HINTS & SOLUTION

1. *(d)* Since, α and β are the roots of the equation $x^2-2x-1=0$, then
sum of roots $=\alpha+\beta=2$ and
product of roots $=\alpha\beta=-1$
Since, $(\alpha+\beta)^2=\alpha^2+\beta^2+2\alpha\beta$
$\Rightarrow\ 4=\alpha^2+\beta^2-2$
$\Rightarrow\ \alpha^2+\beta^2=6$
Now, $\alpha^2\beta^{-2}+\alpha^{-2}\beta^2=\dfrac{\alpha^2}{\beta^2}+\dfrac{\beta^2}{\alpha^2}=\dfrac{\alpha^4+\beta^4}{(\alpha\beta)^2}$
$\Rightarrow\ (\alpha^2+\beta^2)^2=6^2$
$\Rightarrow\ \alpha^4+\beta^4+2\alpha^2\beta^2=36$
$\Rightarrow\ \alpha^4+\beta^4+2(-1)^2=36$
$\Rightarrow\ \alpha^4+\beta^4=34$
$\Rightarrow\ \dfrac{\alpha^4+\beta^4}{(\alpha\beta)^2}=\dfrac{34}{(-1)^2}=34$
Hence, option *(d)* is correct.

2. *(c)* If $x+1$, $4x+1$ and $8x+1$ are in GP, then
$(4x+1)^2=(x+1)(8x+1)$
$\Rightarrow 16x^2+8x+1=8x^2+x+8x+1$
$\Rightarrow 8x^2-x=0$
$\Rightarrow x(8x-1)=0$
$\Rightarrow x=0,\ \dfrac{1}{8}\quad\left[\dfrac{1}{8}\text{ is non trivial value}\right]$
Hence, option *(c)* is correct.

3. *(d)* Given $3^{(x-1)}+3^{(x+1)}=30$
$\Rightarrow\ \dfrac{3^x}{3}+3\cdot3^x=30\quad$ ....(i)
Multiplying both sides by 3 in equation (i)
$\Rightarrow 3^x+3^2\cdot3^x=90\ \Rightarrow\ 3^x+3^{x+2}=90$
Hence, option *(d)* is correct.

4. *(d)* Since, $p=\tan\alpha+\tan\beta$ and $q=\cot\alpha+\cot\beta$
$\Rightarrow\ q=\dfrac{1}{\tan\alpha}+\dfrac{1}{\tan\beta}=\dfrac{\tan\alpha+\tan\beta}{\tan\alpha\tan\beta}$
$\Rightarrow\ q=\dfrac{p}{\tan\alpha\tan\beta}\quad\{\because p=\tan\alpha+\tan\beta\}$
$\Rightarrow\ \dfrac{1}{q}=\dfrac{\tan\alpha\tan\beta}{p}$
Hence, $\dfrac{1}{p}-\dfrac{1}{q}=\dfrac{1}{p}-\dfrac{\tan\alpha\tan\beta}{p}$
$\Rightarrow\ \left(\dfrac{1}{p}-\dfrac{1}{q}\right)=\dfrac{1-\tan\alpha\tan\beta}{p}$
$\Rightarrow\ \left(\dfrac{1}{p}-\dfrac{1}{q}\right)=\dfrac{1-\tan\alpha\tan\beta}{\tan\alpha+\tan\beta}$
$\Rightarrow\ \left(\dfrac{1}{p}-\dfrac{1}{q}\right)=\dfrac{1}{\tan(\alpha+\beta)}=\cot(\alpha+\beta)$
Hence, option *(d)* is correct.

5. *(b)* Here, $\sin\alpha$ and $\cos\alpha$ are the roots of the equation $px^2+qx+r=0$
$\therefore\ \sin\alpha+\cos\alpha=-\dfrac{q}{p}\quad$ ....(i)
and $\sin\alpha\cos\alpha=\dfrac{r}{p}\quad$ ....(ii)

From equation (i), we have
$(\sin\alpha+\cos\alpha)^2=\dfrac{q^2}{p^2}$
$\Rightarrow\ \sin^2\alpha+\cos^2\alpha+2\sin\alpha\cos\alpha=\dfrac{q^2}{p^2}$
$\Rightarrow\ 1+2\left(\dfrac{r}{p}\right)=\dfrac{q^2}{p^2}$
$\Rightarrow\ p^2+2pr=q^2$
$\Rightarrow\ p^2-q^2+2pr=0$
Hence, option *(b)* is correct.

6. *(c)* Length of the latus rectum of a hyperbola is $\frac{2b^2}{a}$ where a is the half of the distance between two vertex of the hyperbola.

$$\text{Latus rectum} = \frac{2b^2}{a} = \frac{10}{3} \Rightarrow b^2 = \frac{5a}{3} \quad ....(i)$$

In case of hyperbola,

$$b^2 = a^2(e^2-1) \quad ....(ii)$$

Putting the value of b^2 from equation (i) and $e = \frac{\sqrt{13}}{3}$ in equation (ii), we get

$$\frac{5a}{3} = a^2\left(\frac{13}{9}-1\right) \Rightarrow \frac{5a}{3} = \frac{4a^2}{9}$$

$$\Rightarrow 4a^2 - 15a = 0 \text{ or } a(4-15a) = 0$$

Since, $a \neq 0$, so $a = \frac{15}{4}$

Length of transverse axis $= 2a = 2\times\frac{15}{4} = \frac{15}{2}$

Hence, option *(c)* is correct.

7. *(b)* Let total number of teams participated in a championship be n.

Since, every team played one match with each other team.

$$\therefore\ {}^nC_2 = 153 \Rightarrow \frac{n!}{2!(n-2)!} = 153$$

$$\Rightarrow \frac{n(n-1)(n-2)!}{2!(n-2)!} = 153 \Rightarrow \frac{n(n-1)}{2} = 153$$

$$\Rightarrow n(n-1) = 306$$

$$\Rightarrow n^2 - n - 306 = 0$$

$$\Rightarrow (n-18)(n+17) = 0$$

$$\Rightarrow n = 18, -17$$

Since, n cannot be negative. So $n = 18$

Hence, option *(b)* is correct.

8. *(a)* From the given options, we have

(a) $111101 = 1\times2^5 + 1\times2^4 + 1\times2^3 + 1\times2^2 + 0\times2^1 + 1\times2^0$

$= 32+16+8+4+1$

$= 61$ (which is a prime number)

(b) $111010 = 1\times2^5 + 1\times2^4 + 1\times2^3 + 0\times2^2 + 1\times2^1 + 0\times2^0$

$= 32+16+8+2$

$= 58$ (which is not a prime number)

(c) $111111 = 1\times2^5 + 1\times2^4 + 1\times2^3 + 1\times2^2 + 1\times2^1 + 1\times2^0$

$= 32+16+8+4+2+1$

$= 63$ (which is not a prime number)

(d) $100011 = 1\times2^5 + 0\times2^4 + 0\times2^3 + 0\times2^2 + 1\times2^1 + 1\times2^0$

$= 32+2+1$

$= 35$ (which is not a prime number)

Thus, option *(a)* is correct.

9. *(c)* Since, the points $A(1,2), B(2,4)$ and $C(3,a)$ are collinear.

$$\Rightarrow \begin{vmatrix} 1 & 2 & 1 \\ 2 & 4 & 1 \\ 3 & a & 1 \end{vmatrix} = 0$$

Expanding the determinant, we have

$$\Rightarrow 1\begin{vmatrix} 4 & 1 \\ a & 1 \end{vmatrix} - 2\begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & 4 \\ 3 & a \end{vmatrix} = 0$$

$$\Rightarrow (4-a) - 2(2-3) + 1(2a-12) = 0$$

$$\Rightarrow 4 - a + 2 + 2a - 12 = 0$$

$$\Rightarrow a - 6 = 0$$

$$\Rightarrow a = 6$$

Thus, the coordinates of C are $(3,6)$

Thus, $BC = \sqrt{(3-2)^2 + (6-4)^2} = \sqrt{1+4} = \sqrt{5}$ units

Hence, option *(c)* is correct.

10. *(d)* The given series $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \cdots$ can be written as

$$1 + \left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^3 + \cdots$$

This is a GP with first term 1 and common ratio $-\frac{1}{2}$

So, the sum of the series is

$$= \frac{1}{1-\left(-\frac{1}{2}\right)} = \frac{1}{1+\frac{1}{2}} = \frac{2}{3}$$

Hence, option *(d)* is correct.

11. *(c)* No. of digits to be filled at one's place $=3$

No. of digits to be filled at 10's place $=5$

No. of digits to be filled at 100's place $=4$

$\therefore$ Total no. of digits formed $= 3\times5\times4 = 60$

If zero is at 100's place, then no. of digits to be filled at one's place $=2$

And no. of digits formed with zero at 10's place $=4$

$\therefore$ No. of digits formed with zero at 100's place $=1\times2\times4=8$

$\therefore$ Required no. of digits formed $60-8=52$

Hence, option *(c)* is correct.

12. *(c)* From the given data, we have

$n(U)=700, n(A)=200, n(B)=300, n(A\cap B)=100$

We know that,

$$n(A\cup B)=n(A)+n(B)-n(A\cap B)$$
$$=200+300-100$$
$$=400$$

Now, $n(A'\cap B')=n(A\cup B)'=\text{n}(U)-n(A\cup B)$
$$=700-400=300$$

Hence, option *(c)* is correct.

13. *(b)* Given that $y=x+e^x$

Differentiate both sides w.r.t. x

$$\frac{dy}{dx}=1+e^x \Rightarrow \frac{dx}{dy}=\frac{1}{1+e^x}$$

Again, differentiate w.r.t. y

$$\frac{d^2x}{dy^2}=\frac{-(1)(e^x)}{(1+e^x)^2}\cdot\frac{dx}{dy}=-\frac{e^x}{(1+e^x)^2}\cdot\frac{1}{1+e^x}$$
$$=-\frac{e^x}{(1+e^x)^3}$$

Hence, option *(b)* is correct.

14. *(c)* Given that $f(x)=\log\left(\frac{1+x}{1-x}\right)$

Therefore,

$$f\left(\frac{2x}{1+x^2}\right)=\log\left(\frac{1+\frac{2x}{1+x^2}}{1-\frac{2x}{1+x^2}}\right)=\log\left(\frac{1+x^2+2x}{1+x^2-2x}\right)$$
$$=\log\left(\frac{(1+x)^2}{(1-x)^2}\right)=\log\left(\frac{1+x}{1-x}\right)^2$$
$$=2\log\left(\frac{1+x}{1-x}\right)=2f(x)$$

Hence, option *(c)* is correct.

15. *(c)* We know from the Sine law that

$$\frac{a}{\sin A}=\frac{b}{\sin B}$$
$$\Rightarrow \frac{2b}{\sin 3B}=\frac{b}{\sin B}$$
$$\Rightarrow 2\sin B=\sin 3B$$
$$\Rightarrow 2\sin B=3\sin B-4\sin^3 B$$
$$\Rightarrow \sin B-4\sin^3 B=0$$
$$\Rightarrow \sin B(1-4\sin^2 B)=0$$
$$\Rightarrow \sin B=0 \text{ or } 1-4\sin^2 B=0 \Rightarrow B=0 \text{ or } B=30°$$
$$\Rightarrow B=30° \text{ and } A=3\times30°=90°$$
$$\Rightarrow B=0 \text{ is not possible, so } B=30° \text{ and } A=90°$$

$\therefore$ The triangle is right angled triangle.

