HINTS & SOLUTION

1. *(a)* Given equation is $(x-p)(x-6)+1=0$.

$\Rightarrow \quad x^2-6x-px+6p+1=0$

$\Rightarrow x^2-(p+6)x+(6p+1)=0$

Now, $b^2-4ac=0$

$a=1,\ b=-(p+6),\ c=6p+1$

$\Rightarrow \quad (p+6)^2-4(6p+1)=0$

$\Rightarrow p^2+36+12p-24p-4=0$

$\Rightarrow \quad p^2+12p+32=0$

$\Rightarrow \quad (p-4)(p-8)=0$

$\Rightarrow \quad p=4,\ 8$

2. *(d)* $(0.101)_2=2^{-1}+2^{-2}\times 0+2^{-3}\times 1$

$=\frac{1}{2}+0+\frac{1}{8}=\frac{5}{8}$

$(0.011)_2=2^{-1}\times 0+2^{-2}\times 1+2^{-3}\times 1$

$=0+\frac{1}{4}+\frac{1}{8}=\frac{3}{8}$

Also, $(11)_2=2^1\times 1+2^0\times 1$

$=2+1=3$

and, $(01)_2=2^1\times 0+2^0\times 1$

$=0+1=1$

$$\therefore \frac{(0.101)_2^{(11)_2}+(0.011)_2^{(11)_2}}{(0.101)_2^{(10)_2}-(0.101)_2^{(01)_2}(0.011)_2^{(01)_2}+(0.011)_2^{(10)_2}}$$

$$=\frac{\left(\frac{5}{8}\right)^3+\left(\frac{3}{8}\right)^3}{\left(\frac{5}{8}\right)^2-\left(\frac{5}{8}\right)\left(\frac{3}{7}\right)+\left(\frac{3}{8}\right)^2}$$

$$=\frac{5}{8}+\frac{3}{8}=\frac{8}{8}=1=(1)_2$$

3. *(b)* Given expression

$\left[\left(-1+i\sqrt{3}\right)/2\right]^{10}+\left[\left(-1-i\sqrt{3}\right)/2\right]^{10}$

We know that $\frac{-1+i\sqrt{3}}{2}=\omega.$

So, $\frac{-1-i\sqrt{3}}{2}=\omega^2$

$\therefore \left[\left(-1+i\sqrt{3}\right)/2\right]^{10}+\left[\left(-1-i\sqrt{3}\right)/2\right]^{10}$

$=\omega^{10}+\omega^{20}$

$=\omega^{3\times 3+1}+\omega^{3\times 6+2}$

$=\left(\omega^3\right)^3\cdot\omega+\left(\omega^3\right)^6\cdot\omega^2$

$=\omega+\omega^2 \quad \left(\because\ \omega^3=1\right)$

$=-1 \quad \left(\because 1+\omega+\omega^2=0\right)$

4. *(d)* COPORATION is a 11 letters word.
It has 5 vowels (O, O, O, A, I) and 6 consonants (C, R, P, R, T, N)

5 vowels can take 5 even places in $\frac{5!}{3!}$ ways. (O is repeated thrice)

Similarly, 6 consonants cam take 6 odd places in $\frac{6!}{2!}$ ways. (R is repeated twice)

So, the total number of ways is given by,

$\frac{5!}{3!}\times\frac{6!}{2!}=20\times 320=7200$

5. *(b)* The given equation

$\left(a^2+b^2\right)x^2-2b(a+c)x+\left(b^2+c^2\right)=0$ has equal roots, so discriminant = 0.

So, $\{2b(a+c)\}^2-4\left(a^2+b^2\right)\left(b^2+c^2\right)=0$

$4b^2\left(a^2+c^2+2ca\right)-4\left(a^2b^2+a^2c^2+b^4+b^2c^2\right)=0$

$\Rightarrow b^2a^2+b^2c^2+2b^2ca-a^2b^2-a^2c^2-b^4-b^2c^2=0$

$\Rightarrow \quad b^4-2b^2ca+a^2c^2=0$

$\Rightarrow \quad (b^2)^2 - 2(b^2)(ca) + (ca)^2 = 0$

$\Rightarrow \quad (b^2 - ca)^2 = 0$

$\Rightarrow \quad b^2 = ca$

This implies that a, b, c are in G.P.

6. *(b)* The first three terms in the expansion of $(1+ax)^n$ are ${}^nC_0,\ {}^nC_1 ax,\ {}^nC_2 a^2 x^2$.

Given ${}^nC_0 = 1,\ {}^nC_1 ax = 12x,\ {}^nC_2 a^2 x^2 = 64x^2$

$\Rightarrow nax = 12x;\ \frac{n(n-1)}{2} a^2 = 64$

Now, $nax = 12x \Rightarrow a = \frac{12}{n}$

$\therefore \frac{n(n-1)}{2} a^2 = 64 \Rightarrow \frac{n(n-1)}{2} \times \frac{144}{n^2} = 64$

$\Rightarrow \frac{n-1}{n} = \frac{64 \times 2}{144} = \frac{8}{9}$

$\Rightarrow n = 9$

7. *(c)* $(1111)_2 = 1\times 2^3 + 1\times 2^2 + 1\times 2^1 + 1\times 2^0$

$= 8+4+2+1 = 15$

$(1001)_2 = 1\times 2^3 + 0\times 2^2 + 0\times 2^1 + 1\times 2^0$

$= 8+1 = 9$

$(1010)_2 = 1\times 2^3 + 0\times 2^2 + 1\times 2^1 + 0\times 2^0$

$= 8+2 = 10$

$\therefore (1111)_2 + (1001)_2 - (1010)_2 = 15+9-10$

$= (14)_{10}$

$= (1110)_2$

8. *(b)* Rationalizing $\frac{i+\sqrt{3}}{-i+\sqrt{3}}$, we get

$$\frac{i+\sqrt{3}}{-i+\sqrt{3}} = \frac{(i+\sqrt{3})^2}{(\sqrt{3}-i)(\sqrt{3}+i)}$$

$$= \frac{i^2+3+2\sqrt{3}i}{3+1}$$

$$= \frac{-1+3+2\sqrt{3}i}{4}$$

$$= \frac{1+\sqrt{3}i}{2}$$

$$= -\frac{(-1-\sqrt{3}i)}{2} = -\omega^2$$

Rationalizing $\frac{i-\sqrt{3}}{i+\sqrt{3}}$, we get

$$\frac{i-\sqrt{3}}{i+\sqrt{3}} = \frac{(i-\sqrt{3})^2}{(i+\sqrt{3})(i-\sqrt{3})}$$

$$= \frac{i^2+3-2\sqrt{3}i}{-1-3}$$

$$= \frac{-1+3-2\sqrt{3}i}{-4}$$

$$= \frac{-1+\sqrt{3}i}{2} = \omega$$

Therefore, $\left(\frac{i+\sqrt{3}}{-i+\sqrt{3}}\right)^{200} + \left(\frac{i-\sqrt{3}}{i+\sqrt{3}}\right)^{200} + 1$

$= (-\omega^2)^{200} + (\omega)^{200} + 1$

$= \omega^{400} + \omega^{200} + 1$

$= (\omega^3)^{133} \cdot \omega + (\omega^3)^{66} \cdot \omega^2 + 1$

$= \omega + \omega^2 + 1 \quad [\because \omega^3 = 1]$

$= 0$

9. *(c)* Equation of line is $ax + by = p$, then length of perpendicular from the origin is,

$$p = \left|\frac{a\times 0 + b\times 0 - p}{\sqrt{a^2+b^2}}\right| \text{ or } p = \left|\frac{-p}{\sqrt{a^2+b^2}}\right|$$

or $\left|\frac{1}{\sqrt{a^2+b^2}}\right| = 1$ or $a^2+b^2 = 1$

Given $b=\frac{\sqrt{3}}{2}$, which implies that,

$$a^2+\frac{3}{4}=1 \Rightarrow a^2=\frac{1}{4}$$

$$\Rightarrow a=\pm\frac{1}{2}$$

Since the angle is with +ve direction of x-axis, we have $a=\frac{1}{2}$

Equation is given by,

$$\frac{1}{2}x+\frac{\sqrt{3}}{2}y=p \quad \text{or } x\cos 60°+y\sin 60°=p$$

Thus, the required angle is $60°$.

10. *(d)* Equation of the parabola is $y^2=2x$. This means that the vertex lies at origin and coordinates of A is $(0,0)$.

Let (x_1, y_1) be the coordinates of the point Q.

Then, $y_1{}^2=2x_1 \quad ...(i)$

and slope of line PQ $=\frac{y_1-2}{x_1-2}$

Also, slope of AP $=\frac{2-0}{2-0}=1$

Since PQ and AP are perpendicular to each other, hence

Slope of AP × Slope of PQ = −1

So,

$$1\times\frac{y_1-2}{x_1-2}=-1$$

$$\Rightarrow y_1-2=-x_1+2$$

$$\Rightarrow x_1+y_1=4$$

$$\Rightarrow x_1=4-y_1$$

Putting value of x_1 in (i)

$$y_1{}^2=2(4-y_1) \Rightarrow y_1{}^2=8-2y_1$$

$$\Rightarrow y_1{}^2+2y_1-8=0$$

$$\Rightarrow y_1=-4,\ 2$$

Hence, coordinates of point Q are $(8,-4)$.

Therefore, required length is,

$$PQ=\sqrt{(8-2)^2+(-4-2)^2}$$

$$=\sqrt{36+36}$$

$$=\sqrt{72}$$

$$=6\sqrt{2}$$

11. *(a)* Given cos A = cos B cos C

tan A − tan B − tan C

$$=\frac{\sin A}{\cos A}-\frac{\sin B}{\cos B}-\frac{\sin C}{\cos C}$$

$$=\frac{\sin A}{\cos A}-\frac{(\sin B\cos C+\cos B\sin C)}{\cos B\cos C}$$

$$=\frac{\sin A}{\cos A}-\frac{\sin(B+C)}{\cos A} \quad [\because \cos B\cos C=\cos A]$$

$$=\frac{\sin A-\sin(B+C)}{\cos A}$$

$$=\frac{\sin A-\sin(\pi-A)}{\cos A} \quad [\because B+C=\pi-A]$$

$$=\frac{\sin A-\sin A}{\cos A}$$

$$=0$$

12. *(c)* Let α and β be the roots of the equation $4x^2+3x+7=0$.

$$\therefore \text{Sum}=\alpha+\beta=-\frac{3}{4} \text{ and Product}=\alpha\beta=\frac{7}{4}$$

Consider $\alpha^{-2}+\beta^{-2}=\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\frac{\alpha^2+\beta^2}{(\alpha\beta)^2}$

$$=\frac{(\alpha+\beta)-2\alpha\beta}{(\alpha\beta)^2}=\frac{\frac{9}{16}-\frac{7}{2}}{\frac{49}{16}}$$

$$=\frac{\frac{9-56}{16}}{\frac{49}{16}}=-\frac{47}{16}\times\frac{16}{49}=-\frac{47}{49}$$

13. *(b)* $\log\left(a+\sqrt{a^2+1}\right)+\log\left(\frac{1}{a+\sqrt{a^2+1}}\right)$

$$=\log\left(a+\sqrt{a^2+1}\right)+\log(1)-\log\left(a+\sqrt{a^2+1}\right)$$

$$=\log\left(a+\sqrt{a^2+1}\right)-\log\left(a+\sqrt{a^2+1}\right)$$

$$=0$$

14. *(a)* Let $S=9+99+999+\cdots$

$$S=\left(10^1-1\right)+\left(10^2-1\right)+\left(10^3-1\right)+\cdots$$

This can be rewritten as,

$$S=\left(10^1+10^2+10^3+\cdots\right)-\left(1+1+1+\cdots 100\text{ times}\right)$$

$$=\frac{10\left(10^{100}-1\right)}{10-1}-100$$

$\left[\because 10^1+10^2+10^3+\cdots \text{ is a GP with } a=10 \text{ and } r=10\right]$

$$=\frac{10}{9}\left(10^{100}-1\right)-100$$

15. *(b)* The expression

$$\frac{\operatorname{cosec}(\pi+\theta)\cot\{(9\pi/2-\theta)\}\operatorname{cosec}^2(2\pi-\theta)}{\cot(2\pi-\theta)\sec^2(\pi-\theta)\sec\{(3\pi/2)+\theta\}}$$

$$=\frac{-\operatorname{cosec}\theta\tan\theta\operatorname{cosec}^2\theta}{-\cot\theta\sec^2\theta\operatorname{cosec}\theta}$$

$$=\frac{\tan^2\theta\operatorname{cosec}^2\theta}{\sec^2\theta}$$

$$=\tan^2\theta\cdot\frac{\cos^2\theta}{\sin^2\theta}$$

$$=\tan^2\theta\cdot\frac{1}{\tan^2\theta}$$

$$=1$$

16. *(d)* In any ΔABC, we have

$$\angle A+\angle B+\angle C=\pi$$

Let $A=\tan^{-1}\frac{1}{2}$ and $B=\tan^{-1}\frac{1}{3}$

$$\Rightarrow \tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}+\angle C=\pi$$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\cdot\frac{1}{3}}\right)+\angle C=\pi$$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{5}{6}}{\frac{5}{6}}\right)+\angle C=\pi$$

$$\Rightarrow \tan^{-1}(1)+\angle C=\pi$$

$$\Rightarrow \frac{\pi}{4}+\angle C=\pi$$

$$\Rightarrow \angle C=\pi-\frac{\pi}{4}$$

$$\Rightarrow \angle C=\frac{3\pi}{4}=135^\circ$$

17. *(a)* Let $u=\sec^2 x,\ v=\tan^2 x$

Differentiate both w.r.t x

$$\frac{du}{dx}=2\sec x\cdot\sec x\tan x=2\sec^2 x\tan x$$

$$\frac{dv}{dx}=2\tan x\cdot\sec^2 x$$

This gives,

$$\frac{du}{dv}=\frac{du}{dx}\times\frac{dx}{dv}$$

$$=\frac{2\sec x\cdot\sec x\tan x}{2\tan x\sec^2 x}$$

$$=1$$

18. *(a)* Let 20 be divided into two parts such that the first part is x and the second part is $20-x$.

