A relation is equivalent if it is reflexive, symmetric and transitive.

So, we check for the same, one-by-one.

$x,y\in N \Rightarrow x>0, y>0$, $R=\left\{(x,y):xy>0, x,y\in N\right\}$

(i) **Reflexive**

$\because x,y\in N$

$\because x,x\in N \Rightarrow x^2>0$

$\therefore\ R$ is reflexive.

(ii) **Symmetric**

$\because x,y\in N$ and $xy>0 \Rightarrow yx>0$

$\therefore\ R$ is also symmetric.

(iii) **Transitive**

$\because x,y,z\in N$

$\therefore\ xy>0, yz>0 \Rightarrow xz>0$

$\therefore\ R$ is also transitive.

Conclusion: R is an equivalence relation.

Hence, option (d) is correct.

100. *(a)* Here α is the root of equation.

$25\cos^2\theta+5\cos\theta-12=0$

$\Rightarrow 25\cos^2\alpha+5\cos\alpha-12=0$

$\Rightarrow 25\cos^2\alpha+20\cos\alpha-15\cos\alpha-12=0$

$\Rightarrow 5\cos\alpha(5\cos\alpha+4)-3(5\cos\alpha+4)=0$

$\Rightarrow (5\cos\alpha-3)(5\cos\alpha+4)=0$

$\Rightarrow \cos\alpha=\frac{3}{5}$ or $\cos\alpha=-\frac{4}{5}$

Here, $\frac{\pi}{2}<\alpha<\pi$

$\therefore \cos\alpha=\frac{-4}{5}$

$\left(\because \text{In } 2^{\text{nd}} \text{ quadrant}, \cos\alpha \text{ value is negative}\right)$

Now, $\sin\alpha=\sqrt{1-\cos^2\alpha}=\sqrt{1-\frac{16}{25}}=\frac{3}{5}$

$\therefore \tan\alpha=\frac{\sin\alpha}{\cos\alpha}=\frac{3}{5}\times\frac{-5}{4}=\frac{-3}{4}$

Hence, option *(a)* is correct.

101. *(b)* Since, $\sin 2\alpha=2\sin\alpha\cdot\cos\alpha$

$\therefore \sin 2\alpha=2\left(\frac{3}{5}\right)\left(\frac{-4}{5}\right)$

$=\frac{6}{5}\times\frac{-4}{5}=\frac{-24}{25}$

Hence, option (b) is correct.

102. *(d)* The given matrix $A=\begin{bmatrix}0 & -4+i\\ 4+i & 0\end{bmatrix}$

Now, from the given options:

From option *(a)*: For symmetric matrix, $A^T = A$

So, $A^T = \begin{bmatrix} 0 & 4+i \\ -4+i & 0 \end{bmatrix} \neq A$

$\therefore$ Given matrix is not symmetric.
Therefore, option *(a)* is wrong.

From option *(b)*: For skew-symmetric, $A^T = -A$

So, $A^T = \begin{bmatrix} 0 & 4+i \\ -4+i & 0 \end{bmatrix} \neq -A$

$\therefore$ Given matrix is not skew-symmetric.
Therefore, option *(b)* is wrong.

From option *(c)*: For Hermitian matrix, $A^T = \overline{A}$, where $\overline{A}$ is conjugate of matrix A.

Now, $\overline{A} = \begin{bmatrix} 0 & -4-i \\ 4-i & 0 \end{bmatrix} \neq A^T$

$\therefore$ Given matrix is not Hermitian matrix.
Therefore, option *(c)* is wrong.

From option *(d)*: For Skew-Hermitian matrix,
The diagonal elements of a skew-Hermitian matrix are pure imaginary or zero.

$A = \begin{bmatrix} 0 & -4+i \\ 4+i & 0 \end{bmatrix}$

Here, diagonal element indicates that the given matrix is skew-Hermitian matrix.
Hence, option *(d)* is correct.