Hence, option *(c)* is correct.

16. *(b)* Let the first instalment be Rs. x and difference of consecutive instalments be Rs. d

$$\Rightarrow \frac{30}{2}(2x+29d)=\frac{3600\times2}{3}$$

$\left(\because 1/3^{\text{rd}} \text{ amount is unpaid, } 2/3^{\text{rd}} \text{ amount is paid}\right)$

$$\Rightarrow 2x+29d=\frac{2400}{15}$$
$$\Rightarrow 2x+29d=160 \qquad ....(\text{i})$$

Since, total amount was 3600 and it was to be paid in 40 instalments

$\Rightarrow \frac{40}{2}(2x+39d)=3600$

$\Rightarrow 2x+39d=180$ ....(ii)

On solving equations (i) and (ii), we get

$x=51$ and $d=2$

$\therefore$ First instalment $=$ Rs. 51

Hence, option *(b)* is correct.

17. *(a)* Let α and β are the roots of the given equation.

$\therefore \alpha+\beta=\frac{-(3p+1)}{3}$

and $\alpha\beta=\frac{-(p+5)}{3}$

Now, $\frac{-(3p+1)}{3}=\frac{-(p+5)}{3}$

$\Rightarrow 3p+1=p+5$

$\Rightarrow 2p=4$

$\Rightarrow p=2$

Hence, option *(a)* is correct.

18. *(d)* Since, A and B are complementary angles, then $A+B=90°$.

Now, we know that

$\cos A\cos B=\cos A\cos(90°-A)$

$=\cos A\sin A=\frac{1}{2}\sin 2A$

Since, $-1\le\sin 2A\le 1$

Hence, $-\frac{1}{2}\le\frac{1}{2}\sin 2A\le\frac{1}{2}$

Thus, the greatest and least values of $\cos A\cos B$ are $\frac{1}{2}$ and $-\frac{1}{2}$.

Hence, option *(d)* is correct.

19. *(c)* The given binary numbers are

$10001100=1\times 2^7+0\times 2^6+0\times 2^5+0\times 2^4+1\times 2^3+1\times 2^2$

$0\times 2^1+0\times 2^0$

$=128+8+4$

$=140$ (decimal numbers)

And,

$1101101=1\times 2^6+1\times 2^5+0\times 2^4+1\times 2^3+1\times 2^2+0\times 2^1$

1×2^1

$=64+32+8+4+1$

$=109$

$\therefore$ Their difference $=140-109=31$

Hence, option *(c)* is correct.

20. *(b)* Equation of the given conic is an equation of ellipse

$\frac{x^2}{a^2+\lambda}+\frac{y^2}{b^2+\lambda}$ $(x\ge 0)$

$\Rightarrow A^2=a^2+\lambda^2$ and $B^2=b^2+\lambda^2$

Eccentricity, $e=\sqrt{1-\frac{B^2}{A^2}}=\sqrt{1-\frac{b^2+\lambda}{a^2+\lambda}}$

$=\sqrt{\frac{a^2+\lambda-b^2-\lambda}{a^2+\lambda}}=\sqrt{\frac{a^2-b^2}{a^2+\lambda}}$

Since, λ is in denominator, so when λ increases, the eccentricity decreases.

Hence, option *(b)* is correct.

21. *(a)* Let $I=\int\sqrt{x}e^{\sqrt{x}}dx$

Put $\sqrt{x}=t\Rightarrow\frac{1}{2\sqrt{x}}dx=dt\Rightarrow dx=2tdt$

$\therefore I=\int te^t\cdot 2tdt=2\int t^2e^tdt$

By parts, let first function is t^2 and second function is e^t. Then,

$I=2\left[t^2e^t-\int 2te^tdt\right]=2\left[t^2e^t-2\left\{te^t-\int e^tdt\right\}\right]$

$=2\left[t^2e^t-2te^t+2e^t\right]+c$

where c is constant of integration.

Since, $t=\sqrt{x}$, so we have

$I=2\left[xe^{\sqrt{x}}-2\sqrt{x}e^{\sqrt{x}}+2e^{\sqrt{x}}\right]+c=2e^{\sqrt{x}}\left(x-2\sqrt{x}+2\right)+c$

Hence, option *(a)* is correct.

22. *(b)* The given expansion is is $\left(x^2+\frac{2}{x}\right)^{15}$

$\therefore\ T_{r+1} = {}^{15}C_r \left(x^2\right)^{15-r}\left(\frac{2}{x}\right)^r$

$= {}^{15}C_r x^{30-2r}\cdot 2^r x^{-r}$

$= {}^{15}C_r x^{30-3r}\cdot 2^r$

Now, above term will be independent of x when $30 = 3r = 0 \Rightarrow r = 10$

$\therefore$ Term independent of $x = {}^{15}C_{10}2^{10}$

Now, coefficient of x^{15}

When $30-3r = 15 \Rightarrow r = 5$

$\therefore$ Required coefficient $= {}^{15}C_5 2^5$

Thus, required ratio $= \dfrac{{}^{15}C_5 2^5}{{}^{15}C_{10}2^{10}}$

$$= \frac{\frac{15!}{5!(10!)}}{\frac{15!}{10!(5!)}\times 2^5} = \frac{1}{2^5} = \frac{1}{32}$$

Hence, option *(b)* is correct.

23. *(d)* The number of diagonals $= \dfrac{n(n-3)}{2}$

where n is the number of sides of polygon.

$\therefore\quad 20 = \dfrac{n(n-3)}{2}$

$\Rightarrow\quad 40 = n^2 - 3n$

$\Rightarrow\quad n^2 - 3n - 40 = 0$

$\Rightarrow\quad n^2 - 8n + 5n - 40 = 0$

$\Rightarrow\quad (n-8)(n+5) = 0$

$\Rightarrow\quad n-8 = 0$ or $n+5 = 0$

Since, the number of diagonals cannot be negative.

$\Rightarrow\quad n = 8$

Hence, option *(d)* is correct.

24. *(a)* Given that the line is $ax\cos\phi + by\sin\phi - ab = 0$.

Let d_1 be the perpendicular distance from $\left(\sqrt{b^2-a^2},0\right)$ to the line $ax\cos\phi + by\sin\phi - ab = 0$ and d_2 from $\left(-\sqrt{b^2-a^2},0\right)$ to the line $ax\cos\phi + by\sin\phi - ab = 0$.

At point $\left(\sqrt{b^2-a^2},0\right)$

$$d_1 = \frac{a\sqrt{b^2-a^2}\cos\phi - ab}{\sqrt{a^2\cos^2\phi + b^2\sin^2\phi}}$$

At point $\left(-\sqrt{b^2-a^2},0\right)$

$$d_2 = \frac{-a\sqrt{b^2-a^2}\cos\phi - ab}{\sqrt{a^2\cos^2\phi + b^2\sin^2\phi}}$$

$$\therefore d_1 d_2 = -\frac{\left[a^2\left(b^2-a^2\right)\cos^2\phi - a^2b^2\right]}{\sqrt{a^2\cos^2\phi + b^2\sin^2\phi}}$$

$$= -\frac{\left[a^2\left(-b^2\sin^2\phi - a^2\cos^2\phi\right)\right]}{\sqrt{a^2\cos^2\phi + b^2\sin^2\phi}} = a^2$$

Hence, option *(a)* is correct.

25. *(b)* Let $I = \int_0^1 x(1-x)^n\,dx$

Put $1-x = t \Rightarrow dx = -dt$

when $x = 0$ then $t = 1$

when $x = 1$ then $t = 0$

$\therefore I = -\int_1^0 (1-t)t^n dx = \int_0^1 \left(t^n - t^{n-1}\right)dx$

$$= \left[\frac{t^{n+1}}{n+1} - \frac{t^{n+2}}{n+2}\right]_0^1$$

$$= \frac{1}{n+1} - \frac{1}{n+2} = \frac{1}{(n+1)(n+2)}$$

Hence, option *(b)* is correct.

26. *(c)* Let $P(15,n-1):P(16,n-2) = 3:4$

$\Rightarrow \dfrac{{}^{15}P_{n-1}}{{}^{16}P_{n-2}} = \dfrac{3}{4}$

$\Rightarrow \dfrac{15!}{(15-n+1)!}\times\dfrac{(16-n+2)!}{16!} = \dfrac{3}{4}$

$\Rightarrow \dfrac{15!}{(16-n)!}\times\dfrac{(18-n)!}{16!} = \dfrac{3}{4}$

$\Rightarrow \frac{(18-n)!}{16(16-n)!}=\frac{3}{4}$

$\Rightarrow \frac{(18-n)(17-n)(16-n)!}{16(16-n)!}=\frac{3}{4}$

$\Rightarrow (18-n)(17-n)=12$

$\Rightarrow 306-17n-18n+n^2=12$

$\Rightarrow n^2-35n+294=0$

$\Rightarrow (n-14)(n-21)=0$

$\Rightarrow n=14 \qquad (\because n\neq 21)$

Hence, option *(c)* is correct.

27. *(a)* We have $\sqrt{i}=\sqrt{\frac{2i}{2}}=\sqrt{\frac{1+2i-1}{2}}$

$=\sqrt{\frac{1+2i+i^2}{2}}=\sqrt{\frac{(1+i)^2}{2}}=\frac{1+i}{\sqrt{2}}$

And, $\sqrt{-i}=\sqrt{-\frac{2i}{2}}=\sqrt{\frac{1-2i-1}{2}}$

$=\sqrt{\frac{1-2i+i^2}{2}}=\sqrt{\frac{(1-i)^2}{2}}=\frac{1-i}{\sqrt{2}}$

Therefore, we have

$\sqrt{i}+\sqrt{-i}=\frac{1+i}{\sqrt{2}}+\frac{1-i}{\sqrt{2}}=\frac{1+i+1-i}{\sqrt{2}}=\frac{2}{\sqrt{2}}=\sqrt{2}$

Hence, option *(a)* is correct.

28. *(c)* Let the geometric progression be $a, ar, ar^2, ar^3, \ldots$ with common ratio r and the first term a.

According to the question, we have

$a+ar=8 \Rightarrow a(1+r)=8 \qquad \ldots\text{(i)}$

And $a+ar+ar^2+ar^3=80$

$\Rightarrow a(1+r)+ar^2(1+r)=80$

$\Rightarrow a(1+r)(1+r^2)=80$

$\Rightarrow 8(1+r^2)=80 \qquad \{\text{from (i)}\}$

$\Rightarrow 1+r^2=10$

$\Rightarrow r^2=10-1=9$

$\Rightarrow r=3 \ (\because r>0)$

From equation (i), we have

$a(1+3)=8 \Rightarrow a=2$

Now, 6^{th} term $=ar^5=2(3)^5=2\times 243=486$

Hence, option *(c)* is correct.