Let $P=x^3(20-x)=20x^3-x^4$

The first and second order derivatives are given by,

$$\frac{dP}{dx}=60x^2-4x^3 \text{ and } \frac{d^2P}{dx^2}=120x-12x^2$$

For maximum value, $\dfrac{dP}{dx}=0$

$$60x^2-4x^3=0\Rightarrow 4x^2(15-x)=0$$

$$\Rightarrow x=0,\ 15$$

$$\left.\frac{d^2P}{dx^2}\right|_{x=15}=120\times15-12\times225$$

$$=1800-2700$$

$$=-900<0$$

Thus, P is maximum for $x=15$.

So, first part is 15 and second part is 5.

Required product is $15\times5=75$.

19. *(a)* Given $y=\cot^{-1}\left[\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]$

Note that,

$$1+\sin x=\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\cos\frac{x}{2}\sin\frac{x}{2}$$

$$=\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2$$

$$\Rightarrow\sqrt{1+\sin x}=\cos\frac{x}{2}+\sin\frac{x}{2}$$

Similarly, $\Rightarrow\sqrt{1-\sin x}=\cos\dfrac{x}{2}-\sin\dfrac{x}{2}$

So, the given expression reduces to,

$$y=\cot^{-1}\left[\frac{\cos\frac{x}{2}+\sin\frac{x}{2}+\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}-\cos\frac{x}{2}+\sin\frac{x}{2}}\right]$$

$$=\cot^{-1}\left[\frac{2\cos\frac{x}{2}}{2\sin\frac{x}{2}}\right]$$

$$=\cot^{-1}\left(\cot\frac{x}{2}\right)$$

$$=\frac{x}{2}$$

Thus, $\dfrac{dy}{dx}=\dfrac{1}{2}$.

20. *(c)* We know that the equation of the straight line passing through the point of intersection of the lines $\dfrac{x}{a}+\dfrac{y}{b}=1$ and $\dfrac{x}{b}+\dfrac{y}{a}=1$ is,

$$\left(\frac{x}{a}+\frac{y}{b}-1\right)+\lambda\left(\frac{x}{b}+\frac{y}{a}-1\right)=0\ \ ...(\text{i})$$

The line passes through the origin, so we have

$$(0-1)+\lambda(0-1)=0\Rightarrow\lambda=-1$$

Substitute the value of λ in eq. (i)

$$\left(\frac{x}{a}+\frac{y}{b}-1\right)-1\left(\frac{x}{b}+\frac{y}{a}-1\right)=0$$

$$\Rightarrow\quad\frac{x}{a}+\frac{y}{b}-1-\frac{x}{b}-\frac{y}{a}+1=0$$

$$\Rightarrow\quad x\left(\frac{1}{a}-\frac{1}{b}\right)-y\left(\frac{1}{a}-\frac{1}{b}\right)=0$$

$$\Rightarrow\quad x-y=0$$

21. *(c)* Given equation of hyperbola is $5x^2-4y^2=k^2$.

$$\frac{x^2}{\frac{k^2}{5}}-\frac{y^2}{\frac{k^2}{4}}=1$$

Compare with the standard equation of hyperbola, we get $a^2=\dfrac{k^2}{5}$ and $b^2=\dfrac{k^2}{4}$.

This gives, $a=\dfrac{k}{\sqrt5}$ and $b=\dfrac{k}{2}$.

The eccentricity is $\dfrac{3}{2}$ and foci at $(\pm2,0)$.

Thus, $e=\dfrac{3}{2}$ and $\pm ae=2$

$$\Rightarrow\frac{k}{\sqrt5}\cdot\frac{3}{2}=2$$

$$\Rightarrow\quad k=\frac{4}{3}\sqrt5$$

22. *(d)* Consider, $\log_{10}\left(\frac{9}{8}\right)-\log_{10}\left(\frac{27}{32}\right)+\log_{10}\left(\frac{3}{4}\right)$

$$=\log_{10}\left(\frac{9}{8}\right)+\log_{10}\left(\frac{32}{27}\right)+\log_{10}\left(\frac{3}{4}\right)$$
$$=\log_{10}\left(\frac{9}{8}\times\frac{32}{27}\right)+\log_{10}\left(\frac{3}{4}\right)$$
$$=\log_{10}\left(\frac{4}{3}\right)+\log_{10}\left(\frac{3}{4}\right)$$
$$=\log_{10}\left(\frac{4}{3}\times\frac{3}{4}\right)$$
$$=\log_{10}(1)$$
$$=0$$

23. *(a)* The given equation is

$$\left(2-\sqrt{3}\right)x^2-\left(7-4\sqrt{3}\right)x+\left(2+\sqrt{3}\right)=0$$

So, sum of roots $=\frac{7-4\sqrt{3}}{2-\sqrt{3}}$

$$=\frac{\left(7-4\sqrt{3}\right)\left(2+\sqrt{3}\right)}{\left(2-\sqrt{3}\right)\left(2+\sqrt{3}\right)}$$
$$=\frac{14+7\sqrt{3}-8\sqrt{3}-12}{4-3}$$
$$=2-\sqrt{3}$$

24. *(c)* Given $x=r\sin\theta\cos\phi$, $y=r\sin\theta\sin\phi$ and $z=r\cos\theta$.

Now, $x^2+y^2+z^2=r^2\sin^2\theta\cos^2\phi$

$$+r^2\sin^2\theta\sin^2\phi+r^2\cos^2\theta$$
$$=r^2\sin^2\theta\left(\sin^2\phi+\cos^2\phi\right)+r^2\cos^2\theta$$
$$=r^2\sin^2\theta+r^2\cos^2\theta$$
$$=r^2\left(\sin^2\theta+\cos^2\theta\right)=r^2$$

Thus, $x^2+y^2+z^2$ is independent of θ and ϕ.

25. *(d)* Given $a+b=3\left(1+\sqrt{3}\right)$ ...(i)

and $a-b=3\left(1-\sqrt{3}\right)$ ...(ii)

By adding (i) and (ii),

$$2a=6\Rightarrow a=3$$
$$\therefore\ b=3\left(1+\sqrt{3}\right)-3=3\sqrt{3}$$

By using sine rule, $\frac{a}{\sin A}=\frac{b}{\sin B}$

$$\Rightarrow \frac{3}{\sin 30^\circ}=\frac{3\sqrt{3}}{\sin B}$$
$$\Rightarrow \frac{1}{\frac{1}{2}}=\frac{\sqrt{3}}{\sin B}$$
$$\Rightarrow \sin B=\frac{\sqrt{3}}{2}=\sin 60^\circ$$
$$\Rightarrow B=60^\circ$$

26. *(a)* $y=(\cos x)^{(\cos x)^{(\cos x)^{\cdot^{\cdot^{\cdot\infty}}}}}$

$$\Rightarrow y=(\cos x)^y$$

Taking logarithm on both sides w.r.t x, we get

$$\Rightarrow \log y=y\cdot\log(\cos x)$$

Differentiating both sides w.r.t x, we get

$$\frac{1}{y}\frac{dy}{dx}=\frac{dy}{dx}\cdot\log(\cos x)+y\cdot(-\tan x)$$
$$\Rightarrow \frac{dy}{dx}\left(\frac{1}{y}-\log\cos x\right)=-y\tan x$$
$$\Rightarrow \frac{dy}{dx}=\frac{-y\tan x}{\frac{1}{y}-\log\cos x}$$
$$\Rightarrow \frac{dy}{dx}=\frac{-y^2\tan x}{1-y\log\cos x}$$

27. *(d)* Given $f(x)=3x^2+6x-9$.

On differentiating, we get

$$f'(x)=6x+6$$

Now, $f'(x)<0\Rightarrow 6x+6<0$

$$\Rightarrow 6x<-6$$
$$\Rightarrow x<-1$$

Hence, $f(x)$ is decreasing in $(-\infty,-1)$.

28. *(b)* Let $u=\sqrt{x^2+16},\ v=x^2$

Differentiate both sides with respect to x,

$$\frac{du}{dx}=\frac{1}{2\sqrt{x^2+16}}\cdot 2x=\frac{x}{\sqrt{x^2+16}}$$

$$\frac{dv}{dx}=2x$$

Thus, $\frac{du}{dv}=\frac{du}{dx}\cdot\frac{dx}{dv}$

$$=\frac{x}{\sqrt{x^2+16}}\cdot\frac{1}{2x}$$

$$=\frac{1}{2\sqrt{x^2+16}}$$

At $x = 3$, we have

$$\left.\frac{du}{dv}\right|_{x=3}=\frac{1}{2\sqrt{9+16}}=\frac{1}{2\sqrt{25}}=\frac{1}{2(5)}=\frac{1}{10}$$

29. *(d)* Let A be the number of students passed in the first semester and B be the number of students passed in the second semester.

Given $n(A)=260,\ n(B)=210$

$n(\overline{A})=$ No of students who did not pass in sem 1

$=500-260=240$

Similarly, $n(\overline{B})=500-210=290$

Given $n(\overline{A}\cup\overline{B})=170$

$\Rightarrow n(\overline{A})+n(\overline{B})-n(\overline{A}\cap\overline{B})=170$

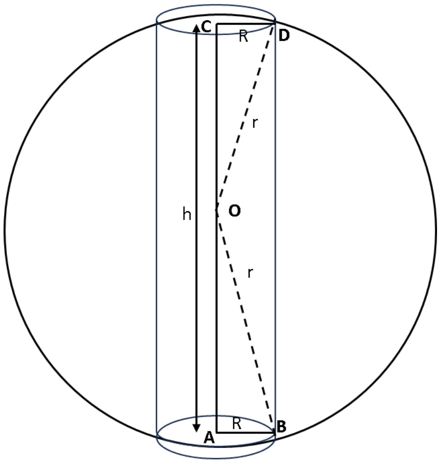
$\Rightarrow \quad 240+290-n(\overline{A}\cap\overline{B})=170$

$\Rightarrow \quad n(\overline{A}\cap\overline{B})=530-170$

$\Rightarrow \quad n(\overline{A}\cap\overline{B})=360$

Thus, students who passed in both semester
= 500 – 360 = 140

30. *(a)* Let h be the height, R be the radius and V be the volume of the cylinder.



In triangle OAB, we have

$$r^2=R^2+\left(\frac{h}{2}\right)^2 \quad \text{...(i)}$$

Clearly, $V=\pi R^2 h$

$$\Rightarrow V(h)=\pi\left(r^2-\frac{h^2}{4}\right)h$$

$$\Rightarrow V(h)=\pi\left(r^2h-\frac{h^3}{4}\right)$$

Now, $V'(h)=\pi\left(r^2-\frac{3h^2}{4}\right)$

For maximum value, $V'(h)=0$

$$\pi\left(r^2-\frac{3h^2}{4}\right)=0\Rightarrow r^2=\frac{3h^2}{4}$$

$$\Rightarrow h^2=\frac{4r^2}{3}$$

$$\Rightarrow h=\frac{2r}{\sqrt{3}}$$

Again, $V''(h)=\pi\left(-\frac{3(2h)}{4}\right)=\frac{-3\pi h}{2}$

$$V''\left(\frac{2r}{\sqrt{3}}\right)=\frac{-3\pi}{2}\frac{2r}{\sqrt{3}}=-\frac{3\pi r}{\sqrt{3}}<0$$

Thus, the volume is maximum when $h=\frac{2r}{\sqrt{3}}$

31. *(a)* Circumference of a circle, $C = 2\pi r$

Differentiating both sides w.r.t t, we have

$$\frac{dC}{dt} = 2\pi \cdot \frac{dr}{dt} = 2\pi \cdot (0.7) = 1.4\pi = 1.4 \times \frac{22}{7} = 4.4 \text{ cm/sec}$$

32. *(c)* $\lim_{x \to a} \frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}}$

$$= \lim_{x \to a} \frac{\left(\sqrt{a+2x}\right)^2 - \left(\sqrt{3x}\right)^2}{\sqrt{a+2x} + \sqrt{3x}} \times \frac{\sqrt{3a+x} + 2\sqrt{x}}{\left(\sqrt{3a+x}\right)^2 - \left(2\sqrt{x}\right)^2}$$

$$= \lim_{x \to a} \frac{a+2x-3x}{\sqrt{a+2x} + \sqrt{3x}} \times \frac{\sqrt{3a+x} + 2\sqrt{x}}{3a+x-4x}$$

$$= \lim_{x \to a} \frac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{a+2x} + \sqrt{3x}} \times \frac{a-x}{3(a-x)}$$

$$= \frac{1}{3} \lim_{x \to a} \frac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{a+2x} + \sqrt{3x}}$$

$$= \frac{1}{3} \frac{\sqrt{4a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$$

$$= \frac{1}{3} \frac{4\sqrt{a}}{2\sqrt{3}\sqrt{a}}$$

$$= \frac{2}{3\sqrt{3}}$$

33. *(c)* Given equations are $x^2 - px + q = 0$ and $x^2 - ax + b = 0$.