103. *(a)* Since, $\left(\vec{a}+\lambda\vec{b}\right)$ perpendicular to $\left(\vec{a}-\lambda\vec{b}\right)$, their dot product is zero. So,

$\left(\vec{a}+\lambda\vec{b}\right)\cdot\left(\vec{a}-\lambda\vec{b}\right)=0$

$\Rightarrow \left|\vec{a}\right|^2 - \lambda^2\left|\vec{b}\right|^2 - \lambda\vec{a}\cdot\vec{b} + \lambda\vec{b}\cdot\vec{a} = 0$

$\Rightarrow \left|\vec{a}\right|^2 - \lambda^2\left|\vec{b}\right|^2 = 0 \qquad \left[\because \vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}\right]$

$\Rightarrow 9 - 16\lambda^2 = 0$

$\Rightarrow \lambda^2 = \frac{9}{16} \Rightarrow \lambda = \pm\frac{3}{4}$

Hence, option *(a)* is correct.

104. *(c)* Given that $\frac{dx}{dt} = x+1 \Rightarrow \frac{dx}{x+1} = dt$

Integrating both sides, we get

$\ln(x+1) = t + c$

At $t = 0,\ x = 0$ (origin point)

Then, $\ln(0+1) = 0 + c \Rightarrow c = 0$

$\therefore \ln(x+1) = t$

When $x-24,\ t = \ln(24+1) = \ln(25) = 2\ln 5$

Hence, option *(c)* is correct.

105. *(d)* For **reflexive**:

$aRa \Rightarrow a$ divides a

$\therefore R$ is reflexive.

For **symmetric**:

$aRb \Rightarrow a$ divides b

$bRa \Rightarrow b$ divides a

which may not be true.

$\therefore R$ is not symmetric.

For **transitive**:

$aRb \Rightarrow a$ divides $b \Rightarrow b = ka$

$bRc \Rightarrow b$ divides $c \Rightarrow c = lb$

Now, $c = lka$

$\Rightarrow a$ divides c

$\Rightarrow aRc$

$\Rightarrow aRb, bRc \Rightarrow cRa$

$\Rightarrow R$ is transitive

Thus, R is only reflexive and transitive.
Hence, option *(d)* is correct.

106. *(b)* Bag I has $5\,\text{white} + 3\,\text{black}$ balls.

Bag II has $2\,\text{white} + 4\,\text{black}$ balls.

$P(\text{Black})_{\text{1st bag}} = \frac{3}{8}$ and $P(\text{White})_{\text{1st bag}} = \frac{5}{8}$

If one ball is drawn from bag I and placed in bag II, bag II will have 7 balls.

If black ball is drawn, then bag II contains $2\,\text{White} + 5\,\text{Black} = 7\,\text{balls}$.

P(black ball from bag I and black ball from bag II)

$= \frac{3}{8} \times \frac{5}{7} = \frac{15}{56}$

If ball is white then bag II has $3\,\text{white} + 4\,\text{black balls}$

P(white ball from bag I and black ball from bag II)

$= \frac{5}{8} \times \frac{4}{7} = \frac{20}{56}$

$\Rightarrow P(\text{black ball})_{\text{bag II}} = \frac{15}{56} + \frac{20}{56} = \frac{35}{56}$

Hence, option *(b)* is correct.

107. *(b)* Line of regression of y on x is:

$y - \bar{y} = b_{yx}(x - \bar{x})$

$\bar{y} = \frac{\sum y}{n};\ \bar{x} = \frac{\sum x}{n} \Rightarrow \bar{y} = \frac{220}{10} = 22;\ \bar{x} = \frac{130}{10} = 13$

$b_{yx} = r \cdot \frac{\sigma_y}{\sigma_x}$

$$r = \frac{n\sum xy - (\sum x)(\sum y)}{\sqrt{\left[n\sum x^2 - (\sum x)^2\right]\left[n\sum y^2 - (\sum y)^2\right]}}$$

$$= \frac{10(3467) - (130)(220)}{\sqrt{\left[10 \times 2288 - (130)^2\right]\left[10 \times 5506 - (220)^2\right]}}$$

$= 0.962$

$\sigma_y = \sqrt{\frac{\sum y^2}{n} - \left(\frac{\sum y}{n}\right)^2} \Rightarrow \sigma_y = 8.2;\ \sigma_x = 7.73$

$\Rightarrow b_{yx} = 0.962 \times \frac{8.2}{7.73} = 1.02$

$\therefore$ Line of regression of y on x is:

$y - 22 = 1.02(x - 13)$

$\Rightarrow y = 1.02x + 8.74$

Hence, option *(b)* is correct.