29. *(c)* Given that $(x+y)(dx-dy)=dx+dy$

Dividing by dx on both the sides

$(x+y)\left(1-\frac{dy}{dx}\right)=1+\frac{dy}{dx}$

Putting $x+y=v$

$1+\frac{dy}{dx}=\frac{dv}{dx}$ and $\frac{dy}{dx}=\frac{dv}{dx}-1$

The equation changes to

$\Rightarrow v\left\{1-\left(\frac{dy}{dx}-1\right)\right\}=\frac{dv}{dx}$

$\Rightarrow v\left(2-\frac{dy}{dx}\right)=\frac{dv}{dx}$

$\Rightarrow 2v-v\frac{dy}{dx}=\frac{dv}{dx}$

$\Rightarrow 2v=(1+v)\frac{dv}{dx}$

$\Rightarrow \left(\frac{1+v}{v}\right)dv=2dx$

$\Rightarrow \left(\frac{1}{v}+1\right)dv=2dx$

Integrate on both the sides, we have

$\int\frac{dv}{v}+\int dv=2\int dx+c$

$\ln v+v=2x+c$

Putting $v=x+y$

$\ln(x+y)+(x+y)=2x+c$

$\ln(x+y)+y-x=c$

$y-x+\ln(x+y)=c$

Hence, option *(c)* is correct.

30. *(a)* Consider $\cos B=\frac{\sin A}{2\sin C}$

$$\Rightarrow \frac{c^2+a^2-b^2}{2ac}=\frac{a}{2c}\quad\left(\because \frac{a}{\sin A}=\frac{c}{\sin C}\Rightarrow\frac{\sin A}{\sin C}=\frac{a}{c}\right)$$

$$\Rightarrow c^2+a^2-b^2=a^2$$

$$\Rightarrow c^2-b^2=0$$

$$\Rightarrow c=b$$

Thus, ΔABC is an isosceles triangle.

Hence option (a) is correct.

31. *(d)* Since $3\in A$ but $(3,3)\notin R$

So, it is not reflexive

and, $(3,4)\in R$ and $(4,3)\in R$ but $(3,3)\notin R$

So, it is not transitive.

Thus, $(4,3)\in R$ is neither reflexive not transitive

Hence, option *(d)* is correct.

32. *(b)* Consider the integral $\int \sin x\log(\tan x)dx$

$$=-\cos x\log\tan x-\int(-\cos x)\frac{1}{\tan x}\cdot\sec^2 x dx$$

$$=-\cos x\log\tan x+\int\frac{1}{\sin x}dx$$

$$=-\cos x\log\tan x+\int\frac{1+\tan^2\frac{x}{2}}{2\tan\frac{x}{2}}dx$$

Let $t=\tan\frac{x}{2}\Rightarrow\frac{dx}{dt}=\frac{2}{1+t^2}\Rightarrow dx=\frac{2}{1+t^2}\cdot dt$

So, $-\cos x\log\tan x+\int\frac{1+\tan^2\frac{x}{2}}{2\tan\frac{x}{2}}dx$

$$=-\cos x\log\tan x+\int\frac{1+t^2}{2t}\cdot\frac{2}{1+t^2}dt$$

$$=-\cos x\log\tan x+\int\frac{1}{t}dt$$

$$=-\cos x\log\tan x+\log(t)+c$$

$$=-\cos x\log\tan x+\log\tan\left(\frac{x}{2}\right)+c$$

Hence, option *(b)* is correct.

33. *(d)* We have $\operatorname{cosec}\left(\frac{13\pi}{12}\right)=\operatorname{cosec}\left(\pi+\frac{\pi}{12}\right)$

$$=-\operatorname{cosec}\left(\frac{\pi}{12}\right)=-\operatorname{cosec}15^\circ$$

$$=-\sqrt{1+\cot^2 15^\circ}$$

$$=-\sqrt{1+\left(2+\sqrt{3}\right)^2}\qquad\left[\because\cot 15^\circ=2+\sqrt{3}\right]$$

$$=-\sqrt{1+4+3+4\sqrt{3}}$$

$$=-\sqrt{8+4\sqrt{3}}\;=-\sqrt{6+2+2\sqrt{12}}$$

$$=-\sqrt{\left(\sqrt{6}\right)^2+\left(\sqrt{2}\right)^2+2\sqrt{6}\sqrt{2}}$$

$$=-\sqrt{\left(\sqrt{6}+\sqrt{2}\right)^2}$$

$$=-\sqrt{6}-\sqrt{2}$$

Hence, option *(d)* is correct.

34. *(b)* A line passes through $(2,2)$ and is perpendicular to the line $3x+y=3$.

Slope of the line $3x+y=3$ is -3

Slope of the line which passes through $(2,2)$ is $\frac{1}{3}$.

$\therefore$ Equation of line passes through $(2,2)$ and having slope $\frac{1}{3}$ is

$$(y-2)=\frac{1}{3}(x-2)$$

$$\Rightarrow 3y-6=x-2$$

$$\Rightarrow x+3y+4=0$$

In order to find the y-intercept of the line, put $x=0$

$\therefore\ -3y=-4\ \Rightarrow\ y=\frac{4}{3}$

Hence, option *(b)* is correct.

35. *(d)* Let $A=\{4n+2\mid n\in N\}$ and $B=\{3n\mid n\in N\}$

$\Rightarrow\ A=\{6,10,14,18,22,26,30,34,38,42,....\}$

and $B=\{3,6,9,12,15,18,21,24,27,30,....\}$

$\therefore A \cap B = \{6, 18, 30, 42, ...\}$

$= 6 + 12n - 12$

$= 12n - 6$

Thus, $\{12n - 6 \mid n \text{ is a natural number}\}$

Hence, option *(d)* is correct.

36. *(b)* Given curve is $x = e^x y$

This can be rewritten as $y = xe^{-x}$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = -xe^{-x} + e^{-x}$$

Put $\frac{dy}{dx} = 0$ for maxima or minima

$\Rightarrow -xe^{-x} + e^{-x} = 0$

$\Rightarrow e^{-x}(1 - x) = 0$

Since, e^{-x} cannot be zero,

$\therefore 1 - x = 0 \Rightarrow x = 1$

Now, $\frac{d^2y}{dx^2} = -e^{-x} + xe^{-x} - e^{-x} = xe^{-x} - 2e^{-x} = e^{-x}(x - 2)$

$\Rightarrow \left(\frac{d^2y}{dx^2}\right) < 0$

$\therefore y$ is maximum at $x = 1$

Thus, when $x = 1$ then $y = e^{-1}$

Hence, maximum point on the curve $x = e^x y$ is $(1, e^{-1})$.

Hence, option *(b)* is correct.

37. *(d)* Number between 100 and 1000 are 3-digit numbers.

It is given that the digits should not be repeated.

Number of given digits $= 5$

In a 3-digit number, first number can be arranged in 5 ways.

Second number is 4 ways.

Third number is 3 ways.

$\therefore$ Numbers that can be formed $= 5 \times 4 \times 3 = 60$

Hence, option *(d)* is correct.

38. *(b)* Given equation of circle is $x^2 + y^2 = 2$

$\Rightarrow y = \sqrt{2 - x^2}$

Required area $= 4 \times$ area of shaded portion

$$= 4\int_0^{\sqrt{2}} \sqrt{2 - x^2}\, dx \qquad ....(i)$$

Let $x = \sqrt{2}\sin t \Rightarrow t = \sin^{-1}\left(\frac{x}{\sqrt{2}}\right)$

Then $dx = \sqrt{2}\cos t\,dt$

$\therefore \int \sqrt{2 - x^2}\,dx = \int \sqrt{2 - 2\sin^2 t} \cdot \sqrt{2}\cos t\,dt$

$= \int \sqrt{2\cos^2 t} \cdot \sqrt{2}\cos t\,dt$

$= \int 2\cos^2 t\,dt$

We know that

$$\int \cos^n x\,dx = \frac{n-1}{n}\int \cos^{n-2} x\,dx + \frac{\cos^{n-1} x \cdot \sin x}{n}$$

$\therefore \int \sqrt{2 - x^2}\,dx = 2\left[\frac{\cos t \cdot \sin t}{2} + \frac{1}{2}\int 1 \cdot t\,dt\right]$

$= 2\left[\frac{\cos t \sin t}{2} + \frac{t}{2}\right]$

$= \cos t \sin t + t$

$= \cos\left(\sin^{-1}\frac{x}{\sqrt{2}}\right)\sin\left(\sin^{-1}\frac{x}{\sqrt{2}}\right) + \sin^{-1}\frac{x}{\sqrt{2}}$

$= \sqrt{1 - \frac{x^2}{2}} \cdot \frac{x}{\sqrt{2}} + \sin^{-1}\frac{x}{\sqrt{2}}$

$= \frac{x}{2}\sqrt{2 - x^2} + \sin^{-1}\frac{x}{\sqrt{2}}$

$\therefore 4\int_0^{\sqrt{2}} \sqrt{2 - x^2}\,dx = 4\left[\frac{x}{2}\sqrt{2 - x^2} + \sin^{-1}\frac{x}{\sqrt{2}}\right]_0^{\sqrt{2}}$

$= 4\left[0 + \sin^{-1}\frac{\sqrt{2}}{\sqrt{2}} - 0 - \sin^{-1}\frac{0}{\sqrt{2}}\right]$

$= 4\left[\sin^{-1} 1 - \sin^{-1} 0\right]$

$= 4\left(\frac{\pi}{2} - 0\right)$

$= 2\pi$

Hence, option *(b)* is correct.

39. *(c)* The given differential equation is

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}=k\frac{d^2y}{dx^2}$$

To express it as a polynomial of derivatives we square both sides,

$$\Rightarrow\left[1+\left(\frac{dy}{dx}\right)^2\right]^3=k\left(\frac{d^2y}{dx^2}\right)^2$$

Highest derivative has power $=2$

Degree of differential equation $=2$

Hence, option *(c)* is correct.

40. *(c)* Given that $(x-2a)(x-2b)+(y-2c)(y-2d)=0$

$$\Rightarrow x^2-2(a+b)x+4ab+y^2-2(c+d)y+4cd=0$$

$$\Rightarrow x^2+y^2-2(a+b)x-2(c+d)y+4(ab+cd)=0$$

From the general equation of circle, we have

$$x^2+y^2+2gx+2fy+c=0$$

Centre $(-g,-f)$ and radius $=\sqrt{(-g)^2+(-f)^2+c}$

Therefore, here $g=-(a+b)$ and $f=-(c+d)$

Hence, centre $(-g,-f)=(a+b,c+d)$

Hence, option *(c)* is correct.

41. *(a)* We have given $\log_{10}(x+1)+\log_{10}5=3$

$$\Rightarrow \log_{10}5(x+1)=3 \quad (\because \log m+\log n=\log mn)$$

$$\Rightarrow 5(x+1)=10^3$$

$$\Rightarrow (x+1)=\frac{1000}{5}$$

$$\Rightarrow x=200-1$$

$$\Rightarrow x=199$$

Hence, option *(a)* is correct.

42. *(c)* Since, r and s are the roots of $Ax^2+Bx+C=0$, then

$$\therefore\ r+s=-\frac{B}{A} \text{ and } rs=\frac{C}{A}$$

Now, given roots of $=\frac{n(n-3)}{2}$ be $=\frac{n(n-3)}{2}$

$$\therefore\ r^2+s^2=-p \text{ and } r^2s^2=q$$

$$\Rightarrow (r+s)^2-2rs=-p$$

$$\Rightarrow\left(-\frac{B}{A}\right)^2-2\left(\frac{C}{A}\right)=-p$$

$$\Rightarrow \frac{B^2}{A^2}-\frac{2C}{A}=-p$$

$$\Rightarrow \frac{B^2-2AC}{A^2}=-p$$

$$\Rightarrow p=\frac{2AC-B^2}{A^2}$$

Hence, option *(c)* is correct.