Root of second equation is

$$x = \frac{a \pm \sqrt{a^2 - 4b}}{2}$$

Since roots of second equation are equal, its discriminant is equal to 0.

$$\sqrt{a^2 - 4b} = 0 \Rightarrow a^2 = 4b \quad ...(\text{i})$$

This further gives $x = \frac{a}{2}$.

As both equations have a common root, $x = \frac{a}{2}$ is the root of the first equation.

Thus, $x = \frac{a}{2}$ satisfies the first equation.

$$\left(\frac{a}{2}\right)^2 - p\left(\frac{a}{2}\right) + q = 0$$

$$\Rightarrow \quad \frac{a^2}{4} = \frac{pa}{2} - q$$

$$\Rightarrow \quad 2b = pa - 2q \quad \left(\text{Using } a^2 = 4b \text{ from (i)}\right)$$

$$\Rightarrow \quad ap = 2(b+q)$$

34. *(c)* Let p_1 be the length of perpendicular from the point $(4, 0)$ to the line $3x\cos\phi + 5\sin\phi = 15$.

$$p_1 = \left| \frac{3(4)\cos\phi + 5(0)\sin\phi - 15}{\sqrt{(3\cos\phi)^2 + (5\sin\phi)^2}} \right|$$

$$= \frac{15 - 12\cos\phi}{\sqrt{(3\cos\phi)^2 + (5\sin\phi)^2}}$$

Let p_2 be the length of perpendicular from the point $(-4, 0)$ to the line $3x\cos\phi + 5\sin\phi = 15$.

$$p_2 = \left| \frac{3(-4)\cos\phi + 5(0)\sin\phi - 15}{\sqrt{(3\cos\phi)^2 + (5\sin\phi)^2}} \right|$$

$$= \left| \frac{-12\cos\phi - 15}{\sqrt{(3\cos\phi)^2 + (5\sin\phi)^2}} \right|$$

$$= \frac{12\cos\phi + 15}{\sqrt{(3\cos\phi)^2 + (5\sin\phi)^2}}$$

Thus, we get

$p_1 \cdot p_2$

$= \dfrac{15-12\cos\phi}{\sqrt{(3\cos\phi)^2+(5\sin\phi)^2}} \cdot \dfrac{15+12\cos\phi}{\sqrt{(3\cos\phi)^2+(5\sin\phi)^2}}$

$= \dfrac{225-144\cos^2\phi}{(3\cos\phi)^2+(5\sin\phi)^2}$

$= \dfrac{225-144\cos^2\phi}{9\cos^2\phi+25\sin^2\phi}$

$= \dfrac{225-144\cos^2\phi}{25-16\cos^2\phi}$

$= \dfrac{9(25-16\cos^2\phi)}{25-16\cos^2\phi}$

$= 9$

35. *(b)* Let x be the number of students who like both music and dance.

Since 5 students likes neither music nor dance, the total number of remaining students is 60 – 5 = 55.

Number of students who like only music = 45 – x

Number of students who like only dance = 50 – x

The sum total of above with the students liking both should be 55.

$(45-x)+x+(50-x)=55$

$\Rightarrow \quad 95-x=55$

$\Rightarrow \quad x=95-55$

$\Rightarrow \quad x=40$

36. *(a)* $\log_2 x, \log_3 x, \log_x 16$ are in G.P.

$\therefore \dfrac{\log_3 x}{\log_2 x} = \dfrac{\log_x 16}{\log_3 x}$

$\Rightarrow (\log_3 x)^2 = \log_2 x \cdot \log_x 16$

$\Rightarrow 2\times\log_3 x = \log_2 x \cdot \log_x 2^4$

$\Rightarrow 2\times\log_3 x = 4\cdot\log_2 x\cdot\log_x 2$

$\Rightarrow \quad \log_3 x = 2(\log_2 x\cdot\log_x 2)$

$\Rightarrow \quad \log_3 x = 2\dfrac{\log_2 x}{\log_2 x} \left(\because \log_a b = \dfrac{1}{\log_b a}\right)$

$\Rightarrow \quad \log_3 x = 2$

$\Rightarrow \quad x = 3^2 = 9$

37. *(a)* $\int \sec x\,dx = \int \dfrac{\sec x(\sec x+\tan x)}{\sec x+\tan x}dx$.

Let $u=\sec x+\tan x$

$\Rightarrow \dfrac{du}{dx} = \sec x\tan x+\sec^2 x$

$\Rightarrow du = \sec x(\sec x+\tan x)dx$

$\int \sec x\,dx = \int \dfrac{du}{u}$

$= \log(u)+C$

$= \log(\sec x+\tan x)+C$

38. *(b)* Given binomial expansion $\left(x^2+\dfrac{1}{x}\right)^{15}$

$T_{r+1} = {}^{15}C_r\left(x^2\right)^{15-r}\left(\dfrac{1}{x}\right)^r$

$= {}^{15}C_r x^{30-2r-r}$

$= {}^{15}C_r x^{30-3r}$

For independent term,

$30-3r=0 \Rightarrow r=10$

Put $r=10$, we get

$T_{10+1} = {}^{15}C_{10} = \dfrac{15!}{10!5!}$

$= \dfrac{15\times14\times13\times12\times11\times10!}{10!\times5\times4\times3\times2\times1}$

$= 3003$

39. *(c)* Given expression is $\dfrac{\cos 10° + \sin 20°}{\cos 20° - \sin 10°}$

$$= \frac{\cos(90° - 80°) + \sin 20°}{\cos(90° - 70°) - \sin 10°}$$

$$= \frac{\sin 80° + \sin 20°}{\sin 70° - \sin 10°}$$

$$= \frac{2\sin\frac{80°+20°}{2}\cos\frac{80°-20°}{2}}{2\cos\frac{70°+10°}{2}\sin\frac{70°-10°}{2}}$$

$$= \frac{2\sin 50° \cos 30°}{2\cos 40° \sin 30°}$$

$$= \frac{\sin(90° - 40°)\cot 30°}{\cos 40°}$$

$$= \frac{\cos 40° \cot 30°}{\cos 40°}$$

$$= \cot 30°$$

$$= \sqrt{3}$$

40. *(c)* Given equation is $\sin\theta = x + \dfrac{a}{x},\ x \in R - \{0\}$

$\Rightarrow \quad x^2 + a = x\sin\theta$

$\Rightarrow x^2 - x\sin\theta + a = 0$

Discriminant $= \sqrt{\sin^2\theta - 4a}$

For x to be real, discriminant ≥ 0

$\Rightarrow \sqrt{\sin^2\theta - 4a} \geq 0$

$\Rightarrow \sin^2\theta - 4a \geq 0$

$\Rightarrow \sin^2\theta \geq 4a$

$\Rightarrow \dfrac{1}{\sin^2\theta} \leq \dfrac{1}{4a}$

$\Rightarrow a \leq \dfrac{\sin^2\theta}{4}$

$\Rightarrow a \leq \dfrac{1}{4} \quad \left(\because \sin^2\theta \in (0,1)\right)$

41. *(b)* Given sequence $20,\ 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \ldots$

This can be rewritten as,

$20,\ \dfrac{77}{4}, \dfrac{37}{2}, \dfrac{71}{4}, \ldots$

This is an A.P. series.

Here, first term = 20, and common difference = $-\dfrac{3}{4}$

nth term $= a + (n-1)d = 20 + (n-1)\left(-\dfrac{3}{4}\right)$

$= \dfrac{83}{4} - \dfrac{3}{4}n$

For first negative term, nth term < 0

$\Rightarrow \dfrac{83}{4} - \dfrac{3}{4}n < 0$

$\Rightarrow \quad 83 < 3n$

$\Rightarrow \quad n > \dfrac{83}{3} = 27.66$

So, n should be 28. Hence, 28th term is first negative term.

42. *(a)* $x^2 + 6x - 7 < 0 \Rightarrow (x+7)(x-1) < 0.$

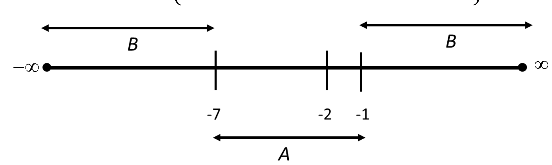
$\Rightarrow x = (-7, 1)$

Let $A = \{-6, -5, -4, -3, -2, -1, 0\}$

$x^2 + 9x + 14 > 0 \Rightarrow (x+7)(x+2) < 0$

$\Rightarrow x \in (-\infty, -7) \cup (-2, \infty)$

Let $B = R - \{-7, -6, -5, -4, -3, -2\}$



So, $A \cap B = (-2, 1)$

43. *(b)* Let $I = \displaystyle\int \frac{dx}{\sin^2 x \cos^2 x}$ which can be rewritten as

$$I = \int \frac{\left(\sin^2 x + \cos^2 x\right)dx}{\sin^2 x \cos^2 x}$$

$$= \int \left(\frac{\sin^2 x}{\sin^2 x \cos^2 x} + \frac{\cos^2 x}{\sin^2 x \cos^2 x} \right) dx$$

$$= \int \left(\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} \right) dx$$

$$= \int \left(\sec^2 x + \operatorname{cosec}^2 x \right) dx$$

$$= \tan x - \cot x + C$$

44. *(a)* Given $f(x) = a + bx + cx^2$

$$\int_0^1 f(x)\,dx = \int_0^1 \left(a + bx + cx^2\right) dx$$

$$= \left[ax + \frac{bx^2}{2} + \frac{cx^3}{3} \right]_0^1$$

$$= a + \frac{b}{2} + \frac{c}{3} \quad \ldots(\text{i})$$

Here, $f(0) = a,\ f\left(\frac{1}{2}\right) = a + \frac{b}{2} + \frac{c}{4}$

and $f(1) = a + b + c$

Now, $\left[f(0) + 4f(1/2) + f(1)\right]/6$

$$= \frac{a + 4\left(a + \frac{b}{2} + \frac{c}{4}\right) + a + b + c}{6}$$

$$= \frac{a + 4\left(\frac{4a + 2b + c}{4}\right) + a + b + c}{6}$$

$$= \frac{a + 4a + 2b + c + a + b + c}{6}$$

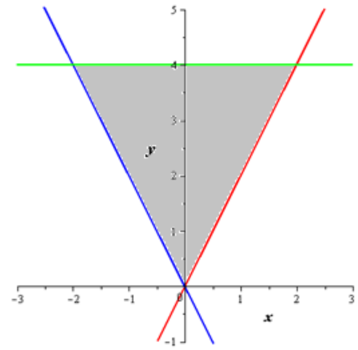
$$= \frac{6a + 3b + 2c}{6}$$

$$= a + \frac{b}{2} + \frac{c}{3} \quad \ldots(\text{ii})$$

From equations (i) and (ii),

$$\int_0^1 f(x)\,dx = \left[f(0) + 4f(1/2) + f(1)\right] \Big/ 6$$

45. *(c)* The shaded region is shown below:



$$\text{Required area } = 2\int_0^4 \frac{y}{2}\,dy$$

$$= \left.\frac{y^2}{2}\right|_0^4$$

$$= \frac{16}{2} - 0$$

$$= 8$$

46. *(b)* The given equation is

$$k\frac{d^2y}{dx^2} = \left[1 + \left(\frac{dy}{dx}\right)^3\right]^{3/2}$$

Squaring both sides,

$$k^2\left(\frac{d^2y}{dx^2}\right)^2 = \left[1 + \left(\frac{dy}{dx}\right)^3\right]^3$$

Degree of a differential equation is the highest power of the highest order derivative when the derivatives are expressed as polynomial.
Here, degree of the differential equation is 2.

47. *(d)* The given expression is

$$\left(1 + \cos 67\frac{1^\circ}{2}\right)\left(1 + \cos 112\frac{1^\circ}{2}\right)$$

This can be written as,

$$\left(1+\cos 67\frac{1^\circ}{2}\right)\left(1+\cos\left(180^\circ-67\frac{1^\circ}{2}\right)\right)$$
$$=\left(1+\cos 67\frac{1^\circ}{2}\right)\left(1-\cos 67\frac{1^\circ}{2}\right)$$
$$=1-\cos^2 67\frac{1^\circ}{2}$$
$$=\sin^2 67\frac{1^\circ}{2}$$
$$=\frac{1-\cos 135^\circ}{2}\quad\left(\because \sin^2 A=\frac{1-\cos 2A}{2}\right)$$
$$=\frac{\sqrt{2}+1}{2\sqrt{2}}$$

48. *(d)* Let a and d be the first term and common difference of an AP respectively.

pth term $=a+(p-1)d$

and qth term $=a+(q-1)d$

According to the question,
$$p\left[a+(p-1)d\right]=q\left[a+(q-1)d\right]$$
$$\Rightarrow pa+\left(p^2-p\right)d=qa+\left(q^2-q\right)d$$
$$\Rightarrow (p-q)a=\left(q^2-p^2+p-q\right)d$$
$$\Rightarrow (p-q)a=(p-q)(-p-q+1)d$$
$$\Rightarrow a=-(p+q-1)d$$

Now, $(p+q)$th term is
$$a+(p+q-1)d=-(p+q-1)d+(p+q-1)d$$
$$=0$$

49. *(d)* Intersecting lines are: $x+2y=5$ and $3x+7y=17$

On solving these equations, we get $x=1$ and $y=2$.