108. *(c)* Mean of 100 items $= \bar{x}_{100} = 50$

Mean of 150 items $= \bar{x}_{150} = 40\ x_n$

Standard deviation of 100 items $= \sigma_{100} = 5$

Standard deviation of 150 items $= \sigma_{150} = 6$

$\therefore \bar{x}_{250} = \frac{n_1\bar{x}_{100} + n_2\bar{x}_{150}}{n_1 + n_2} = \frac{(100 \times 50) + (150 \times 40)}{100 + 150}$

$= \frac{11000}{250} = 44$

Now, $d_1 = 50 - 44 = 6 \qquad d_1^2 = 36$

And, $d_2 = 40 - 44 = -4 \qquad d_2^2 = 16$

$$\therefore \sigma_{250} = \frac{\sqrt{n_1\left(\sigma_{100}^2 + d_1^2\right) + n_2\left(\sigma_{150}^2 + d_2^2\right)}}{n_1 + n_2}$$

$= \frac{\sqrt{390}}{5} = \frac{37.28}{5} \approx 7.5$

Hence, option *(c)* is correct.

109. *(d)* Given $np = 4$ and $npq = \frac{4}{3}$

$\therefore 4q = \frac{4}{3} \Rightarrow q = \frac{1}{3}$

$\therefore p = 1 - \frac{1}{3} = \frac{2}{3} \Rightarrow n = \frac{4 \times 3}{2} = 6$

Now, $P(X \geq 5) = {}^6C_5 p^5 q^1 + {}^6C_6 p^6 q^0$

$= {}^6C_5\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right) + {}^6C_6\left(\frac{2}{3}\right)^6$

$= \frac{6 \times 32}{3^6} + \frac{64}{3^6} = \frac{256}{3^6} = \frac{2^8}{3^6}$

Hence, option *(d)* is correct.

110. *(c)* Given that $E(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, therefore,

$E(\alpha) = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ and $E(\beta) = \begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix}$

Now, $E(\alpha)E(\beta) = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix}$

$$= \begin{bmatrix} \cos\alpha\cdot\cos\beta - \sin\alpha\cdot\sin\beta & \cos\alpha\cdot\sin\beta + \sin\alpha\cdot\cos\beta \\ -\sin\alpha\cdot\cos\beta - \cos\alpha\cdot\sin\beta & -\sin\alpha\cdot\sin\beta + \cos\alpha\cdot\cos\beta \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\alpha+\beta) & \sin(\alpha+\beta) \\ -\sin(\alpha+\beta) & \cos(\alpha+\beta) \end{bmatrix}$$

$$= E(\alpha+\beta)$$

Hence, option *(c)* is correct.

111. *(d)* Given that $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + \hat{j} + 2\hat{k}$

$\therefore \vec{a} + \vec{b} = 4\hat{i} - \hat{j} + 5\hat{k}$

Then, $\vec{c} = \lambda(\vec{a} + \vec{b}) = \lambda(4\hat{i} - \hat{j} + 5\hat{k})$

$\Rightarrow l = \sqrt{16\lambda^2 + \lambda^2 + 25\lambda^2}$

$\Rightarrow l = \sqrt{42\lambda^2}$

$\Rightarrow \lambda = \dfrac{l}{\sqrt{42}}$

$\therefore \vec{c} = \dfrac{l}{\sqrt{42}}(4\hat{i} - \hat{j} + 5\hat{k}) = \dfrac{1}{\sqrt{42}}(4, -1, 5)$

Hence, option *(d)* is correct.

112. *(a)* The given lines are $\dfrac{x-2}{1} = \dfrac{y-(-1)}{-2} = \dfrac{z-(-2)}{1}$

and $\dfrac{x-1}{1} = \dfrac{y-\left(-\frac{3}{2}\right)}{\frac{3}{2}} = \dfrac{z-(-5)}{2}$

Dr's of 1st lines are $a_1 = 1, b_1 = -2, c_1 = 1$

Dr's of 2nd lines are $a_2 = 2, b_2 = 3, c_2 = 4$

Let θ be the angle b/w two lines given by

$$\cos\theta = \left|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}\right|$$

$$\Rightarrow \cos\theta = \left|\frac{1\times 2 + (-2)\times 3 + 1\times 4}{\sqrt{1+4+1}\sqrt{4+9+16}}\right| = 0$$

$$\Rightarrow \theta = \frac{\pi}{2}$$

Hence, option *(a)* is correct.