43. *(c)* Given that $z=1+\cos\frac{\pi}{5}+i\sin\frac{\pi}{5}$

Therefore, we have

$$|z|=\sqrt{1^2+\cos^2\frac{\pi}{5}+2\cos\frac{\pi}{5}+\sin^2\frac{\pi}{5}}$$

$$=\sqrt{1+1+2\cos\frac{\pi}{5}}$$

$$=\sqrt{2\left(1+\cos\frac{\pi}{5}\right)}$$

$$=\sqrt{2\left(1+2\cos^2\frac{\pi}{10}-1\right)}$$

$$=\sqrt{2\left(2\cos^2\frac{\pi}{10}\right)}=2\cos\frac{\pi}{10}$$

Hence, option *(c)* is correct.

44. *(c)* The given equation is $4\left(\sin^2x+\cos x\right)=1$

$$\Rightarrow\ 4\sin^2x+4\cos x=1$$

$$\Rightarrow\ 4\sin^2x+4\cos x-1=0$$

$$\Rightarrow\ 4\left(1-\cos^2x\right)+4\cos x-1=0$$

$$\Rightarrow\ 4-4\cos^2x+4\cos x-1=0$$

$$\Rightarrow\ -4\cos^2x+4\cos x+3=0$$

$$\Rightarrow\ 4\cos^2x-4\cos x-3=0$$

This is a quadratic equation in $\cos x$

$\Rightarrow\ 4\cos^2 x - 6\cos x + 2\cos x - 3 = 0$

$\Rightarrow\ (2\cos x - 3)(2\cos x + 1) = 0$

$\Rightarrow\ \cos x = \frac{3}{2}$ and $\cos x = -\frac{1}{2}$

But $\cos x = \frac{3}{2}$ is not possible, therefore, $\cos x = -\frac{1}{2}$

$\Rightarrow \cos A = -\frac{1}{2} = \cos 210°$

$\Rightarrow A = 210°$

Hence, option *(c)* is correct.

45. *(b)* The given differential equation is

$3e^x \tan y dx + (1+e^x)\sec^2 y dy = 0$

By separating the variables, we get

$$3e^x dx = \frac{-(1+e^x)\sec^2 y}{\tan y} dy$$

$$\Rightarrow \frac{3e^x}{1+e^x} dx = -\frac{\sec^2 y}{\tan y} dy$$

Integrate on both the sides, we have

$$\Rightarrow \int \frac{3e^x}{1+e^x} dx + \int \frac{\sec^2 y}{\tan y} dy = 0$$

$\Rightarrow 3\log(1+e^x) + \log\tan y = \log c$

$\Rightarrow \log(1+e^x)^3 \tan y = \log c \quad \{\because \log m + \log n = \log mn\}$

$\Rightarrow (1+e^x)^3 \tan y = c$

Hence, option *(b)* is correct.

46. *(a)* Consider the following $\lim\limits_{x\to 0} \frac{\sin^2 ax}{bx}$

Multiply and divide by $= \log\left(\frac{(1+x)^2}{(1-x)^2}\right)$

$$\Rightarrow \lim_{x\to 0} \frac{\sin^2 ax}{bx} = \lim_{x\to 0}\left[\frac{\sin^2 ax}{bx} \times \frac{a^2x^2}{a^2x^2}\right]$$

$$= \lim_{x\to 0}\left[\frac{\sin^2 ax}{a^2x^2} \times \frac{a^2x^2}{bx}\right]$$

$$= \lim_{x\to 0}\left[\left(\frac{\sin ax}{ax}\right)^2 \times \frac{a^2 x}{b}\right]$$

$$= \lim_{x\to 0}\left(\frac{\sin ax}{ax}\right)^2 \cdot \lim_{x\to 0}\frac{a^2x}{b}$$

$$= 1\times 0 \qquad \left(\because \lim_{x\to 0}\frac{\sin x}{x} = 1\right)$$

$= 0$

Hence, option *(a)* is correct.

47. *(c)* Number of 5 digits numbers with all distinct digit is same as filling of 5 vacant placed out of 10 boxes.

First digit of any number can be chosen in 9 ways.

Remaining 4 digits can be chosen in remaining 9 digits in ${}^9P_4$ ways.

$\therefore$ Total no. of such number $9\times 9\times 8\times 7\times 6 = 27216$

Hence, option *(c)* is correct.

48. *(a)* Given that $\frac{x}{a} + \frac{y}{b} = 1$ ....(i)

and $\frac{x}{b} + \frac{y}{a} = 1$ ....(ii)

From solving equations (i) and (ii), we get the intersection point.

$bx + ay = ax + by$

$\Rightarrow (b-a)x = (b-a)y$

$\therefore\ x = y$ ....(iii)

$\Rightarrow \frac{x}{a} + \frac{y}{b} = 1$

$\therefore x(a+b) = ab$

$\therefore x = \frac{ab}{a+b}$ and $y = \frac{ab}{a+b}$ from equation (iii)

Hence, option *(a)* is correct.

49. *(d)* Let a be the first term and ar be the second term of GP with common ratio r.

Given that $S_\infty = 6$ and $a + ar = \frac{9}{2} \Rightarrow \frac{a}{1-r} = 6$

$\Rightarrow a = 6(1-r)$ ....(i)

And $a+ar=\frac{9}{2}$

$\Rightarrow 6(1-r)+6r(1-r)=\frac{9}{2}$ [from (i)]

$\Rightarrow 12-12r+12r-12r^2=9$

$\Rightarrow r^2=\frac{3}{12}=\frac{1}{4} \Rightarrow r=\frac{1}{2}$ or $-\frac{1}{2}$

$\Rightarrow a=3$ or 9

Hence, option *(d)* is correct.

50. *(d)* $xRy \Leftrightarrow x$ and y are graduates of the same university

Reflexive $xRx \Leftrightarrow x$ and x are graduates of the same university.

$\therefore$ Relation is reflexive.

Symmetric $xRy \Leftrightarrow x$ and y are graduates of the same university

$\Rightarrow yRx \Leftrightarrow y$ and x are graduates of the same university.

$\therefore$ Relation is symmetric.

Transitive $xRy, yRz \Leftrightarrow xRz$

It means x and y, y and z are graduates of the same university, then x and z are also graduates of the same university.

$\therefore$ Relation is transitive.

Thus, relation is reflexive, symmetric and transitive.

Hence, option *(d)* is correct.

51. *(d)* Let $f(x)=\frac{\sin^5 x\cos^3 x}{x^4}$

Then,

$$f(-x)=\frac{\sin^5(-x)\cos^3(-x)}{(-x)^4}$$

$$=\frac{-\sin^5 x\cos^3 x}{x^4}$$

$$=-f(x)$$

$\therefore f(x)$ is an odd function.

Thus, $\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\frac{\sin^5 x\cos^3 x}{x^4}dx=0$

Hence, option *(d)* is correct.

52. *(c)* Let ABC is an equilateral triangle with $A(0,0)$ and $B(3,\sqrt{3})$ and C to be known.

$\therefore AB=\sqrt{(3-0)^2+(\sqrt{3}-0)^2}=\sqrt{9+3}=\sqrt{12}$

Take option (a) i.e. $C(0,2\sqrt{3})$

$CA=\sqrt{(0)^2+(2\sqrt{3})^2}=\sqrt{12}$

$CB=\sqrt{(3)^2+(\sqrt{3})^2}=\sqrt{12}$

Take option (b) i.e. $C(3,-\sqrt{3})$

$CA=\sqrt{(3)^2+(\sqrt{3})^2}=\sqrt{12}$

$CB=\sqrt{(0)^2+(2\sqrt{3})^2}=\sqrt{12}$

$\therefore$ Both options (a) and (b) are correct.

Hence, option *(c)* is correct.

53. *(c)* Let $\sin\alpha$ and $\cos\alpha$ be the roots of $ax^2+bx+c=0$.

Now, $\sin\alpha+\cos\alpha=\frac{-b}{a}$ and $\sin\alpha\cos\alpha=\frac{c}{a}$

Consider $\sin\alpha+\cos\alpha=\frac{-b}{a}$

Squaring both sides, $(\sin\alpha+\cos\alpha)^2=\frac{b^2}{a^2}$

$\Rightarrow \sin^2\alpha+\cos^2\alpha+2\sin\alpha\cos\alpha=\frac{b^2}{a^2}$

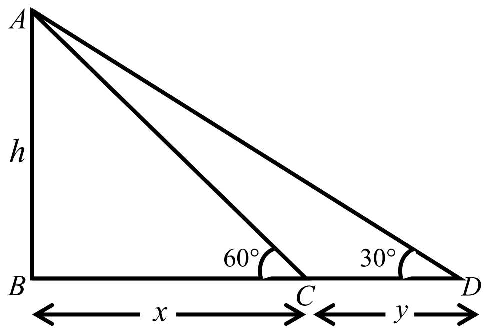
$\Rightarrow 1+\frac{2c}{a}=\frac{b^2}{a^2}$

$\Rightarrow \frac{a+2c}{a}=\frac{b^2}{a^2} \Rightarrow a+2c=\frac{b^2}{a}$

$\Rightarrow a^2+2ac=b^2 \Rightarrow b^2-a^2=2ac$

Hence, option *(c)* is correct.

54. *(b)* Let AB be the tower and AC and AD be the shadows of the tower. Let h be the height of the tower.



In ΔABC, $\tan 60° = \frac{h}{x}$ ....(i)

In ΔABD, $\tan 30° = \frac{h}{x+50}$ ....(ii)

$(i) \div (ii) \Rightarrow \frac{\sqrt{3}}{\frac{1}{\sqrt{3}}} = \frac{x+50}{x}$

$\Rightarrow 3x = x + 50$

$\Rightarrow x = 25$

$\therefore\ h = x\sqrt{3} = 25\sqrt{3}$

Hence, option *(b)* is correct.

55. *(b)* Let $N =$ National savings certificates and $S =$ Shares

Total number of persons $= 32$

No. of persons who invest in National savings certificates $= 30$

No. of persons who invest in shares $= 17$

Therefore, $n(N \cup S) = 32, n(N) = 30, n(S) = 17$

We know that,

$n(N \cup S) = n(N) + n(S) - n(N \cap S)$

$\Rightarrow 32 = 30 + 17 - n(N \cap S)$

$\Rightarrow n(N \cap S) = 47 - 32 = 15$

Hence, option *(b)* is correct.

56. *(c)* Given that $P(n,r) = 2520$ and $C(n,r) = 21$

${}^nP_r = \frac{n!}{(n-r)!} = 2520$ ....(i)

${}^nC_r = \frac{n!}{r!(n-r)!} = 21$ ....(ii)

Divide equation (i) by (ii), we have

$\Rightarrow r! = \frac{2520}{21} = 120$

$\Rightarrow r! = 5! \Rightarrow r = 5$

$\because {}^nC_r = 21 \Rightarrow {}^nC_5 = 21 \Rightarrow n = 7$

$\therefore C(n+1, r+1) = C(8,6) = \frac{8!}{6!(8-6)!} = \frac{8 \times 7}{2} = 28$

Hence, option *(c)* is correct.

57. *(c)* Given, angles of triangle are in ratio $1:2:3$

Consider, $A = 30°, B = 60°$ and $C = 90°$

We know, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$\Rightarrow \frac{a}{\sin 30°} = \frac{b}{\sin 60°} = \frac{c}{\sin 90°}$

$\Rightarrow \frac{a}{\frac{1}{2}} = \frac{b}{\frac{\sqrt{3}}{2}} = \frac{c}{1}$

$\Rightarrow a:b:c = \frac{1}{2} : \frac{\sqrt{3}}{2} : 1 = 1 : \sqrt{3} : 2$

Hence, option *(c)* is correct.