Equation of perpendicular line is,
$$3x+4y=10 \text{ or } y=-\frac{3}{4}x+\frac{10}{4}$$

So, slope is $-\frac{3}{4}$.

This further gives the slope of the required line as $\frac{4}{3}$.

So, equation of the required line is,
$$y-2=\frac{4}{3}(x-1) \text{ or } 4x-3y+2=0$$

50. *(b)* $B=\int_0^\pi \frac{\sin x\,dx}{\sin x-\cos x}$
$$=\int_0^\pi \frac{\sin x}{\sin x-\cos x}\times\frac{\sin x+\cos x}{\sin x+\cos x}dx$$
$$=\int_0^\pi \frac{\sin^2 x+\sin x\cos x}{\sin^2 x-\cos^2 x}dx$$
$$=\int_0^\pi \frac{\sin^2 x+\sin x\cos x}{\cos 2x}dx$$
$$=-\frac{1}{2}\int_0^\pi \frac{2\sin^2 x}{\cos 2x}dx-\frac{1}{2}\int_0^\pi \frac{2\sin x\cos x}{\cos 2x}dx$$
$$=-\frac{1}{2}\int_0^\pi \frac{1-\cos 2x}{\cos 2x}dx-\frac{1}{2}\int_0^\pi \frac{\sin 2x}{\cos 2x}dx$$
$$=-\frac{1}{2}\int_0^\pi \sec 2x\,dx+\frac{1}{2}\int_0^\pi dx-\frac{1}{2}\int_0^\pi \tan 2x\,dx$$

Integrating, we get
$$B=-\frac{1}{2}\left[\frac{\log|\sec 2x+\tan 2x|}{2}\right]_0^\pi+\frac{1}{2}[x]_0^\pi$$
$$-\frac{1}{2}\left[\frac{\log|\sec 2x|}{2}\right]_0^\pi$$
$$=-\frac{1}{4}\left[\log(1+0)-\log(1+0)\right]+\frac{1}{2}(\pi-0)$$
$$-\frac{1}{2}\left[\log 1-\log 1\right]$$
$$=0+\frac{\pi}{2}-0=\frac{\pi}{2}$$

51. *(b)* Let a, ar, and ar^2 be three positive terms of G.P.

According to the question,

$$a=\frac{1}{3}\left(ar+ar^2\right)$$
$$\Rightarrow 3=r+r^2$$
$$\Rightarrow r^2+r-3=0$$
$$\Rightarrow \quad r=\frac{-1\pm\sqrt{1+4\times 3}}{2}$$
$$\Rightarrow \quad r=\frac{-1\pm\sqrt{13}}{2}$$
$$\Rightarrow \quad r=\frac{\sqrt{13}-1}{2},-\left(\frac{1+\sqrt{13}}{2}\right)$$

Since r cannot be negative, we get $r=\dfrac{\sqrt{13}-1}{2}$.

52. *(d)* The universal set is given by,

$$U=\{HHH,HHT,HTH,HTT,THH,THT,TTH,TTT\}$$

The set A, B, and C are given by,

$$A=\{TTT\}$$
$$B=\{HTT,THH,THT,TTH\}$$
$$C=\{HHH,\ HHT,\ HTH,THH\}$$

This implies,

$$B'=\{HHH,HHT,HTH,THH,TTT\}$$
$$C'=\{HTT,THT,TTH,TTT\}$$

This gives,

$$B'\cup C'=\left\{\begin{matrix}HHH,HHT,HTH,HTT,THH,\\ THT,TTH,TTT\end{matrix}\right\}=U$$
$$B'\cap C'=\{TTT\}$$

Now, $A\cup(B'\cup C')=\{TTT\}=B'\cap C'$

53. *(c)* Since α, β are roots of $x^2+px-q=0$, then

$$\alpha+\beta=-p \quad ...(\text{i})$$

and $\quad \alpha\beta=-q \quad ...(\text{ii})$

Put the value of α from (ii) in (i).

$$\frac{-q}{\beta}+\beta=-p\Rightarrow -q+\beta^2=-p\beta$$
$$\Rightarrow \beta^2=q-p\beta \quad ...(\text{iii})$$

Since γ, δ are roots of $x^2-px+r=0$, then

$$\gamma+\delta=p \quad \text{and} \quad \gamma\delta=r$$

Now,

$$(\beta+\gamma)(\beta+\delta)=\beta^2+\beta\delta+\beta\gamma+\gamma\delta$$
$$=\beta^2+\beta(\delta+\gamma)+\gamma\delta$$
$$=q-p\beta+p\beta+r$$
$$=q-r$$

54. *(c)* Given equation is,

$$xy=ae^x+be^{-x} \quad ...(\text{i})$$

Differentiate both sides w.r.t x

$$x\frac{dy}{dx}+y=ae^x-be^{-x}$$

Again, differentiating both sides w.r.t x

$$x\frac{d^2y}{dx^2}+\frac{dy}{dx}+\frac{dy}{dx}=ae^x+be^{-x}$$
$$\Rightarrow x\frac{d^2y}{dx^2}+2\frac{dy}{dx}=xy \quad (\text{from (i)})$$

This is the required differential equation of second order and first degree.

55. *(d)* Given $xRy\Rightarrow x^2-4xy+3y^2=0$.

$$\Rightarrow \quad x^2-xy-3xy+3y^2=0$$
$$\Rightarrow x(x-y)-3y(x-y)=0$$
$$\Rightarrow \quad (x-y)(x-3y)=0$$

Reflexive property:

$$xRx\Rightarrow(x-x)(x-3x)=0$$

So, R is reflexive.

Symmetric property: Let us check an example $(1,2)$ and $(2,1)$.

For $(1,2)\Rightarrow(1-2)(1-6)=(-1)(-5)=10$

For $(2,1)\Rightarrow(2-1)(2-3)=(-1)(-1)=-1$

So, R is not symmetric.

Transitive property:

For $(9x,3x) \Rightarrow (9x-3x)(9x-9x)=0$

For $(3x,x) \Rightarrow (3x-x)(3x-3x)=0$

For $(9x,x) \Rightarrow (9x-x)(9x-3x) \neq 0$

So, R is not transitive.

Thus, R is reflexive, but not symmetric and transitive.

56. *(a)* Let $\sin\theta = \frac{5}{13}$ and $\sin\phi = \frac{99}{101}$

$\therefore \cos(\pi-(\theta+\phi)) = -\cos(\theta+\phi)$

$= -\{\cos\theta\cos\phi - \sin\theta\sin\phi\}$

Substitute the values to get,

$\therefore \cos(\pi-(\theta+\phi))$

$= -\left\{\sqrt{1-\frac{25}{169}}\sqrt{1-\left(\frac{99}{101}\right)^2} - \frac{5}{13}\times\frac{99}{101}\right\}$

Simply further to get,

$= -\left\{\frac{12}{13}\times\frac{20}{101} - \frac{5}{13}\times\frac{99}{101}\right\}$

$= -\left\{\frac{240}{1313} - \frac{495}{1313}\right\}$

$= \frac{255}{1313}$

57. *(b)* We have $A = \tan^{-1}2$ and $B = \tan^{-1}3$

$\Rightarrow \tan A = 2$ and $\tan B = 3$

Since A, B, C are angles of a triangle,

$A+B+C=\pi$

$\Rightarrow C = \pi - (A+B) \quad ...(i)$

Now, $A+B = \tan^{-1}2 + \tan^{-1}3$

$= \tan^{-1}\left(\frac{2+3}{1-2\cdot 3}\right)$

$= \tan^{-1}\left(\frac{5}{-5}\right)$

$= \tan^{-1}(-1) = \tan^{-1}\left(\tan\frac{3\pi}{4}\right) = \frac{3\pi}{4}$

So, from (i),

$C = \pi - \frac{3\pi}{4} = \frac{\pi}{4}$

58. *(b)* Consider $\cos 10° + \cos 110° + \cos 130°$

$= \cos 130° + \cos 10° + \cos 110°$

$= 2\cos\left(\frac{130°+10°}{2}\right)\cos\left(\frac{130°-10°}{2}\right) + \cos 110°$

$= 2\cos 70°\cos 60° + \cos 110°$

$= \cos 70° + \cos 110° \quad \left[\because \cos 60° = \frac{1}{2}\right]$

$= \cos(180°-110°) + \cos 110°$

$= -\cos 110° + \cos 110°$

$= 0$

59. *(c)* Given equation is,

$y^2 = 4a(x-a) \quad ...(i)$

Differentiate both sides w.r.t. x

$2yy' = 4a \Rightarrow a = \frac{yy'}{2}$

Substitute this value of a in equation (i)

$y^2 = 4\left(\frac{yy'}{2}\right)\left(x - \frac{yy'}{2}\right)$

$\Rightarrow y^2 = yy'(2x - yy')$

$\Rightarrow y'(yy'-2x) + y^2 = 0$

60. *(b)* Given that $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2+6 \\ 0 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 8 \\ 0 & 9 \end{bmatrix}$

Since $f(x) = x^2 - x + 2$, putting A in place of x,

$f(A) = A^2 - A + 2I$

$= \begin{bmatrix} 1 & 8 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1-1+2 & 8-2+0 \\ 0+0+0 & 9-3+2 \end{bmatrix}$

$= \begin{bmatrix} 2 & 6 \\ 0 & 8 \end{bmatrix}$

61. *(c)* Given matrix equation is,

$$\begin{vmatrix} x^2 & -2x & -2\omega^2 \\ 2 & \omega & -\omega \\ 0 & \omega & 1 \end{vmatrix} = 0$$

By $C_2 \to C_2 + C_3$, we get

$$\begin{vmatrix} x^2 & -2x-2\omega^2 & -2\omega^2 \\ 2 & 0 & -\omega \\ 0 & 1+\omega & 1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x^2 & -2x-2\omega^2 & -2\omega^2 \\ 2 & 0 & -\omega \\ 0 & -\omega^2 & 1 \end{vmatrix} = 0$$

$[\because 1+\omega = \omega^2]$

Expanding along row 3,

$$\omega^2 \begin{vmatrix} x^2 & -2\omega^2 \\ 2 & -\omega \end{vmatrix} + 1 \begin{vmatrix} x^2 & -2x-2\omega^2 \\ 2 & 0 \end{vmatrix} = 0$$

$\Rightarrow \omega^2(-\omega x^2 + 4\omega^2) - (-4x - 4\omega^2) = 0$

$\Rightarrow \omega^3(-x^2 + 4\omega) - (-4x - 4\omega^2) = 0$

$\Rightarrow -x^2 + 4\omega + 4x + 4\omega^2 = 0 \quad (\because \omega^3 = 1)$

$\Rightarrow -x^2 + 4\omega + 4x - 4 - 4\omega = 0 \quad (\because -\omega^2 = 1 + \omega)$

$\Rightarrow x^2 - 4x + 4 = 0$

$\Rightarrow (x-2)^2 = 0$

$\Rightarrow x = 2$

62. *(a)* Since *a, b, c* are in GP, we get

$b^2 = ac$

Expanding the given determinant, we get

$$\begin{vmatrix} a & b & a+b \\ b & c & b+c \\ a+b & b+c & 0 \end{vmatrix}$$

$= a\left[0 - (b+c)^2\right] - b\left[0 - (a+b)(b+c)\right]$

$+ (a+b)\left[b(b+c) - c(a+b)\right]$

$= -a(b^2 + c^2 + 2bc) + b(a+b)(b+c)$

$+ (a+b)(b^2 + bc - ac - bc)$

$= -a(b^2 + c^2 + 2bc) + b(ab + ac + b^2 + bc)$

$[\because b^2 = ac]$

$= -a(b^2 + c^2 + 2bc) + b(ab + 2ac + bc)$

$= -ab^2 - ac^2 - 2abc + ab^2 + 2abc + b^2c$

$= -ac^2 + b^2c$

$= -ac^2 + ac \cdot c = -ac^2 + ac^2 = 0$

63. *(d)* Given $P(A) = 0.6$, $P(B) = 0.5$ and $P(A \cap B) = 0.4$

$P(\overline{A}) = 1 - P(A) = 1 - 0.6 = 0.4$

$P(\overline{B}) = 1 - P(B) = 1 - 0.5 = 0.5$

Now,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.6 + 0.5 - 0.4$

$= 0.7$

$P(\overline{A} \cup B) = P(\overline{A}) + P(B) - P(\overline{A} \cap B)$

$= P(\overline{A}) + P(B) - \{P(B) - P(A \cap B)\}$

$= 0.4 + 0.5 - (0.5 - 0.4)$

$= 0.8$

Hence, statement (1) is incorrect.
Also,

$$P\left(\overline{B}/\overline{A}\right)=\frac{P\left(\overline{B}\cap\overline{A}\right)}{P\left(\overline{A}\right)}$$

$$=\frac{P\left(B\cup A\right)'}{P\left(\overline{A}\right)}$$

$$=\frac{1-P\left(A\cup B\right)}{P\left(\overline{A}\right)}$$

$$=\frac{1-0.7}{0.4}$$

$$=0.75$$

Hence, statement (2) is incorrect.