113. *(c)* By Bayes' Theorem, we have

Required probability $= P(A_1/B)$

$$= \frac{P(A_1)P(B/A_1)}{P(A_1)P(B/A_1) + P(A_2)P(B/A_2) + P(A_3)P(B/A_3)}$$

$$= \frac{0.4\times 0.25}{0.4\times 0.25 + 0.4\times 0.4 + 0.2\times 0.125}$$

$$= \frac{0.1}{0.285} = \frac{20}{57}$$

Hence, option *(c)* is correct.

114. *(b)* We know that $|\vec{a}\times\vec{b}|^2 + |\vec{a}\cdot\vec{b}|^2 = |\vec{a}|^2 \times |\vec{b}|^2$

$\therefore\ 64 + |\vec{a}\cdot\vec{b}|^2 = 4\times 25$

$\Rightarrow |\vec{a}\cdot\vec{b}|^2 = 100 - 64 = 36$

$\Rightarrow \vec{a}\cdot\vec{b} = 6$

Hence, option *(b)* is correct.

115. *(b)* A leap year has 366 days in which 2 days may be any one of the following pairs:

(Sunday, Monday), (Monday, Tuesday), (Tuesday, Wednesday), (Wednesday, Thursday), (Thursday, Friday), (Friday, Saturday) and (Saturday, Sunday)

$\therefore$ Required probability $= \dfrac{2}{7} + \dfrac{2}{7} - \dfrac{1}{7} = \dfrac{3}{7}$

Hence, option *(b)* is correct.

116. *(d)* $r = 0.6$, covariance $= 27, \sigma^2_{(y)} = 25 \Rightarrow \sigma(y) = 5$

We know $r = \dfrac{\text{covariance}(x,y)}{\sigma(x)\cdot\sigma(y)}$

$\Rightarrow \sigma(x) = \dfrac{\text{covariance}(x,y)}{r\cdot\sigma(y)} = \dfrac{27}{0.6\times 5} = 9$

$\Rightarrow \sigma^2(x) = 81$

Hence, option *(d)* is correct.

117. *(c)* Possible samples are as follows:

$\{HHH, HTH, HHT, THH, TTH, THT, HTT, TTT\}$

Let A be the event of getting one head.

Let B be the event of getting no head.

Favourable outcome for $A=\{TTH, THT, HTT\}$

Favourable outcome for $B=\{TTT\}$

Total no. of outcomes $=8$

$\therefore P(A)=\frac{3}{8}, P(B)=\frac{1}{8}$

$\therefore$ Required probability $=P(A)+P(B)$

$=\frac{3}{8}+\frac{1}{8}=\frac{4}{8}=\frac{1}{2}$

Hence, option *(c)* is correct.

118. *(a)* Since, the AM of numbers $x_1, x_2, \ldots x_n$ is μ

$\therefore n\mu=x_1+x_2+\cdots+x_n$

Sum of new numbers

$=(x_1+1)+(x_2+2)+\cdots+(x_n+n)$

$=(x_1+x_2+\cdots+x_n)+(1+2+3+\cdots+n)$

$=n\mu+\frac{n(n+1)}{2}$

$\therefore$ AM $=\mu+\frac{(n+1)}{2}$

Hence, option *(a)* is correct.

119. *(d)* The standard deviation of n observations $x_1, x_2, \ldots x_n$ is 6 and of $y_1, y_2, \ldots y_n$ is 8.

Then, the standard deviation of n observations $x_1-y_1, x_2-y_2, \ldots x_n-y_n$ is $8-6=2$

Hence, option *(d)* is correct.

120. *(d)* Given regression lines are $6x+y=30$ and $3x+2y=25$

Point of intersection of both lines is

$(\bar{x}, \bar{y})=\left(\frac{35}{9}, \frac{20}{3}\right)$

From the given line $6x+y=30 \Rightarrow x=-\frac{1}{6}y+5$

And, $3x+2y=25 \Rightarrow y=-\frac{3}{2}x+\frac{25}{2}$

$\therefore r^2=\left(-\frac{1}{6}\right)\left(-\frac{3}{2}\right) \Rightarrow r=\pm\frac{1}{2}$

$\because$ sign of $\bar{x}, \bar{y}$ and r is same.

$\therefore r=\frac{1}{2}$

Hence, option *(d)* is correct.