58. *(a)* Let $I = \int \ln(x^2)\,dx$

$= 2\int 1 \cdot \ln(x)\,dx$ $\{\because \ln(m^n) = n\ln(m)\}$

$= 2\left[\ln x \cdot x - \int \frac{1}{x} \cdot x dx\right]$ {using by parts}

$= 2[x \ln x - x] + c$

$= 2x \ln x - 2x + c$

Hence, option *(a)* is correct.

59. *(a)* The general equation of all parabolas where axes are parallel to y-axis is

$y = Ax^2 + Bx + C$ ....(i)

where A, B and C are arbitrary constants.

On differentiating equation (i) w.r.t. x, we get

$$\frac{dy}{dx} = 2Ax + B \qquad ....(ii)$$

On differentiating equation (ii) w.r.t. x, we get

$$\frac{d^2y}{dx^2} = 2A \qquad ....(iii)$$

On differentiating equation (iii) w.r.t. x, we get

$$\frac{d^3y}{dx^3} = 0$$

Hence, option *(a)* is correct.

60. *(a)* The line $4x + y = 4$ can be written as $y = -4x + 4$

So, slope is -4

The line parallel to $4x + y = 4$ will have slope -4 only.

Given point $= (1,3)$

Equation of line passing through $(1,3)$ and slope -4 is

$$y - 3 = -4(x - 1)$$

$$\Rightarrow y - 3 = -4x + 4$$

$$\Rightarrow 4x + y = 7$$

Solving the two equations, we get

$x = \frac{3}{2}$ and $y = 1$

The distance between the points $(1,3)$ and $\left(\frac{3}{2}, 1\right)$ is

$$= \sqrt{\left(\frac{3}{2} - 1\right)^2 + (1-3)^2} = \sqrt{\left(\frac{1}{2}\right)^2 + (-2)^2} = \sqrt{\frac{1}{4} + 4} = \frac{\sqrt{17}}{2}$$

Hence, option *(d)* is correct.

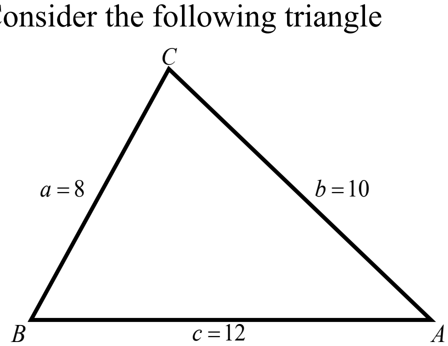
61. *(c)* Let $A = \{a, b, c\} \Rightarrow O(A) = 3$

Now, number of proper subsets of

$A = 2^{O(A)} - 1 = 2^3 - 1 = 7$

Hence, option *(c)* is correct.

62. *(b)* Consider the following triangle



We know from the cosine law that, we have

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{64 + 100 - 144}{2(8)(10)} = \frac{1}{8}$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{100 + 144 - 64}{2(10)(12)} = \frac{3}{4}$$

$\sin C = \frac{3\sqrt{7}}{8}$ and $\sin A = \frac{\sqrt{7}}{4}$

$$\frac{\cos C}{\cos A} = \frac{1}{6} < 1 \Rightarrow \cos C < \cos A \Rightarrow C > A$$

$$\cos(C - A) = \cos C \cos A + \sin C \sin A$$

$$= \frac{1}{8} \times \frac{3}{4} + \frac{3\sqrt{7}}{8} \times \frac{\sqrt{7}}{4} = \frac{3}{4} = \cos A$$

$$\Rightarrow C - A = A \Rightarrow C = 2A$$

Hence, option *(b)* is correct.

63. *(c)* Given matrix is $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

So, $A^2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix} = 3\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

and

$$A^3 = 3\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= 3\begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix} = 9\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = 3^2 A$$

similarly $A^4 = 3^3 A$. Hence $A^n = 3^{n-1} A$

Hence, option *(c)* is correct.

64. *(b)* Consider the following quotient

$$\frac{1+x+iy}{1+x-iy}=\frac{(1+x+iy)(1+x+iy)}{(1+x-iy)(1+x+iy)} \quad \{\text{by rationalizing}\}$$

$$=\frac{(1+x)^2+iy(1+x)+iy(1+x)-y^2}{1+x^2+2x+y^2} \quad \{\because i^2=-1\}$$

$$=\frac{1+x^2+2x-y^2+2iy(1+x)}{2(1+x)} \quad \{\because x^2+y^2=1\}$$

$$=\frac{1-y^2+2x+x^2+2iy(1+x)}{2(1+x)}$$

$$=\frac{2x^2+2x+2iy(1+x)}{2(1+x)}$$

$$=x+iy \quad \{\because 1-y^2=x^2\}$$

Hence, option *(b)* is correct.

65. *(a)* Let the required vector be $\hat{i}+\hat{j}$

Since, the vector $\hat{i}+\hat{j}$ is equally inclined to the vectors $\hat{i}+3\hat{j}$ and $3\hat{i}+\hat{j}$. Therefore,

Angle between $\hat{i}+\hat{j}$ and $\hat{i}+3\hat{j}=\theta_1$ is equal to the angle between $\hat{i}+\hat{j}$ and $3\hat{i}+\hat{j}=\theta_2$

$\therefore$ Angle between $\hat{i}+\hat{j}$ and $\hat{i}+3\hat{j}$

$$=\cos^{-1}\left[\frac{(1)(1)+(1)(3)}{\sqrt{(1)^2+(1)^2}\sqrt{(1)^2+(3)^2}}\right]$$

$$=\cos^{-1}\left[\frac{1+3}{\sqrt{2}\sqrt{10}}\right]$$

$$=\cos^{-1}\left[\frac{4}{\sqrt{20}}\right]=\cos^{-1}\left[\frac{2}{\sqrt{5}}\right]$$

And, angle between $\hat{i}+\hat{j}$ and $3\hat{i}+\hat{j}$

$$=\cos^{-1}\left[\frac{(1)(2)+(1)(1)}{\sqrt{(1)^2+(3)^2}\sqrt{(1)^2+(1)^2}}\right]$$

$$=\cos^{-1}\left[\frac{1+3}{\sqrt{10}\sqrt{2}}\right]$$

$$=\cos^{-1}\left[\frac{4}{\sqrt{20}}\right]=\cos^{-1}\left[\frac{2}{\sqrt{5}}\right]$$

Hence, required vector is $\hat{i}+\hat{j}$

Hence, option *(a)* is correct.

66. *(d)* Given equations are $x^2+kx+64=0$ ....(i)

and $x^2-8x+k=0$ ....(ii)

Since, both equations have real roots, discriminant ≥ 0

$\Rightarrow b^2 \geq 4ac$

From equation (i), we have

$k^2 \geq 4(64) \Rightarrow k^2 \geq 256 \Rightarrow k \geq 16$ ....(A)

And from equation (ii), we have

$64 \geq 4k \Rightarrow 4k \leq 64 \Rightarrow k \leq 16$ ....(B)

Hence, from equation (A) and (B), we have

$k=16$

Hence, option *(d)* is correct.

67. *(c)* The given system of equations are:

$p^2x+(p+1)^3y=(p+2)^3$ ....(i)

$px+(p+1)y=p+2$ ....(ii)

$x+y=1$ ....(iii)

This system is consistent, if values of x and y from first two equation satisfy the third equation.

which $\Rightarrow \begin{vmatrix} p^3 & (p+1)^3 & (p+2)^3 \\ p & (p+1) & (p+2) \\ 1 & 1 & 1 \end{vmatrix}=0$

Applying $C_2 \to C_2-C_1$

$$\Rightarrow \begin{vmatrix} p^3 & (p+1)^3-p^3 & (p+2)^3-p^3 \\ p & 1 & 2 \\ 1 & 0 & 0 \end{vmatrix} = 0$$

$\Rightarrow 2(p+1)^3-2p^3-(p+2)^3+p^3=0$

$\Rightarrow 2(p^3+1+3p^2+3p)-2p^3-(p^3+8+12p+6p^2)+p^3=$

$\Rightarrow 2p^3+2+6p^2+6p-2p^3-p^3-8-12p-6p^2+p^3=0$

$\Rightarrow -6-6p=0$

$\Rightarrow p=-1$

Hence, option *(c)* is correct.

68. *(c)* $S=$ Set of all integers and

$R=\{(a,b),\ a,b\in S \text{ and } ab\geq 0\}$

For reflexive:

$aRa \Rightarrow a.a=a^2\geq 0$ for all integers $a.a\geq 0$

For symmetric:

$aRb \Rightarrow ab\geq 0\ \forall\ a,b\in S$

If $ab\geq 0$, then $ba\geq 0 \Rightarrow bRa$

For transitive:

If $ab\geq 0, bc\geq 0$, then also $ac\geq 0$

Relation R is reflexive, symmetric and transitive. Therefore, relation is equivalence.

Hence, option *(c)* is correct.

69. *(a)* **Statement 1:**

A non-leap year has 365 days. i.e., 52 weeks and 1 day.

1 day can be {Sunday}, {Monday}, {Tuesday}, {Wednesday}, {Thursday}, {Friday}, {Saturday}.

In total, there are 7 possibilities and 1 possibility is Sunday.

$\therefore$ Required probability $=\dfrac{1}{7}$

A leap year has 366 days. i.e., 52 weeks and 2 days.

2 days can be {Sun, Mon}, {Mon, Tue}, {Tue, Wed}, {Wed, Thu}, {Thu, Fri}, {Fri, Sat}, {Sat, Sun}.

In total, there are 7 possibilities and 2 possibilities have Sundays.

$\therefore$ Required probability $=\dfrac{2}{7}$

So, statement 1 is correct.

Statement 2:

March has 31 days. i.e., 4 complete weeks and 3 days.

3 days can be {S, M, T}, {M, T, W}, {T, W, Th}, {W, Th, F}, {Th, F, Sa}, {F, Sa, S}, {Sa, S, M}.

In total 7 possibilities, Monday can come in 3 possibilities.

$\therefore$ Required probability $=\dfrac{3}{7}$

April has 30 days, i.e., 4 complete weeks and 2 days.

2 days can be {S, M}, {M, T}, {T, W}, {W, Th}, {Th, F}, {F, Sa}, {Sa, s}.

In total 7 possibilities, Monday can come in 2 possibilities.

$\therefore$ Required probability $=\dfrac{2}{7}$

So, statement 2 is wrong.

Hence, option *(a)* is correct.

70. *(c)* Given that $f(x)=|x|+x^2$

$$\Rightarrow f(x)=\begin{cases} x^2+x, & x\geq 0 \\ x^2-x, & x<0 \end{cases}$$

Consider the left hand limit $\lim\limits_{x\to 0^-} f(x)$

$\lim\limits_{x\to 0^-} f(x)=\lim\limits_{h\to 0} f(0-h)=\lim\limits_{h\to 0} f(0-h)^2-(0-h)$

$=\lim\limits_{h\to 0} h^2+h$

$=0$

And the right hand limit $\lim\limits_{x\to 0^+} f(x)$

$\lim\limits_{x\to 0^+} f(x)=\lim\limits_{h\to 0} f(0+h)=\lim\limits_{h\to 0} f(0+h)^2-(0+h)$

$=\lim\limits_{h\to 0} h^2+h$

$=0$

Since LHL=RHL $=f(0)$

$\Rightarrow f(x)$ is continuous at $x=0$.