64. *(c)* A = Event of showing 5 on at least one dice

$$=\left\{\begin{matrix}(1,5),(2,5),(3,5),(4,5),(5,5),(6,5)\\(5,1),(5,2),(5,3),(5,4),(5,6)\end{matrix}\right\}$$

$\therefore\ n(A)=11$

and $n(S)=6\times 6=36$

B = Event of showing sum 10 or more when at least one dice shows 5

$$=\{(5,5),(5,6),(6,5)\}$$

$\therefore\ n(B)=3$

Now $n(A\cap B)=3$

Thus, $P\left(\frac{B}{A}\right)=\frac{P(A\cap B)}{P(A)}$

$$=\frac{\frac{n(A\cap B)}{n(S)}}{\frac{n(A)}{n(S)}}$$

$$=\frac{\frac{3}{36}}{\frac{11}{36}}=\frac{3}{11}$$

65. *(b)* Given $\begin{vmatrix} y & x & y+z \\ z & y & x+y \\ x & z & z+x \end{vmatrix}=0$

Applying $R_1\to R_1+R_2+R_3$

$$\begin{vmatrix} x+y+z & x+y+z & 2(x+y+z) \\ z & y & x+y \\ x & z & z+x \end{vmatrix}=0$$

$$(x+y+z)\begin{vmatrix} 1 & 1 & 2 \\ z & y & x+y \\ x & z & z+x \end{vmatrix}=0$$

Applying $C_2\to C_1-C_2,\ C_3\to C_3-2C_1$

$$(x+y+z)\begin{vmatrix} 1 & 0 & 0 \\ z & z-y & x+y-2z \\ x & x-z & z-x \end{vmatrix}=0$$

$$(x+y+z)\left[(z-y)(z-x)-(x-z)(x+y-2z)\right]=0$$

$$(x+y+z)(z-x)(z-y+x+y-2z)=0$$

$$(x+y+z)(z-x)(x-z)=0$$

This gives, $x+y=-z$ or $x=z$.

66. *(c)* Let the angles A, B, and C of a triangle are $2x$, $5x$, and $5x$ respectively.

So,

$$2x+5x+5x=180^\circ\Rightarrow 12x=180^\circ$$

$$\Rightarrow\quad x=15^\circ$$

Angles are 30°, 75°, 75°

$\angle B=75^\circ$, $\angle C=75^\circ$

$\therefore\ \tan B\tan C=\left(\tan 75°\right)^2$

$=\left(\tan\left(45°+30°\right)\right)^2$

$=\left(\dfrac{\tan 45°+\tan 30°}{1-\tan 45°\tan 30°}\right)^2$

$=\left(\dfrac{1+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}}\right)^2$

$=\left(\dfrac{\sqrt{3}+1}{\sqrt{3}-1}\right)^2$

$=\left(\dfrac{\left(\sqrt{3}+1\right)^2}{3-1}\right)^2$

$=\dfrac{1}{4}\left(4+2\sqrt{3}\right)^2$

$=\dfrac{1}{4}\left(28+16\sqrt{3}\right)$

$=7+4\sqrt{3}$

67. *(b)* Let $\cos^{-1}\dfrac{4}{5}=A\Rightarrow\cos A=\dfrac{4}{5}\Rightarrow\sin A=\dfrac{3}{5}$

Let $\cos^{-1}\dfrac{12}{13}=B\Rightarrow\cos B=\dfrac{12}{13}\Rightarrow\sin B=\dfrac{5}{13}$

Now, $\cos\left\{\cos^{-1}\dfrac{4}{5}+\cos^{-1}\dfrac{12}{13}\right\}$

$=\cos\left(A+B\right)$

$=\cos A\cos B-\sin A\sin B$

$=\dfrac{4}{5}\cdot\dfrac{12}{13}-\dfrac{3}{5}\cdot\dfrac{5}{13}$

$=\dfrac{48-15}{65}$

$=\dfrac{33}{65}$

68. *(d)* Given $\left|\vec{a}\right|=7$, $\left|\vec{b}\right|=11$, and $\left|\vec{a}+\vec{b}\right|=10\sqrt{3}$

Now, $\left|\vec{a}+\vec{b}\right|^2=\left|\vec{a}\right|^2+\left|\vec{b}\right|^2+2\left|\vec{a}\right|\left|\vec{b}\right|\cos\theta$

$\therefore\ \left(10\sqrt{3}\right)^2=49+121+2\times7\times11\cos\theta$

$300=170+154\cos\theta$

$\dfrac{65}{77}=\cos\theta$

Now, $\left|\vec{a}-\vec{b}\right|^2=\left|\vec{a}\right|^2+\left|\vec{b}\right|^2-2\left|\vec{a}\right|\left|\vec{b}\right|\cos\theta$

$\left|\vec{a}-\vec{b}\right|^2=49+121-2\times7\times11\times\dfrac{65}{77}$

$=170-130$

$=40$

$\Rightarrow\left|\vec{a}-\vec{b}\right|=\sqrt{40}=2\sqrt{10}$

69. *(a)* Given $f\left(x\right)=\dfrac{x}{1+|x|}$

$$f\left(x\right)=\begin{cases}\dfrac{x}{1-x},\ x<0\\[2mm]\dfrac{x}{1+x},\ x\ge 0\end{cases}$$

$\therefore$ LHD $=f'\left(0^-\right)=\lim\limits_{h\to0}\dfrac{f\left(0-h\right)-f\left(0\right)}{-h}$

$=\lim\limits_{h\to0}\dfrac{f\left(-h\right)-f\left(0\right)}{-h}$

$=\lim\limits_{h\to0}\dfrac{\frac{-h}{1+h}-0}{-h}=\lim\limits_{h\to0}\dfrac{1}{1+h}=1$

$\therefore$ RHD $=f'\left(0^+\right)=\lim\limits_{h\to0}\dfrac{f\left(0+h\right)-f\left(0\right)}{h}$

$=\lim\limits_{h\to0}\dfrac{f\left(h\right)-f\left(0\right)}{h}$

$=\lim\limits_{h\to0}\dfrac{\frac{h}{1+h}-0}{h}=\lim\limits_{h\to0}\dfrac{1}{1+h}=1$

As, LHD = RHD, the given function is differentiable at $x=0$.

Hence, $f\left(x\right)$ is differentiable in $\left(-\infty,\infty\right)$.

70. *(b)* Given equation is $y^2=8x$.

Take an arbitrary point on this curve. If we take *y* as *p*, then point is $\left(\frac{p^2}{8}, p\right)$.

The distance between $\left(\frac{p^2}{8}, p\right)$ and $(4,2)$ is,

$$d^2 = \left(\frac{p^2}{8} - 4\right)^2 + (p-2)^2 \quad ...(i)$$

$$= \frac{1}{64}\left(p^2 - 32\right)^2 + (p-2)^2$$

Differentiate both sides w.r.t. to *p*.

$$2d\frac{dd}{dp} = \frac{1}{64} \times 2\left(p^2-32\right)(2p) + 2(p-2)$$

$$= \frac{1}{16}\left(p^2-32\right)p + 2(p-2)$$

$$= \frac{1}{16}\left(p^3 - 32p + 32p - 64\right)$$

$$= \frac{1}{16}\left(p^3 - 64\right)$$

For minimum value, $\frac{dd}{dp} = 0$

$$\frac{1}{16}\left(p^3-64\right) = 0 \Rightarrow p^3 = 64$$

$$\Rightarrow \quad p = 4$$

It can be shown that the second derivative of *d* w.r.t to *p* at $p = 4$ is positive.

From (i),

$$d^2 = \frac{1}{64}(16-32)^2 + (4-2)^2 \Rightarrow d^2 = \frac{1}{64} \times 256 + 4$$

$$\Rightarrow d^2 = 4 + 4$$

$$\Rightarrow d^2 = 8$$

$$\Rightarrow d = 2\sqrt{2}$$

71. *(c)* Given sequence is 1, 5, 9, 13, 17, …

This is an A.P.

Here $a = 1$, $d = 4$

The *n*th term is given by,

$$a_n = a + (n-1)d$$

$$= 1 + (n-1)4$$

$$= 1 + 4n - 4$$

$$= 4n - 3$$

72. *(b)* Number of students who like music, $n(M) = 680$

Number of students who like dance, $n(D) = 215$

Total number of students, $n(M \cup D) = 850$

$$\Rightarrow n(M) + n(D) - n(M \cap D) = 850$$

$$\Rightarrow \quad 680 + 215 - n(M \cap D) = 850$$

$$\Rightarrow \quad n(M \cap D) = 895 - 850$$

$$\Rightarrow \quad n(M \cap D) = 45$$

73. *(c)* Let $\log_7 \log_7 \sqrt{7\sqrt{7\sqrt{7}}} = x$

$$7^x = \log_7 \sqrt{7\sqrt{7\sqrt{7}}}$$

$$= \frac{1}{2}\log_7 7\sqrt{7\sqrt{7}}$$

$$= \frac{1}{2}\left[\log_7 7 + \frac{1}{2}\log_7 7\sqrt{7}\right]$$

$$= \frac{1}{2}\left[1 + \frac{1}{2}\log_7 7 + \frac{1}{2}\log_7 \sqrt{7}\right]$$

$$= \frac{1}{2}\left[1 + \frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2}\log_7 7\right]$$

$$= \frac{1}{2}\left[1 + \frac{1}{2} + \frac{1}{4}\right] = \frac{7}{8}$$

$$\Rightarrow x = \log_7\left(\frac{7}{8}\right) = \log_7 7 - \log_7 8$$

$$\Rightarrow x = 1 - \log_7 2^3$$

$$\Rightarrow x = 1 - 3\log_7 2$$

74. *(b)* Given equation $x^2 - 4x + [x] = 0$ in $[0,2]$.

Case 1: Let $0 \le x < 1$

$[x] = 0$

$\therefore x^2 - 4x + 0 = 0 \Rightarrow x(x-4) = 0$

$\Rightarrow \quad x = 0, 4$

As x can't take value 4 in $0 \le x < 1$, so $x = 0$.

Case 2: Let $1 \le x < 2$

$[x] = 1$

$\therefore x^2 - 4x + 1 = 0$

Roots are $x = \dfrac{4 \pm \sqrt{16-4}}{2} = \dfrac{4 \pm \sqrt{12}}{2} = 2 \pm \sqrt{3}$

In interval, $1 \le x < 2$, $2 \pm \sqrt{3}$ are not the roots.

Case 3: Let $x = 2$

$[x] = 2$

$\therefore x^2 - 4x + 2 = 0$

Roots are $x = \dfrac{4 \pm \sqrt{16-8}}{2} = \dfrac{4 \pm \sqrt{8}}{2} = 2 \pm \sqrt{2}$

Since $x = 2$, roots can't be $2 \pm \sqrt{2}$.

So, there is only one solution, $x = 0$.

75. *(b)* Consider $\dfrac{1 - 3\tan^2 A}{3\tan A - \tan^3 A}$.

This is the reciprocal of the tangent triple angle formula.

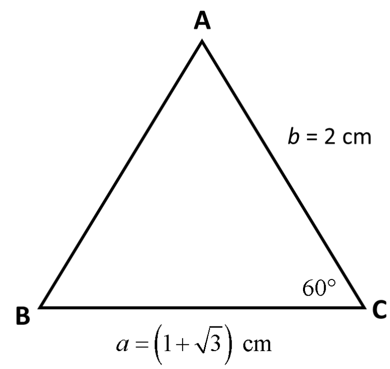
So, $\dfrac{1 - 3\tan^2 A}{3\tan A - \tan^3 A} = \dfrac{1}{\tan 3A}$

$= \dfrac{1}{\tan \frac{41\pi}{4}} \quad \left[\because A = \dfrac{41\pi}{12}\right]$

$= \dfrac{1}{\tan\left(10\pi + \frac{\pi}{4}\right)}$

$= \dfrac{1}{\tan \frac{\pi}{4}} = 1$

76. *(a)*



Now as $a > b$, we have $\angle A > \angle B$.

Now from Sine Rule, $\dfrac{\sin A}{a} = \dfrac{\sin B}{b}$

$\Rightarrow \dfrac{\sin A}{1+\sqrt{3}} = \dfrac{\sin B}{2}$

Consider option (a) 45° and 75°

This gives, $\Rightarrow \dfrac{\sin 75°}{1+\sqrt{3}} = \dfrac{\sin 45°}{2}$

$\Rightarrow \dfrac{\sqrt{6}+\sqrt{2}}{4(1+\sqrt{3})} = \dfrac{1}{2\sqrt{2}}$

$\Rightarrow 2\sqrt{12} + 4 = 4 + 4\sqrt{3}$

$\Rightarrow \; 4 + 4\sqrt{3} = 4 + 4\sqrt{3}$

$\therefore$ option (a) is correct

77. *(d)* Given $X = \{1, 2, 3, 4\}$.

$R = \{(1,1), (2,2), (3,3), (1,2), (2,1), (2,3), (3,2)\}$

R is reflexive if aRa for all $a \in X$

As $(4,4) \notin X$, so R is not reflexive.

R is transitive if $aRb, bRc \Rightarrow aRc$ for all $a, b, c \in X$

As $(1,2), (2,3) \in X$ but $(1,3) \notin X$, so R is not transitive.