Now,

$\text{LHD}=\lim\limits_{h\to 0}\dfrac{f(0-h)-f(0)}{-h}=\lim\limits_{h\to 0}\dfrac{h^2+h}{-h}=-\lim\limits_{h\to 0} h+1=-1$

And,

$$\text{RHD} = \lim_{h\to 0}\frac{f(0+h)-f(0)}{h} = \lim_{h\to 0}\frac{h^2+h}{h} = \lim_{h\to 0} h+1 = 1$$

Thus, LHD $\neq$ RHD

$\Rightarrow f(x)$ is not differentiable at $x=0$

Hence, option *(c)* is correct.

71. *(a)* Let $R=\{(1,1),(2,2),(3,3),(1,2),(2,3),(1,3)\}$

Reflexive

Since, $1R1$, $2R2$, $3R3$ in the set R

$\therefore$ R is reflexive relation.

Symmetric

Since, $1R2$ but 2 is not related to 1 in R

$\therefore$ R is not symmetric relation.

Transitive

$1R2$, $2R3 \Rightarrow 1R3$

$\therefore$ R is transitive relation.

Hence, R is reflexive and transitive but not symmetric.

Hence, option *(a)* is correct option.

72. *(a)* The general term of GP $= ar^{n-1}$

Given that $a_{10}=9$ and $a_4=4$

$\Rightarrow$ $ar^9=9$ and $ar^3=4$

On dividing, we get

$$\frac{ar^9}{ar^3}=\frac{9}{4} \Rightarrow r^6=\frac{9}{4} \qquad ....(i)$$

Therefore,

$ar^3=4 \Rightarrow (ar^3)^2=4^2$

$\Rightarrow a^2r^6=16$

$\Rightarrow a^2\times\frac{9}{4}=16$

$\Rightarrow a^2=\frac{64}{9} \Rightarrow a=\frac{8}{3}$

Thus, $a_7=ar^6=\frac{8}{3}\times\frac{9}{4}=6$

Hence, option *(a)* is correct.

73. *(b)* Let $A=\{1,2,5,6\}$ and $B=\{1,2,3\}$

$\therefore A\times B=\{(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(5,1),(5,2),(5,3),(6,1),(6,2),(6,3)\}$

And

$B\times A=\{(1,1),(1,2),(1,5),(1,6),(2,1),(2,2),(2,5),(2,6),(3,1),(3,2),(3,5),(3,6)\}$

$\Rightarrow (A\times B)\cap(B\times A)=\{(1,1),(1,2),(2,1),(2,2)\}$

Hence, option *(b)* is correct.

74. *(c)* Let the two equations be $x^2+mx+1=0$ and $x^2+x+m=0$

Let given equations have a common root α.

Therefore, 'α' satisfies the both equations.

Then, $\alpha^2+m\alpha+1=0$ and $\alpha^2+\alpha+m=0$

$$\Rightarrow \frac{\alpha^2}{m^2-1}=\frac{\alpha}{1-m}=\frac{1}{1-m}$$

By equation 2nd and 3rd, we get

$$\Rightarrow \frac{\alpha}{1-m}=\frac{1}{1-m} \Rightarrow \alpha=1$$

Also, by equating 1st and 3rd, we get

$$\frac{\alpha^2}{m^2-1}=\frac{1}{1-m}$$

$\Rightarrow 1-m=m^2-1 \quad (\because \alpha=1)$

$\Rightarrow m^2+m-2=0$

$\Rightarrow m=1$ and $m=-2$

Hence, option *(c)* is correct.

75. *(b)* Let $I=\int_0^{\pi/2}\frac{dx}{3\cos x+5}$

$$\Rightarrow I=\int_0^{\pi/2}\frac{dx}{3\left[\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right]+5}$$

$$\Rightarrow I=\int_0^{\pi/2}\frac{\left(1+\tan^2\frac{x}{2}\right)dx}{3-3\tan^2\frac{x}{2}+5+5\tan^2\frac{x}{2}}$$

$$\Rightarrow I=\int_0^{\pi/2}\frac{\sec^2\frac{x}{2}dx}{2\tan^2\frac{x}{2}+8}=\frac{1}{2}\int_0^{\pi/2}\frac{\sec^2\frac{x}{2}dx}{\tan^2\frac{x}{2}+2^2}$$

Put $\tan\frac{x}{2}=y\Rightarrow\frac{1}{2}\sec^2\frac{x}{2}dx=dy$

$$\therefore I=\frac{1}{2}\int_0^1\frac{dy}{y^2+2^2}=\frac{1}{2}\left[\tan^{-1}\left(\frac{y}{2}\right)\right]_0^1$$

$$=\frac{1}{2}\left[\tan^{-1}\left(\frac{1}{2}\right)-0\right]=\frac{1}{2}\tan^{-1}\left(\frac{1}{2}\right)$$

Also, $I=\frac{1}{2}\tan^{-1}\frac{1}{2}=k\cot^{-1}(2)$

$$\Rightarrow\frac{1}{2}\tan^{-1}\frac{1}{2}=k\tan^{-1}\left(\frac{1}{2}\right)\quad\left[\because\tan^{-1}(x)=\cot^{-1}(x)\right]$$

$$\therefore\quad k=\frac{1}{2}$$

Hence, option *(b)* is correct.

76. *(a)* We know from the properties of triangle that

$$\cos\frac{A}{2}=\sqrt{\frac{s(s-a)}{bc}}\Rightarrow\cos^2\frac{A}{2}=\frac{s(s-a)}{bc}$$

and $\cos\frac{C}{2}=\sqrt{\frac{s(s-c)}{ab}}\Rightarrow\cos^2\frac{C}{2}=\frac{s(s-c)}{ab}$

So, $a\cos^2\frac{C}{2}+c\cos^2\frac{A}{2}$

$$=a\left(\frac{s(s-c)}{ab}\right)+c\left(\frac{s(s-c)}{bc}\right)=\frac{s(s-c+s-a)}{b}$$

$$=\frac{s(2s-a-c)}{b}=\frac{s(a+b+c-a-c)}{b}=\frac{sb}{b}=s$$

$$=\frac{30}{2}=15\,\text{cm}\ \left[\text{given that } 2s=30\right]$$

Hence, option *(a)* is correct.

77. *(b)* If (l_1,m_1,n_1) and (l_2,m_2,n_2) are the direction ratios then the angle between the lines is

$$\cos\theta=\frac{l_1l_2+m_1m_2+n_1n_2}{\sqrt{l_1^2+m_1^2+n_1^2}\sqrt{l_2^2+m_2^2+n_2^2}}$$

Here, $l_1=1,m_1=1,n_1=1$ and $l_2=1,m_2=-1,n_2=n$ and $\theta=60°$

$$\therefore\cos 60°=\frac{1\times1+1\times(-1)+1\times n}{\sqrt{(1)^2+(1)^2+(1)^2}\sqrt{(1)^2+(-1)^2+(n)^2}}$$

$$\Rightarrow\frac{1}{2}=\frac{n}{\sqrt{3}\sqrt{2+n^2}}\Rightarrow 3\left(2+n^2\right)=4n^2$$

$$\Rightarrow n^2=6\Rightarrow n=\pm\sqrt{6}$$

Hence, option *(b)* is correct.

78. *(b)* The given equation is

$$y=\frac{2}{3C}(Cx-1)^{3/2}+B$$

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx}=\frac{2}{3C}\cdot\frac{3}{2}(Cx-1)^{\frac{1}{2}}\cdot C+0=(Cx-1)^{\frac{1}{2}}$$

On squaring both sides, we get

$$\left(\frac{dy}{dx}\right)^2=Cx-1$$

$$\Rightarrow\left(\frac{dy}{dx}\right)^2+1=Cx \qquad \text{....(i)}$$

Now, on differentiating w.r.t. x, we get

$$2\left(\frac{dy}{dx}\right)\cdot\frac{d^2y}{dx^2}=C$$

From equation (i), we get

$$\left(\frac{dy}{dx}\right)^2+1=2x\left(\frac{dy}{dx}\right)\frac{d^2y}{dx^2}$$

Hence, option *(b)* is correct.

79. *(a)* Let $A(x,y)$ be the point that divides $(4,3)$ and $(5,7)$ internally in the ratio $2:3$

$$A=(x,y)=\left(\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n}\right)$$

$$=\left(\frac{10+12}{2+3},\frac{14+9}{2+3}\right)=\left(\frac{22}{5},\frac{23}{5}\right)$$

Let $B(x',y')$ be the point that divides $(4,3)$ and $(5,7)$ externally in the ratio $2:3$

$$B=(x',y')=\left(\frac{mx_2-nx_1}{m-n},\frac{my_2-ny_1}{m-n}\right)$$

$$=\left(\frac{10-12}{2-3},\frac{14-9}{2-3}\right)=(2,-5)$$

$\therefore$ Distance between A and $=B=\sqrt{(x'-x)^2+(y'-y)^2}$

$$=\sqrt{\left(2-\frac{22}{5}\right)^2+\left(-5-\frac{23}{5}\right)^2}$$

$$=\sqrt{\frac{(-12)^2}{25}+\frac{(-48)^2}{25}}$$

$$=\frac{12}{5}\sqrt{1+4^2}=\frac{12\sqrt{17}}{5}$$

Hence, option *(a)* is correct.

80. *(c)* The given equation of sphere is

$ax^2+by^2+cz^2-2x+4y+2z-3=0$

This equation represents a equation of sphere, if coefficient of x^2, y^2 and z^2 is same. i.e. $a=b=c$

$\therefore$ Equation of sphere can be re-written as $bx^2+by^2+bz^2-2x+4y+2z-3=0$

$$\Rightarrow x^2+y^2+z^2-\frac{2x}{b}+\frac{4y}{b}+\frac{2z}{b}-\frac{3}{b}=0$$

The centre of this sphere is $\left(\frac{1}{b},-\frac{2}{b},-\frac{1}{b}\right)$

Given that the centre of sphere is $\left(\frac{1}{2},-1,-\frac{1}{2}\right)$

$$\therefore \frac{1}{b}=\frac{1}{2}\Rightarrow b=2$$

Hence, option *(c)* is correct.

81. *(d)* Let $A=\hat{i}+\hat{j}+\hat{k}$, $B=2\hat{i}+4\hat{j}-5\hat{k}$ and $C=\hat{i}+2\hat{j}+3\hat{k}$

Now, $B+C=\left(2\hat{i}+4\hat{j}-5\hat{k}\right)+\left(\hat{i}+2\hat{j}+3\hat{k}\right)$

$$=(2+b)\hat{i}+6\hat{j}-2\hat{k}$$

The unit vector parallel to $B+C$ is

$$\hat{n}=\frac{(2+b)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{(2+b)^2+(6)^2+(-2)^2}}=\frac{(2+b)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{b^2+4b+44}}$$

Now, $\left(\hat{i}+\hat{j}+\hat{k}\right)\cdot\hat{n}=1\Rightarrow \frac{2+b+6-2}{\sqrt{b^2+4b+44}}=1$

$$\Rightarrow 2+b+6-2=\sqrt{b^2+4b+44}$$

$$\Rightarrow 8b=8\Rightarrow b=1$$

Hence, option *(d)* is correct.

82. *(a)* Let $\begin{vmatrix} k & b+c & b^2+c^2 \\ k & c+a & c^2+a^2 \\ k & a+b & a^2+b^2 \end{vmatrix}=\Delta$

$$=k\begin{vmatrix} 1 & 1 & 1 \\ b+c & c+a & a+b \\ b^2+c^2 & c^2+a^2 & a^2+b^2 \end{vmatrix}$$

Applying $C_2\to C_2-C_1,\ C_3\to C_3-C_1$

$$=k\begin{vmatrix} 1 & 0 & 0 \\ b+c & a-b & a-c \\ b^2+c^2 & a^2-b^2 & a^2-c^2 \end{vmatrix}$$

$$=k\begin{vmatrix} 1 & 0 & 0 \\ b+c & a-b & a-c \\ b^2+c^2 & (a-b)(a+b) & (a-c)(a+c) \end{vmatrix}$$

$$=k(a-b)(a-c)\begin{vmatrix} 1 & 1 \\ a+b & a+c \end{vmatrix}$$

$$=k(a-b)(a-c)(a+c-a-b)$$

$$=k(a-b)(b-c)(c-a)$$

But given $\Delta=(a-b)(b-c)(c-a)$

Thus, $k=1$

Hence, option *(a)* is correct.

83. *(c)* From the properties of a triangle, we know that

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

Given that $a=2b$ and $A=3B$

$$\Rightarrow \frac{\sin 3B}{\sin B}=2 \Rightarrow \frac{3\sin B-4\sin^3 B}{\sin B}=2$$

$$\Rightarrow 3-4\sin^2 B=2$$

$$\Rightarrow \sin^2 B=\left(\frac{1}{2}\right)^2$$

$$\Rightarrow \sin B=\frac{1}{2}=\sin\frac{\pi}{6}$$

$$\Rightarrow B=\frac{\pi}{6}$$

Since, $A=3B$, so $\angle A=\frac{\pi}{2}$

Thus, triangle is right angled triangle.

Hence, option *(c)* is correct.

84. *(c)* Let a and b be two numbers such that

$$\text{A.M.}=\frac{a+b}{2}=10 \Rightarrow a+b=20 \qquad ....(\text{i})$$

and $\text{G.M.}=\sqrt{ab}=8 \Rightarrow ab=64 \qquad ....(\text{ii})$

From (i) and (ii), we have

$$a^2-20a+64=0 \Rightarrow (a-4)(a-16)=0$$

$$\Rightarrow a=4,\ 16$$

Thus, when $a=4,\ b=16$ and when $a=16,\ b=4$

Thus one number exceeds the other number by 12.

Hence, option *(c)* is correct.

85. *(c)* Let $D=\begin{vmatrix}1 & z & -y\\ -z & 1 & x\\ y & -x & 1\end{vmatrix}$

Expanding along R_1

$$\Rightarrow D=1\begin{vmatrix}1 & x\\ -x & 1\end{vmatrix}-z\begin{vmatrix}z & x\\ y & 1\end{vmatrix}-y\begin{vmatrix}-z & 1\\ y & -x\end{vmatrix}$$

$$=(1+x^2)-z(-z-xy)-y(xz-y)$$

$$=1+x^2+z^2+xyz-xyz+y^2$$

$$=1+x^2+y^2+z^2$$

$$=1+1=2$$

Hence, option *(c)* is correct.

86. *(d)* **For reflexive:**

$(a,a)=a-a=0$ is divisible by 5.

For symmetry:

If $(a-b)$ is divisible by 5, then $b-a=-(a-b)$ is also divisible by 5.

Thus relation is symmetric.

For transitive:

If $(a-b)$ and $(b-c)$ is divisible by 5.

Then $(a-c)$ is also divisible by 5

Thus relation is transitive.

$\therefore$ R is an equivalent relation.

Hence, option *(d)* is correct.

87. *(a)* Consider the quadratic equation $ax^2+bx+c=0$ whose one root is $\frac{1}{2-\sqrt{-2}}$

Consider $\frac{1}{2-\sqrt{-2}}=\frac{1}{2-\sqrt{2}i}\times\frac{2+\sqrt{2}i}{2+\sqrt{2}i}$

$$=\frac{2+\sqrt{2}i}{4+2i}=\frac{2+\sqrt{2}i}{6}$$

$\therefore$ Another root will be $\frac{2-\sqrt{2}i}{6}$

$(\because \text{complex roots always occurs in pair})$

Thus, sum of roots $=\frac{2+\sqrt{2}i}{6}+\frac{2-\sqrt{2}i}{6}=\frac{4}{6}$

And product of roots $=\left(\frac{2+\sqrt{2}i}{6}\right)\left(\frac{2-\sqrt{2}i}{6}\right)=\frac{4+2}{36}=\frac{1}{6}$

$\therefore$ Required equation is

$$x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0$$

$$\Rightarrow x^2 - \frac{4}{6}x + \frac{1}{6} = 0$$

$$\Rightarrow 6x^2 - 4x + 1 = 0$$

Thus, the values of a, b, c are $6, -4, 1$ respectively.

Hence, option *(a)* is correct.

88. *(b)* Given, $\begin{vmatrix} 2a & 3r & x \\ 4b & 6s & 2y \\ -2c & -3t & -z \end{vmatrix} = \lambda \begin{vmatrix} a & r & x \\ b & s & y \\ c & t & z \end{vmatrix}$

Taking 2 common from C_1 and 3 from C_2 in LHS

$$\therefore 2\times 3\begin{vmatrix} a & r & x \\ 2b & 2s & 2y \\ -c & -t & -z \end{vmatrix} = \lambda \begin{vmatrix} a & r & x \\ b & s & y \\ c & t & z \end{vmatrix}$$

Taking 2 common from R_2 and -1 from R_3 in LHS.

$$\therefore -12\begin{vmatrix} a & r & x \\ b & s & y \\ c & t & z \end{vmatrix} = \lambda \begin{vmatrix} a & r & x \\ b & s & y \\ c & t & z \end{vmatrix}$$

$$\Rightarrow \lambda = -12$$

Hence, option *(b)* is correct.

89. *(d)* Events when a coin is flipped and head occurs are {HT, HH}.

Events when a coin is flipped and tail occurs are {T1, T2, T3, T4, T5, T6}.

($\because$ dice are rolled after tail appears)

So, total number of events $= 8$

Favourable event $= \{\text{HT}\} = 1$

$$\therefore \text{Required probability} = \frac{1}{8}$$

Hence, option *(d)* is correct.

90. *(a)* Let $\overrightarrow{OA} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\overrightarrow{OB} = 2\hat{i} + 5\hat{j} - \hat{k}$ and $\overrightarrow{OC} = -\hat{i} + \hat{j} + 2\hat{k}$ be three position vectors.

Now, $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \hat{i} + 3\hat{j} - 4\hat{k}$

And $\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = -2\hat{i} - \hat{j} - \hat{k}$

$$\therefore \text{Area of } \Delta ABC = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|$$

Now, $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & -4 \\ -2 & -1 & -1 \end{vmatrix}$

$$= \hat{i}(-3-4) - \hat{j}(-1-8) + \hat{k}(-1+6)$$

$$= -7\hat{i} + 9\hat{j} + 5\hat{k}$$

$$\therefore \left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = \sqrt{(-7)^2 + (9)^2 + (5)^2} = \sqrt{155}$$

$$\therefore \text{Required area} = \frac{\sqrt{155}}{2}$$

Hence, option *(a)* is correct.

91. *(c)* As A and B are independent events.

So, $P(A \cap B) = P(A) \cdot P(B)$

$$\Rightarrow P(A \cap B) = \frac{1}{5}P(B)$$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\Rightarrow \frac{7}{10} = \frac{1}{5} + P(B) - \frac{1}{5}P(B)$$

$$\Rightarrow P(B)\left(1 - \frac{1}{5}\right) = \frac{7}{10} - \frac{1}{5}$$

$$\Rightarrow \frac{4}{5}P(B) = \frac{1}{2} \Rightarrow P(B) = \frac{5}{8}$$

Now, $P(\overline{B}) = 1 - P(B) = 1 - \frac{5}{8} = \frac{3}{8}$

Hence, option *(c)* is correct.

92. *(d)* Given points are $A(k, 1, -1)$, $B(2k, 0, 2)$ and $C(2+2k, k, 1)$

Let $r_1 =$ length of line

$$AB = \sqrt{(2k-1)^2 + (0-1)^2 + (2+1)^2} = \sqrt{k^2 + 10}$$

Let $r_2 =$ length of line

$$BC = \sqrt{(2)^2 + k^2 + (-1)^2} = \sqrt{k^2 + 5}$$

Now, let l_1, m_1, n_1 be the direction-cosines of line AB and l_2, m_2, n_2 be the direction cosines of BC.

Since AB is perpendicular to BC

$\therefore l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$

Now, $l_1 = \dfrac{k}{\sqrt{k^2+10}}, m_1 = \dfrac{-1}{\sqrt{k^2+10}}, n_1 = \dfrac{3}{\sqrt{k^2+10}}$

And $l_2 = \dfrac{2}{\sqrt{k^2+5}}, m_2 = \dfrac{k}{\sqrt{k^2+5}}, n_2 = \dfrac{-1}{\sqrt{k^2+5}}$

So, $l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$

$\Rightarrow \dfrac{2k}{\sqrt{k^2+10}\sqrt{k^2+5}} - \dfrac{k}{\sqrt{k^2+10}\sqrt{k^2+5}} - \dfrac{3}{\sqrt{k^2+10}\sqrt{k^2+5}}$

$\Rightarrow 2k - k - 3 = 0$

$\Rightarrow k = 3$

For $k = 3, AB$ is perpendicular to BC.

Hence, option *(d)* is correct.

93. *(c)* The relation S is defined in the set of integers Z and xSy, if integer x divides integer y.

Reflexive: Since, every integer divides itself.

$\therefore$ integer x divides integer y

$\Rightarrow xSx$

Hence, S is reflexive.

Symmetric: Let $x, y \in Z$ such that xSy

i.e., integer x divides integer y.

Now, this does not implies that integer y divided integer x.

e.g. take $x = 2$ and $y = 4$

Then, 2 divided 4 but 4 does not divides 2.

Thus, S is not symmetric.

Transitive: Let $x, y, z \in Z$ such that xSy and ySz.

This implies integer x divides integer y and integer y divides integer z.

Then, integer x divides integer z.

$\Rightarrow xSz$

Hence, S is transitive.

Hence, option *(c)* is correct.

94. *(c)* Given $\begin{vmatrix} a-b & b+c & a \\ b-c & c+a & b \\ c-a & a+b & c \end{vmatrix}$

Applying $C_1 \to C_1 - C_3$

$= \begin{vmatrix} -b & c & a \\ -c & c+a & b \\ -a & a+b & c \end{vmatrix}$

Applying $C_2 \to C_2 + C_1$

$= \begin{vmatrix} -b & c & a \\ -c & a & b \\ -a & b & c \end{vmatrix} = (-1)\begin{vmatrix} b & c & a \\ c & a & b \\ a & b & c \end{vmatrix}$

$= -1\left[b\left(ac-b^2\right) - c\left(c^2-ab\right) + a\left(bc-a^2\right)\right]$

$= -\left[abc - b^3 - c^3 + abc + abc - a^3\right]$

$= a^3 + b^3 + c^3 - 3abc$

Hence, option *(c)* is correct.