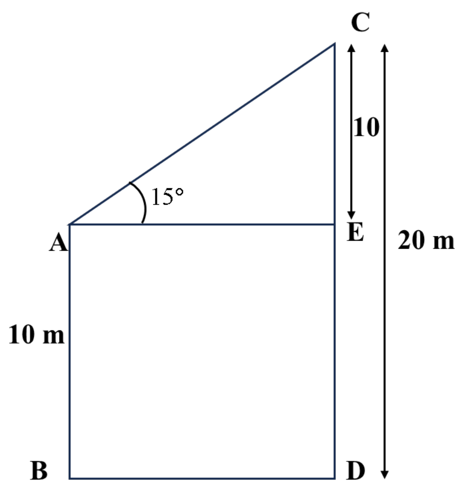
R is symmetric if $aRb \Rightarrow bRa$ for all $a, b \in X$

So, R is symmetric.

Thus, R is neither reflexive nor transitive but symmetric.

78. *(b)* Let AB and CD be two poles of height 10 m and

20 m respectively.



In triangle AEC,

$$\frac{CE}{AE} = \tan 15°$$

$$\Rightarrow \frac{10}{AE} = \tan(45° - 30°)$$

$$= \frac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°}$$

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

This further gives,

$$\Rightarrow AE = 10\frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= 10\frac{4 + 2\sqrt{3}}{3 - 1}$$

$$= 5(4 + 2\sqrt{3})$$

79. *(a)* Given $\cot\theta = 2\cos\theta$, where $(\pi/2) < \theta < \pi$

$$\Rightarrow \frac{\cos\theta}{\sin\theta} = 2\cos\theta$$

$$\Rightarrow \frac{\cos\theta}{2\cos\theta} = \sin\theta$$

$$\Rightarrow \frac{1}{2} = \sin\theta$$

This further implies that,

$$\sin\frac{\pi}{6} = \sin\theta$$

$$\Rightarrow \theta = \frac{\pi}{6}$$

But $\frac{\pi}{2} < \theta < \pi, \therefore \theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

80. *(b)* Number of students who play chess, $n(A) = 60$

Number of students who play tennis, $n(B) = 50$

Number of students who play carrom, $n(C) = 48$

Given $n(A \cap B) = 20, n(B \cap C) = 15, n(A \cap C) = 12$

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

$$= 60 + 50 + 48 - 20 - 15 - 12 + n(A \cap B \cap C)$$

$$= 111 + n(A \cap B \cap C)$$

So, minimum number of students is 111.

81. *(b)* $2x + 3y = 20 \Rightarrow y = \frac{20 - 2x}{3}$

As, $(x, y) \in N$

For $x = 1,\ y = \frac{20-2}{3} = 6,\ (x, y) = (1, 6) \in N$

For $x = 4,\ y = \frac{20-8}{3} = 4,\ (x, y) = (4, 4) \in N$

For $x = 7,\ y = \frac{20-14}{3} = 2,\ (x, y) = (7, 2) \in N$

So, number of elements (x, y) in relation R is 3.

82. *(c)* Let $\tan A = 1/2$ and $\tan B = 1/3$

We know that,

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$= \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \times \frac{1}{3}} = \frac{\frac{5}{6}}{\frac{5}{6}} = 1$$

$\Rightarrow A+B=\tan^{-1}(1)=\frac{\pi}{4}$

Multiply both sides by 4, we have

$4A+4B=\pi$

83. *(d)* Let $\begin{vmatrix} a & b & c \\ l & m & n \\ p & q & r \end{vmatrix}=2$

$$\begin{vmatrix} 6a & 3b & 15c \\ 2l & m & 5n \\ 2p & q & 5r \end{vmatrix}=2\times5\begin{vmatrix} 3a & 3b & 3c \\ l & m & n \\ p & q & r \end{vmatrix}$$

$$=2\times5\times3\begin{vmatrix} a & b & c \\ l & m & n \\ p & q & r \end{vmatrix}$$

$$=2\times5\times3\times2$$

$$=60$$

84. *(c)* Given determinant $\begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix}$

Take out a, b, and c from row 1, row 2, and row 3 respectively.

$$=abc\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix}$$

Take out a, b, and c from column 1, column 2, and column 3 respectively.

$$=a^2b^2c^2\begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

Applying $R_2 \to R_2+R_1,\ R_3 \to R_3+R_1$

$$=a^2b^2c^2\begin{vmatrix} -1 & 1 & 1 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{vmatrix}$$

$$=a^2b^2c^2(-1)(0-4)$$

$$=4a^2b^2c^2$$

85. *(b)* Given expression $\frac{\sin x}{1+\cos x}+\frac{1+\cos x}{\sin x}$.

This can be rewritten as,

$$\frac{\sin x}{1+\cos x}+\frac{1+\cos x}{\sin x}$$

$$=\frac{\sin^2 x+1+\cos^2 x+2\cos x}{(1+\cos x)(\sin x)}$$

$$=\frac{(\sin^2 x+\cos^2 x)+1+2\cos x}{(1+\cos x)(\sin x)}$$

$$=\frac{2+2\cos x}{(1+\cos x)(\sin x)}$$

$$=\frac{2(1+\cos x)}{(1+\cos x)(\sin x)}=\frac{2}{\sin x}=2\operatorname{cosec} x$$

86. *(c)* Given $A=\begin{bmatrix} 2 & 7 \\ 1 & 5 \end{bmatrix}$

$$A^{-1}=\frac{1}{|A|}\operatorname{adj}(A)$$

$$=\frac{1}{(10-7)}\begin{bmatrix} 5 & -7 \\ -1 & 2 \end{bmatrix}$$

$$=\frac{1}{3}\begin{bmatrix} 5 & -7 \\ -1 & 2 \end{bmatrix}$$

So, $3A^{-1}=\begin{bmatrix} 5 & -7 \\ -1 & 2 \end{bmatrix}$

Thus, $A+3A^{-1}=\begin{bmatrix} 2 & 7 \\ 1 & 5 \end{bmatrix}+\begin{bmatrix} 5 & -7 \\ -1 & 2 \end{bmatrix}$

$$=\begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}=7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}=7I$$

87. *(b)* $\cos^4\frac{7\pi}{8}+\cos^4\frac{5\pi}{8}=\left(\cos^2\frac{7\pi}{8}\right)^2+\left(\cos^2\frac{5\pi}{8}\right)^2$

$$=\left(\cos^2\frac{7\pi}{8}-\cos^2\frac{5\pi}{8}\right)^2+2\cos^2\frac{7\pi}{8}\cos^2\frac{5\pi}{8}$$

$$=\left[(-1)\sin\left(\frac{7\pi}{8}+\frac{5\pi}{8}\right)\sin\left(\frac{7\pi}{8}-\frac{5\pi}{8}\right)\right]^2+\frac{1}{2}\left[2\cos\frac{7\pi}{8}\cos\frac{5\pi}{8}\right]^2$$

$$=\left[\sin\frac{3\pi}{2}\sin\frac{\pi}{4}\right]^2+\frac{1}{2}\left[\cos\frac{3\pi}{2}+\cos\frac{\pi}{4}\right]^2$$

$$=\left[(-1)\cdot\frac{1}{\sqrt{2}}\right]^2+\frac{1}{2}\left[0+\frac{1}{\sqrt{2}}\right]^2$$

$$=\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$$

88. *(c)* X follow $\text{B}(6,p)$, $16P(X=4)=P(X=2)$

$$\Rightarrow 16\,{}^6C_4p^4(1-p)^{6-4}={}^6C_2p^2(1-p)^{6-2}$$

$$\Rightarrow 16\frac{6!}{2!4!}p^4(1-p)^2=\frac{6!}{2!4!}\,p^2(1-p)^4$$

$$\Rightarrow \quad 16p^2=(1-p)^2$$

$$\Rightarrow \quad 16p^2=1+p^2-2p$$

$$\Rightarrow \quad 15p^2+2p-1=0$$

Solve further,

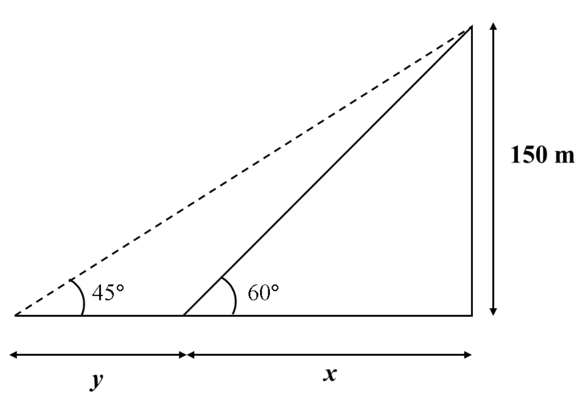
$$15p^2+5p-3p-1=0$$

$$(5p-1)(3p+1)=0$$

$$p=\frac{1}{5},-\frac{1}{3}$$

As $p>0,\ p=\frac{1}{5}$

89. *(b)*



$$\tan 60^\circ=\frac{150}{x}\Rightarrow x=\frac{150}{\sqrt{3}}$$

Also,

$$\tan 45^\circ=\frac{150}{x+y}\Rightarrow x+y=150$$

$$\Rightarrow \quad y=150-x$$

$$\Rightarrow \quad y=150-\frac{150}{\sqrt{3}}$$

$$\Rightarrow \quad y=150\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right)$$

This represents the distance travelled.

Speed (in m/hr) = $150\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right)\times\frac{60}{2}=\frac{4500(\sqrt{3}-1)}{\sqrt{3}}$

90. *(c)* Given $\tan\theta+\sec\theta=4$

$$\Rightarrow \frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}=4$$

$$\Rightarrow \quad 1+\sin\theta=4\cos\theta$$

Squaring both sides,

$$(1+\sin\theta)^2=16\cos^2\theta$$

$$\Rightarrow (1+\sin\theta)^2=16\left(1-\sin^2\theta\right)$$

$$\Rightarrow (1+\sin\theta)^2=16(1+\sin\theta)(1-\sin\theta)$$

$$\Rightarrow \quad 1+\sin\theta=16-16\sin\theta$$

$$\Rightarrow \quad 17\sin\theta=15$$

$$\Rightarrow \quad \sin\theta=\frac{15}{17}$$

91. *(c)* Let $p(x,y)$, $A(2a,0)$ and $B(0,3a)$.

According to the question $PA=PB$

$$\Rightarrow PA^2=PB^2$$

This further gives,

$$(x-2a)^2+(y-0)^2=(x-0)^2+(y-3a)^2$$

$$\Rightarrow x^2-4ax+4a^2+y^2=x^2+y^2-6ay+9a^2$$

$$\Rightarrow \quad 4ax-6ay+5a^2=0$$

$$\Rightarrow \quad 4x-6y+5a=0$$

92. *(d)* We know that if $R=\{(x,y),\ x\in A,\ y\in B\}$, then

$R^{-1}=\{(y,x),\ y\in B, x\in A\}$

Statement 1: Let R be reflexive.

$(x,x)\in R \Rightarrow (x,x)\in R^{-1}$

So, R^{-1} is also reflexive.

Statement 2: Let R be symmetric.

$(x,y)\in R \Rightarrow (y,x)\in R$

Let $(y,x)\in R^{-1} \Rightarrow (x,y)\in R^{-1}$

So, R^{-1} is also symmetric.

Statement 3: Let R be transitive.

So, $(x,y),(y,x)\in R \Rightarrow (x,z)\in R$

Now

$(x,y)\in R \Rightarrow (y,x)\in R^{-1}$

$(y,z)\in R \Rightarrow (z,y)\in R^{-1}$

$(x,z)\in R \Rightarrow (z,x)\in R^{-1}$

So, $(z,y),(y,x)\in R^{-1} \Rightarrow (z,x)\in R^{-1}$

So, R^{-1} is also transitive.

93. *(c)* Given function $f(x)=|x-3|$

$\therefore$ LHL $= \lim_{h\to 0} f(0-h) = \lim_{h\to 0}|0-h-3|$

$= \lim_{h\to 0}(h+3)=3$

RHL $= \lim_{h\to 0} f(0+h) = \lim_{h\to 0}|0+h-3|$

$= \lim_{h\to 0}|h-3| = 3$

Since LHL = RHL at $x = 0$, $f(x)=|x-3|$ is continuous at $x = 0$.

Now LHD $= f'(0^-) = \lim_{h\to 0}\frac{f(0-h)-f(0)}{-h}$

$= \lim_{h\to 0}\frac{3-h-3}{-h}$

$= \lim_{h\to 0} 1 = 1$

RHD $= f'(0^+) = \lim_{h\to 0}\frac{f(0+h)-f(0)}{h}$

$= \lim_{h\to 0}\frac{h-3+3}{h}$

$= \lim_{h\to 0} 1 = 1$

Since LHD = RHD at $x = 0$, $f(x)=|x-3|$ is differentiable at $x = 0$.

Hence, both statements (1) and (2) are correct.

94. *(a)* $X=\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}$, $B=\begin{bmatrix}5 & 2\\ -2 & 1\end{bmatrix}$ and $A=\begin{bmatrix}p & q\\ r & s\end{bmatrix}$

Now, $AX = B$

$$\begin{bmatrix}p & q\\ r & s\end{bmatrix}\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}=\begin{bmatrix}5 & 2\\ -2 & 1\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}3p+q & -4p-q\\ 3r+s & -4r-s\end{bmatrix}=\begin{bmatrix}5 & 2\\ -2 & 1\end{bmatrix}$$

This gives,

$3p+q=5 \quad ...(i)$

$-4p-q=2 \quad ...(ii)$

$3r+s=-2 \quad ...(iii)$

$-4r-s=1 \quad ...(iv)$

Adding equations (i) and (ii), we get

$-p=7 \Rightarrow p=-7$

$\Rightarrow q=5-3(-7)=26$

Adding equations (iii) and (iv), we get

$-r=-1 \Rightarrow r=1$

$\Rightarrow s=-2-3(1)=-5$

Hence, $A=\begin{bmatrix}p & q\\ r & s\end{bmatrix}=\begin{bmatrix}-7 & 26\\ 1 & -5\end{bmatrix}$

95. *(c)* $\because\ 2\tan\alpha = 1 \Rightarrow \tan\alpha = \frac{1}{2}$

Also, $\alpha+\beta=\frac{\pi}{4} \Rightarrow \tan(\alpha+\beta)=\tan\frac{\pi}{4}$

$\Rightarrow \dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}=1$

$\Rightarrow \quad \tan\alpha+\tan\beta=1-\tan\alpha\tan\beta$

$\Rightarrow \quad \dfrac{1}{2}+\tan\beta=1-\dfrac{1}{2}\tan\beta$

$\Rightarrow \quad \dfrac{3}{2}\tan\beta=\dfrac{1}{2}$

$\Rightarrow \quad \tan\beta=\dfrac{1}{3}$

Now, $\tan 2\beta=\dfrac{2\tan\beta}{1-\tan^2\beta}=\dfrac{2\times\frac{1}{3}}{1-\frac{1}{9}}=\dfrac{2}{3}\times\dfrac{9}{8}=\dfrac{3}{4}$

96. *(c)* Given $\alpha+\beta=90°$

$$\begin{vmatrix}\cos^2\frac{\alpha}{2} & \sin^2\frac{\alpha}{2}\\ \sin^2\frac{\beta}{2} & \cos^2\frac{\beta}{2}\end{vmatrix}=\cos^2\frac{\alpha}{2}\cos^2\frac{\beta}{2}-\sin^2\frac{\alpha}{2}\sin^2\frac{\beta}{2}$$

$=\left(\cos\frac{\alpha}{2}\cos\frac{\beta}{2}+\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\right)$

$\times\left(\cos\frac{\alpha}{2}\cos\frac{\beta}{2}-\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\right)$

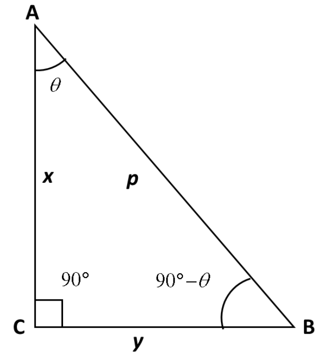
$=\cos\dfrac{\alpha-\beta}{2}\times\cos\dfrac{\alpha+\beta}{2}$

$=\cos\dfrac{\alpha-\beta}{2}\times\cos\dfrac{90°}{2}$

$=\cos\dfrac{\alpha-\beta}{2}\times\dfrac{1}{\sqrt{2}}$

Maximum value of $\cos\dfrac{\alpha-\beta}{2}$ is 1. So maximum value of determinant is $\dfrac{1}{\sqrt{2}}$.

97. *(b)*



$\overrightarrow{AB}\cdot\overrightarrow{AC}+\overrightarrow{BC}\cdot\overrightarrow{BA}+\overrightarrow{CA}\cdot\overrightarrow{CB}$

$=\left(AB\cdot AC\cdot\cos\theta\right)+\left(BC\cdot BA\cdot\cos(90-\theta)\right)$

$+\left(CA\cdot CB\cdot\cos 90\right)$

$=px\cos\theta+py\sin\theta+0$

$=p\left(x\cos\theta+y\sin\theta\right)$

By projection formula,

$p=x\cos\theta+y\cos(90-\theta)$

$=x\cos\theta+y\sin\theta$

$\therefore\ \overrightarrow{AB}\cdot\overrightarrow{AC}+\overrightarrow{BC}\cdot\overrightarrow{BA}+\overrightarrow{CA}\cdot\overrightarrow{CB}$

$=p\times p=p^2$

98. *(c)* Let the coordinates of A, B, C, and D are $(0,4,1),(2,3,-1),(4,5,0),(2,6,2)$ respectively.

The sides and diagonals are as follows:

$AB=\sqrt{(2-0)^2+(3-4)^2+(-1-1)^2}$

$=\sqrt{4+1+4}=\sqrt{9}=3$

$BC=\sqrt{(4-2)^2+(5-3)^2+(0+1)^2}$

$=\sqrt{4+4+1}=\sqrt{9}=3$

$CD=\sqrt{(2-4)^2+(6-5)^2+(2-0)^2}$

$=\sqrt{4+1+4}=\sqrt{9}=3$

$DA=\sqrt{(0-2)^2+(4-6)^2+(1-2)^2}$

$=\sqrt{4+4+1}=\sqrt{9}=3$

$AC=\sqrt{(4-0)^2+(5-4)^2+(0-1)^2}$

$=\sqrt{16+1+1}=\sqrt{18}$

$BD = \sqrt{(2-2)^2 + (6-3)^2 + (2+1)^2}$

$= \sqrt{9+9} = \sqrt{18}$

Since $AB = BC = CD = DA$, sides are equal.

Also, $AC = BD$, diagonals are equal.

Hence, *A*, *B*, *C*, and *D* are vertices of a square.

99. *(a)* Given $\sin 2\theta = \cos 3\theta$.

$\Rightarrow \sin 2\theta = \sin(90° - 3\theta)$

$\Rightarrow \quad 2\theta = 90° - 3\theta$

$\Rightarrow \quad \theta = 18$

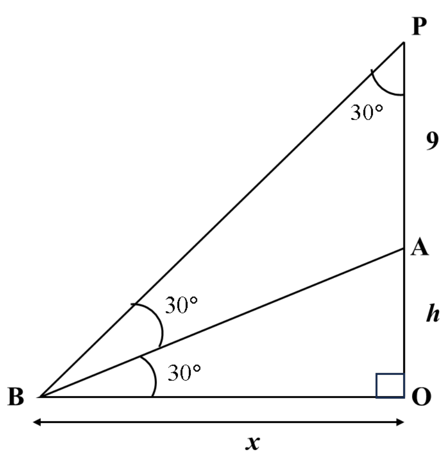
Now, $1 + 4\sin\theta = 1 + 4\sin 18°$

$= 1 + 4\left(\frac{\sqrt{3}-1}{4}\right)$

$= 1 + \sqrt{3} - 1$

$= \sqrt{3}$

100. *(c)*



ΔPAB is isosceles triangle.

So, $\theta = \angle PBA = 30°$

In ΔABO,

$\tan 30° = \frac{h}{x} \Rightarrow x = h\sqrt{3} \quad ...(i)$

In ΔPBO,

$\tan 60° = \frac{9+h}{x} \Rightarrow \sqrt{3} = \frac{9+h}{x}$

$\Rightarrow \sqrt{3} = \frac{9+h}{h\sqrt{3}}$ (from (i))

$\Rightarrow 3h = 9 + h$

$\Rightarrow 2h = 9$

$\Rightarrow \quad h = 4.5\,\text{m}$

So, total height = 9 + 4.5 = 13.5 m

101. *(c)* Given $\operatorname{cosec}\theta - \cot\theta = \frac{1}{\sqrt{3}}$, where $\theta \neq 0$

$\Rightarrow \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta} = \frac{1}{\sqrt{3}} \Rightarrow \frac{1-\cos\theta}{\sin\theta} = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{1 - \left(1 - 2\sin^2\frac{\theta}{2}\right)}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}} = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad \frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}} = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad \tan\frac{\theta}{2} = \frac{1}{\sqrt{3}} = \tan 30°$

$\Rightarrow \quad \theta = 60°$

Thus, $\cos\theta = \cos 60° = \frac{1}{2}$.

102. *(a)* Since $\vec{c}$ is parallel to $\vec{a}$, we have

$\vec{c} = \lambda\,\vec{a}$

Now $\vec{b} = \vec{c} + \vec{d}$

$= \lambda\vec{a} + \vec{d}$

$= \lambda\left(\hat{i} + \hat{j}\right) + x\hat{i} + y\hat{j} + z\hat{k}$

This gives,

$3\hat{i} + 4\hat{k} = (\lambda + x)\hat{i} + (\lambda + y)\hat{j} + z\hat{k}$

Comparing, we get

$z = 4,\ \lambda + y = 0,\ \lambda + x = 3$

$\Rightarrow \lambda = -y$ and $x - y = 3 \quad ...(i)$

Now $\vec{d}$ is perpendicular to $\vec{a}$, so $\cos\theta = 0$

$$\left(\hat{i}+\hat{j}\right)\cdot\left(x\hat{i}+y\hat{j}+z\hat{k}\right)=0$$

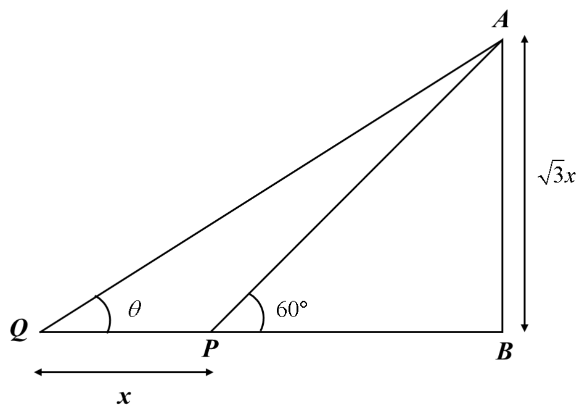
$$x+y=0 \quad ...(\text{ii})$$

Solving equations (i) and (ii), we get

$$x=\frac{3}{2},\ y=-\frac{3}{2}$$

$$\therefore\ \vec{c}=\lambda\vec{a}=\frac{3}{2}\left(\hat{i}+\hat{j}\right)$$

103. *(b)*



In ΔAPB,

$$\tan 60^\circ=\frac{\sqrt{3}x}{BP}\Rightarrow\sqrt{3}=\frac{\sqrt{3}x}{BP}$$

$$\Rightarrow BP=x \qquad ...(\text{i})$$

In ΔAQB,

$$\tan\theta=\frac{\sqrt{3}x}{x+BP}\Rightarrow\tan\theta=\frac{\sqrt{3}x}{x+x}$$

$$\Rightarrow\tan\theta=\frac{\sqrt{3}}{2}$$

$$\because\ \frac{1}{\sqrt{3}}<\frac{\sqrt{3}}{2}<1,\text{ we have } 30^\circ<\theta<45^\circ$$

104. *(a)* Let the equation of the line passing through $(1,2,3)$ and having direction ratios $\langle 1,2,3\rangle$ is,

$$\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}=a$$

$$\Rightarrow x-1=a,\ y-2=2a,\ z-3=3a$$

$$\Rightarrow x=a+1,\ y=2a+2,\ z=3a+3$$

At x-axis, $y=0$ and $z=0$

$$\Rightarrow 2a+2=0,\ 3a+3=0$$

$$\Rightarrow a=-1$$

$$\therefore\ x=-1+1=0$$

105. *(b)* $4y-15x+410=0$

$$y-\frac{15}{4}x+\frac{410}{4}=0$$

$$y=\frac{15}{4}x-\frac{410}{4}$$

$$\therefore\ b_{yx}=\frac{15}{4}$$

Also, $30x-2y-825=0$

$$x-\frac{2}{30}y-\frac{825}{30}=0$$

$$x=\frac{1}{15}y+\frac{165}{6}$$

$$\therefore\ b_{xy}=\frac{1}{15}$$

Correlation coefficient $=\sqrt{b_{yx}\,b_{xy}}$

$$=\sqrt{\frac{15}{4}\times\frac{1}{15}}$$

$$=\sqrt{\frac{1}{4}}$$

$$=\frac{1}{2}$$

106. *(b)* Equation of the line passing through $(1,-2,4)$ and whose normal passes through $(2,1,2)$ is,

$$2(x-1)+1(y+2)+2(z-4)=0$$

$$\Rightarrow \quad 2x-2+y+2+2z-8=0$$

$$\Rightarrow \quad 2x+y+2z-8=0$$

So, distance of the plane $2x+y+2z-8=0$ from the point $(3,2,3)$ is,

$$=\frac{2(3)+2+2(3)-8}{\sqrt{4+1+4}}=\frac{6}{3}=2$$

107. *(b)* Given that the regression coefficients of Y on X is -6, and the correlation coefficient between X and Y is $-\frac{1}{2}$.

This implies that $b_{yx} = -6,\ r = -\frac{1}{2}$.

$$\Rightarrow \left(-\frac{1}{2}\right)^2 = -6 \times b_{xy}$$

$$\Rightarrow \frac{1}{4} = -6 \times b_{xy}$$

$$\Rightarrow -\frac{1}{24} = b_{xy}$$

108. *(b)* Given vectors are

$\vec{\alpha} = \hat{i} + 2\hat{j} - \hat{k},\ \vec{\beta} = 2\hat{i} - \hat{j} + 3\hat{k}$, and $\vec{\gamma} = 2\hat{i} + \hat{j} + 6\hat{k}$

Let $\vec{\delta} = a\hat{i} + b\hat{j} + c\hat{k}$.

Since $\vec{\alpha}$ and $\vec{\beta}$ are both perpendicular to the vector $\vec{\delta}$,

$$\vec{\alpha} \cdot \vec{\delta} = 0 \Rightarrow a + 2b - c = 0 \quad \text{...(i)}$$

and $\vec{\beta} \cdot \vec{\delta} = 0 \Rightarrow 2a - b + 3c = 0 \quad \text{...(ii)}$

From (i) and (ii),

$$\frac{a}{5} = \frac{b}{-5} = \frac{c}{-5} \Rightarrow \frac{a}{1} = \frac{b}{-1} = \frac{c}{-1} = x(\text{say})$$

So, $a = x,\ b = -x,\ c = -x$

Also, it is given that $\vec{\delta} \cdot \vec{\gamma} = 10$

$$\Rightarrow 2a + b + 6c = 10$$

$$\Rightarrow 2x - x - 6x = 10$$

$$\Rightarrow -5x = 10$$

$$\Rightarrow x = -2$$

So, $\vec{\delta} = -2\hat{i} + 2\hat{j} + 2\hat{k}$

and $|\vec{\delta}| = \sqrt{4+4+4} = \sqrt{12} = 2\sqrt{3}$

109. *(d)* Given expression is $\sin 18^\circ \cos 36^\circ$.

We know that $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$

Now,

$$\cos 36^\circ = 1 - 2\sin^2 18^\circ$$

$$= 1 - \frac{2}{16}\left(\sqrt{5}-1\right)^2$$

$$= 1 - \frac{1}{8}\left(5 + 1 - 2\sqrt{5}\right)$$

$$= 1 - \frac{1}{8} \times 2\left(3 - \sqrt{5}\right)$$

$$= 1 - \frac{1}{4}\left(3 - \sqrt{5}\right)$$

$$= \frac{1+\sqrt{5}}{4}$$

$$\sin 18^\circ \cos 36^\circ = \frac{\sqrt{5}-1}{4} \times \frac{1+\sqrt{5}}{4}$$

$$= \frac{5-1}{16}$$

$$= \frac{1}{4}$$

110. *(c)* Let $\vec{a} = 2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} - 3\hat{k}$

Two diagonals of the parallelogram are given by,

$$\vec{a} + \vec{b} = \left(2\hat{i} - 4\hat{j} + 5\hat{k}\right) + \left(\hat{i} - 2\hat{j} - 3\hat{k}\right)$$

$$= 3\hat{i} - 6\hat{j} + 2\hat{k}$$

$$\vec{a} - \vec{b} = \left(2\hat{i} - 4\hat{j} + 5\hat{k}\right) - \left(\hat{i} - 2\hat{j} - 3\hat{k}\right)$$

$$= \hat{i} - 2\hat{j} + 8\hat{k}$$

Dot product of the diagonals is,

$$\left(\vec{a} + \vec{b}\right) \cdot \left(\vec{a} - \vec{b}\right) = \left(3\hat{i} - 6\hat{j} + 2\hat{k}\right) \cdot \left(\hat{i} - 2\hat{j} + 8\hat{k}\right)$$

$$= 3 + 12 + 16 = 31 \text{ units}$$

111. *(a)* As $\left(\vec{a} + \vec{b}\right)$ is perpendicular to $\vec{a}$, so we have

$$\left(\vec{a} + \vec{b}\right) \cdot \vec{a} = 0 \Rightarrow |\vec{a}|^2 + \vec{b} \cdot \vec{a} = 0$$

$$\Rightarrow \vec{a} \cdot \vec{b} = -|\vec{a}|^2 \quad \text{...(i)}$$

Given magnitude of $\vec{b}$ is twice that of $\vec{a}$,

$$|\vec{b}| = 2|\vec{a}| \Rightarrow |\vec{b}|^2 = 4|\vec{a}|^2 \quad \text{...(ii)}$$

Now, $\left(4\vec{a} + \vec{b}\right) \cdot \vec{b} = 4\vec{a} \cdot \vec{b} + |\vec{b}|^2$

$= 4\left(-\left|\vec{a}\right|^2\right) + 4\left|\vec{a}\right|^2$

$= -4\left|\vec{a}\right|^2 + 4\left|\vec{a}\right|^2$

$= 0$

112. *(b)* Given function $f(x) = \begin{cases} 2+x & \text{if } x \geq 0 \\ 2-x & \text{if } x < 0 \end{cases}$

For $x \geq 1$

$\lim_{x\to1^+} f(x) = \lim_{x\to1^+} 2+x = 2+1 = 3$

For $x < 1$

$\lim_{x\to1^-} f(x) = \lim_{x\to1^-} 2+x = 2+1 = 3$

So, $\lim_{x\to1} f(x)$ exist.

At $x = 0$

LHL $= \lim_{h\to0^-} f(0-h) = \lim_{h\to0^-} 2+h = 2$

RHL $= \lim_{h\to0^+} f(0+h) = \lim_{h\to0^+} 2+h = 2$

For $f(0) = 2+0 = 2$

Since LHL = RHL = $f(0)$, $f(x)$ is continuous at $x = 0$.

Now LHD $= f'(0^-) = \lim_{h\to0} \frac{f(0-h)-f(0)}{-h}$

$= \lim_{h\to0} \frac{2+h-2}{-h}$

$= \lim_{h\to0} -1 = -1$

RHD $= f'(0^+) = \lim_{h\to0} \frac{f(0+h)-f(0)}{h}$

$= \lim_{h\to0} \frac{2+h-2}{h}$

$= \lim_{h\to0} 1 = 1$

Since LHD $\neq$ RHD at $x = 0$, $f(x)$ is not differentiable at $x = 0$.

113. *(d)* The given equation of planes are $x+y+2z=3$ and $-2x+y-z=11$

We know that the angle between the planes $a_1x+b_1y+c_1z+d_1=0$ and $a_2x+b_2y+c_2z+d_2=0$ is given by,

$$\cos\theta = \left|\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|$$

Here, $a_1=1,\ a_2=-2,\ b_1=1,\ b_2=1,\ c_1=2,\ c_2=-1$

This gives,

$$\cos\theta = \left|\frac{1\times(-2)+1\times1+2\times(-1)}{\sqrt{1+1+4}\sqrt{4+1+1}}\right|$$

$$= \left|\frac{-2+1-2}{\sqrt{6}\sqrt{6}}\right| = \frac{3}{6} = \frac{1}{2}$$

$$\Rightarrow \cos\theta = \cos\frac{\pi}{3} \Rightarrow \theta = \frac{\pi}{3}$$

114. *(d)* Given $r_{xy} = 0.6$

$Z = X+5,\ W = \frac{Y}{3}$

$\Rightarrow b_{zx} = 1 \Rightarrow b_{wy} = \frac{1}{3}$

Now $b_{zx}b_{wy} = 1\left(\frac{1}{3}\right) = \frac{1}{3}$

$\Rightarrow \frac{r_{zw}}{r_{xy}} = \frac{1}{3} \Rightarrow r_{zw} = \frac{r_{xy}}{3} = \frac{0.6}{3} = 0.2$

115. *(a)* Given $t_n = \sin^n\theta + \cos^n\theta$.

$$\therefore \frac{t_3 - t_5}{t_5 - t_7} = \frac{\sin^3\theta + \cos^3\theta - \sin^5\theta - \cos^5\theta}{\sin^5\theta + \cos^5\theta - \sin^7\theta - \cos^7\theta}$$

$$= \frac{\sin^3\theta(1-\sin^2\theta) + \cos^3\theta(1-\cos^2\theta)}{\sin^5\theta(1-\sin^2\theta) + \cos^5\theta(1-\cos^2\theta)}$$

$$= \frac{\sin^3\theta\cos^2\theta + \cos^3\theta\sin^2\theta}{\sin^5\theta\cos^2\theta + \cos^5\theta\sin^2\theta}$$

$$= \frac{\sin^2\theta\cos^2\theta(\sin\theta + \cos\theta)}{\sin^2\theta\cos^2\theta(\sin^3\theta + \cos^3\theta)}$$

$$= \frac{\sin\theta + \cos\theta}{\sin^3\theta + \cos^3\theta}$$

$$= \frac{t_1}{t_3}$$

116. *(a)* Given linear equations are:

$$kx + y + z = 1$$
$$x + k + z = 1$$
$$x + y + kz = 1$$

This can be rewritten as,

$$\begin{bmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$AX = B$$
$$X = A^{-1}B$$

Linear equations will have a unique solution when A^{-1} exist, i.e. $|A| \neq 0$

$$\begin{vmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{vmatrix} \neq 0$$

This gives,

$$k(k^2 - 1) - 1(k-1) + 1(1-k) \neq 0$$
$$k^3 - k - k + 1 + 1 - k \neq 0$$
$$k^3 - 3k + 2 \neq 0$$
$$(k-1)(k^2 + k - 2) \neq 0$$
$$(k-1)(k-1)(k+2) \neq 0$$
$$k \neq 1, -2$$

117. *(d)* Given expression

$$\tan 9° - \tan 27° - \tan 63° + \tan 81°$$
$$= \tan 9° - \tan 27° - \tan(90° - 27°) + \tan(90° - 9°)$$
$$= \tan 9° - \tan 27° - \cot 27° + \cot 9°$$
$$= \tan 9° + \cot 9° - (\tan 27° + \cot 27°)$$

We know that,

$$\tan\theta + \cot\theta = \frac{1}{\sin\theta\cos\theta} = \frac{2}{2\sin\theta\cos\theta} = \frac{2}{\sin 2\theta}$$

$$\therefore \tan 9° + \cot 9° - (\tan 27° + \cot 27°)$$
$$= \frac{2}{\sin 18°} - \frac{2}{\sin 54°}$$
$$= 2\left(\frac{\sin 54° - \sin 18°}{\sin 18° \sin 54°}\right)$$

We know that,

$$\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$$

$$\therefore 2\left(\frac{\sin 54° - \sin 18°}{\sin 18° \sin 54°}\right) = 2 \cdot 2\frac{\cos 36° \sin 18°}{\sin 18° \sin 54°}$$
$$= 4\frac{\cos 36° \sin 18°}{\sin 18° \sin(90° - 36°)}$$
$$= 4\frac{\cos 36° \sin 18°}{\sin 18° \cos 36°}$$
$$= 4$$

118. *(c)* Given $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplaner.

$$\therefore \vec{a} \cdot (\vec{b} \times \vec{c}) = 0 \quad ...(i)$$

We have $(\vec{a} \times \vec{b}) \times \vec{c} = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{b} \cdot \vec{c})\vec{a}$

Now, $[(\vec{a} \times \vec{b}) \times \vec{c}] \cdot (\vec{a} \times \vec{b})$

$$=\left[(\vec{a}\cdot\vec{c})\vec{b}-(\vec{b}\cdot\vec{c})\vec{a}\right]\cdot(\vec{a}\times\vec{b})$$
$$=(\vec{a}\cdot\vec{c})\left[\vec{b}\cdot(\vec{a}\times\vec{b})\right]-(\vec{b}\cdot\vec{c})\left[\vec{a}\cdot(\vec{a}\times\vec{b})\right]$$
$$=0-0$$
$$=0$$

So, $(\vec{a}\times\vec{b})\times\vec{c}$ is coplaner with $\vec{a}$ and $\vec{b}$ and perpendicular to $\vec{a}\times\vec{b}$.

119. *(b)* Given $f(x)=2x-3$ and $g(x)=x^3+5$

$$(f\circ g)(x)=f(x^3+5)$$
$$=2(x^3+5)-3$$
$$=2x^3+7$$

Let $2x^3+7=y\Rightarrow x=\left(\frac{y-7}{2}\right)^{\frac{1}{3}}$

$$\Rightarrow (f\circ g)^{-1}(y)=\left(\frac{y-7}{2}\right)^{\frac{1}{3}}$$
$$\Rightarrow (f\circ g)^{-1}(x)=\left(\frac{x-7}{2}\right)^{\frac{1}{3}}$$

So, coordinates of Q are $(3\times2-2,\ -2\times2+1,\ 2\times2-5)$ or $(4,-3,-1)$ ...(2)

Also, the midpoint of PQ is

$L\left(\frac{3\times2}{2}-2,\ 1-2,\ 2-5\right)$ or $L(1,\ -1,\ -3)$ ...(3)

$$PQ=\sqrt{(-2-4)^2+(1+3)^2+(-5+1)^2}$$
$$=\sqrt{36+16+16}$$
$$=\sqrt{68}$$
$$\Rightarrow PQ=2\sqrt{17}>8$$

120. *(d)* Let $Q(x_1,y_1,z_1)$ be the image of the point $P(-2,1,-5)$ in the plane $3x-2y+2z+1=0$.

Direction ratios of PQ are $3,-2,2$ ...(i)

The equation of the line PQ is,

$$\frac{x+2}{3}=\frac{y-1}{-2}=\frac{z+5}{2}=r$$

Then, coordinates of any point on the line PQ is given by $3r-2,\ -2r+1,\ 2r-5$.

Let $Q(3r-2,\ -2r+1,\ 2r-5)$ be such a point.

Let L be the midpoint of PQ. So, we have $L\left(\frac{3r}{2}-2,\ 1-r,\ r-5\right)$.

Since L lies on the given plane.

$$3\left(\frac{3r}{2}-2\right)-2(1-r)+2(r-5)+1=0$$
$$\Rightarrow \frac{17r}{2}=17$$
$$\Rightarrow \quad r=2$$