95. *(a)* Given that $L_1 \equiv x + y - 4 = 0$

$L_2 \equiv 3x + y - 4 = 0$

$L_3 \equiv x + 3y - 4 = 0$

Slope of $L_1 = m_1 = \dfrac{-1}{1} = -1$

Slope of $L_2 = m_2 = \dfrac{-3}{1} = -3$

Slope of $L_3 = m_3 = \dfrac{-1}{3}$

Angle between L_1 and L_2

$\Rightarrow \tan\theta_1 = \left|\dfrac{-1-(-3)}{1+(-1)(-3)}\right| = \dfrac{-1+3}{1+3} = \dfrac{1}{2}$

Angle between L_2 and L_3

$\Rightarrow \tan\theta_2 = \left|\dfrac{-3-\left(-\frac{1}{3}\right)}{1+(-3)\left(-\frac{1}{3}\right)}\right| = \left|\dfrac{-9+1}{3+3}\right| = \dfrac{4}{3}$

Angle between L_1 and L_3

$$\Rightarrow \tan\theta_3 = \left|\frac{-1-\left(-\frac{1}{3}\right)}{1+(-1)\left(-\frac{1}{3}\right)}\right| = \left|\frac{-3+1}{3+1}\right| = \frac{1}{2}$$

$\therefore$ The triangle formed is an isosceles triangle.
Hence, option *(a)* is correct.

96. *(d)* Let $\vec{a} = p\left(-3\hat{i} - 2\hat{j} + 13\hat{k}\right)$

$$= (-3p)\hat{i} + (-2p)\hat{j} + (13p)\hat{k}$$

It is given that $\vec{a}$ is of unit length.

$\therefore \ |\vec{a}| = 1 \Rightarrow |\vec{a}|^2 = 1$

$\Rightarrow (-3p)(-3p) + (-2p)(-2p) + (13p)(13p) = 1$

$\Rightarrow 9p^2 + 4p^2 + 169p^2 = 1$

$\Rightarrow p^2 = \frac{1}{182}$

$\Rightarrow p = \frac{1}{\sqrt{182}}$

Hence, option *(d)* is correct.

97. *(b)* The total number of cards in the deck is 52.
Number of aces in the deck $= 4$
There are 4 aces out of 52 cards.

$\therefore$ Probability of first ace $= \frac{4}{52}$

Now, probability of drawing the second ace, without replacement (one ace has already been drawn) so,
Remaining cards $= 51$
Remaining aces $= 3$

$\therefore$ Probability of second ace $= \frac{3}{51}$

$\therefore$ Probability of two aces is

$P(2\text{ aces}) = \frac{4}{52} \times \frac{3}{51} = \frac{1}{221}$

Hence, option *(b)* is correct.

98. *(a)* Given that $AB = A$ and $BA = B$
Consider $B = BA = B(AB) \qquad \{\because AB = A\}$

$= (BA) \cdot B = B \cdot B = B^2$

Thus, $B^2 = B$
Hence, option (a) is correct.

99. *(b)* Given quadratic equation $x^2 - bx + 1 = 0$
It has no real roots, it means equation has imaginary roots, which is possible when $B^2 - 4AC < 0$
Here, $B = -b,\ A = 1,\ C = 1$

$\Rightarrow b^2 - 4 < 0 \Rightarrow b^2 < 4 \Rightarrow -2 < b < 2$

Hence, option *(b)* is correct.

100. *(c)* Consider $\begin{vmatrix} m & n & p \\ p & m & n \\ n & p & m \end{vmatrix}$

Applying $C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} m+n+p & n & p \\ p+m+n & m & n \\ n+p+m & p & m \end{vmatrix}$$

Take $m+n+p$ common from C_1.

$$= (m+n+p)\begin{vmatrix} 1 & n & p \\ 1 & m & n \\ 1 & p & m \end{vmatrix}$$

$$= (m+n+p)\left[\left(m^2 + n^2 + p^2\right) - mn - np - pm\right]$$

Hence, value of the determinant has linear factor.
Hence, option *(c)* is correct.

101. *(c)* $(a,b)R(c,d) \Leftrightarrow a + d = b + c$

(i) $a + a = a + a$

$\therefore \ (a,a)R(a,a) \Rightarrow R$ is reflexive

(ii) $(a,b)R(c,d) \Rightarrow a + d = b + c$

$(c,d)R(a,b) \Rightarrow c + b = d + a$

$\therefore \ R$ is symmetric

(iii) Let $(a,b)R(c,d)$ and $(c,d)R(e,f)$

$\Rightarrow a+d=b+c$ and $c+f=d+e$

$\Rightarrow a+d+c+f=b+c+d+e$

$\Rightarrow a+f=b+e$

$\Rightarrow (a,b)R(e,f)$

$\therefore R$ is transitive

From (i), (ii) and (iii) R is an equivalence relation.

Hence option *(c)* is correct.

102. *(d)* Equation of line passing through the points $(2,1,3)$ and $(4,-2,5)$ is

$$\frac{x-2}{4-2}=\frac{y-1}{-2-1}=\frac{z-3}{5-3}=\lambda \Rightarrow \frac{x-2}{2}=\frac{y-1}{-2}=\frac{z-3}{2}=\lambda$$

$\Rightarrow x=2\lambda+2,\ y=-3\lambda+1$ and $z=2\lambda+3$

Since, this line cuts the plane $2x+y-z=3$

So, $(2\lambda+2,\ -3\lambda+1,\ 2\lambda+3)$ satisfies the equation of the plane.

$\therefore\ 2\lambda+2-3\lambda+1-2\lambda-3=3$

$\Rightarrow -3\lambda=3 \Rightarrow \lambda=-1$

Therefore, the points are given by

$[2(-1)+2,-3(-1)+1,2(-1)+3]=(0,4,1)$

Hence, option *(d)* is correct.

103. *(d)* Let the ratio is $k:1$

Then, $0=\frac{4k+2}{k+1} \Rightarrow 4k+2=0 \Rightarrow k=-\frac{1}{2}$

And $4=\frac{-2k+1}{k+1} \Rightarrow 4k+4=-2k+1 \Rightarrow k=-\frac{1}{2}$

Hence, the plane divides the line is the ratio $1:2$ externally.

Hence, option *(d)* is correct.

104. *(c)* For $2x^2-2(k-2)x+(k+1)=0$

Let roots be α and β

$\therefore \alpha+\beta=-2$ and $\alpha\beta=-\frac{k+1}{2}$

$\because \alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$\Rightarrow \alpha^2+\beta^2=(k-2)^2+(k+1)$

$\Rightarrow \alpha^2+\beta^2=k^2-4k+4+k+1$

$\Rightarrow \alpha^2+\beta^2=k^2-3k+5$

$\Rightarrow \alpha^2+\beta^2=\left(k-\frac{3}{2}\right)^2+\left(5-\frac{9}{4}\right)$

For minimum value $k=\frac{3}{2}$

Hence, option *(c)* is correct.

105. *(c)* We have $r=\sqrt{b_{xy}\cdot b_{yx}}$

$=\sqrt{\left(-\frac{1}{6}\right)\times\left(-\frac{3}{2}\right)}$

$=\sqrt{\frac{1}{2}\times\frac{1}{2}}=\pm\frac{1}{2}$

Since, b_{xy} and b_{yx} both have negative sign, therefore, we have to take the negative sign.

Hence, correlation coefficient $=-\frac{1}{2}$

Hence, option *(c)* is correct.

106. *(a)* Let $A=\begin{bmatrix} x+y & y \\ 2x & x-y \end{bmatrix}, B=\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ and $C=\begin{bmatrix} 3 \\ 2 \end{bmatrix}$

Here $AB=C$

$\therefore \begin{bmatrix} x+y & y \\ 2x & x-y \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix}=\begin{bmatrix} 3 \\ 2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2(x+y) & -y \\ 4x & -x+y \end{bmatrix}=\begin{bmatrix} 3 \\ 2 \end{bmatrix}$

$2x+y=3$ ....(i)

$3x+y=2$ ....(ii)

From equations (i) and (ii), we get

$x=-1$ and $y=5$

$\therefore A=\begin{bmatrix} 4 & 5 \\ -2 & -6 \end{bmatrix}$

Now, $A^2=\begin{bmatrix} 4 & 5 \\ -2 & -6 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ -2 & -6 \end{bmatrix}$

$$=\begin{bmatrix}16-10 & 20-30\\ -8+12 & -10+36\end{bmatrix}=\begin{bmatrix}6 & -10\\ 4 & 26\end{bmatrix}$$

Hence, option *(a)* is correct.

107. *(d)* We know that $\vec{a}\times\vec{b}=|\vec{a}||\vec{b}|\sin\theta\hat{n}$

$\therefore |\vec{a}\times\vec{b}|=|\vec{a}||\vec{b}|\sin\theta=2\times3\sin\theta=6\sin\theta$

And, $\therefore |\vec{a}\cdot\vec{b}|=|\vec{a}||\vec{b}|\cos\theta=2\times3\cos\theta=6\cos\theta$

$\therefore |\vec{a}\times\vec{b}|^2+|\vec{a}\cdot\vec{b}|^2=(6\sin\theta)^2+(6\cos\theta)^2$

$=36(\sin^2\theta+\cos^2\theta)=36$

Hence, option *(d)* is correct.

108. *(a)* We have Mean $=(\bar{x})=\dfrac{\sum x_i}{N}$

Here, $\bar{x}=\dfrac{4+7+8+9+10+12+13+17}{8}=10$

$\therefore$ Mean deviation about mean $=\dfrac{\sum|x_i-\bar{x}|}{N}$

$$=\frac{|4-10|+|7-10|+|8-10|+|9-10|+|10-10|+|12-10|+|13-10|+|17-10|}{8}$$

$$=\frac{6+3+2+1+0+2+3+7}{8}$$

$$=\frac{24}{8}=3$$

Hence, option *(b)* is correct.

109. *(a)* Let x be the side of first square and y be the side of second square.

$\therefore$ Area of first square, $A_1=x^2$

and, area of second square, $A_2=y^2$

$=(x+x^2)^2 \quad (\because y=x+x^2)$

$=x^2+x^4+2x^3$

Now, $\dfrac{dA_1}{dx}=2x$ and $\dfrac{dA_2}{dx}=2x+4x^3+6x^2$

Hence, the rate of change of area of the second square with respect to the area of the first square

$=\dfrac{dA_2}{dA_1}=\dfrac{2x+4x^3+6x^2}{2x}=1+3x+2x^2$

Hence, option *(a)* is correct.

110. *(b)* The given curve is $y=e^x$

Slope of the tangent $=\dfrac{dy}{dx}=e^x$

$\therefore \left(\dfrac{dy}{dx}\right)_{(0,1)}=e^0=1$

Therefore, equation of tangent line is

$y-y_0=\dfrac{dy}{dx}(x-x_0)$

$\Rightarrow y-1=1(x-0)$

$\Rightarrow y-1=x$

$\Rightarrow y=x+1$

when tangent meet *x*-axis, *y*-coordinates becomes zero.

$\therefore y=x+1\Rightarrow 0=x+1\Rightarrow x=-1$

Hence, the point is $(-1,0)$

Hence option *(b)* is correct.

111. *(a)* Let angle between $\vec{a}$ and $\vec{b}$ be θ

$|\vec{a}+\vec{b}|=\sqrt{|\vec{a}|^2+|\vec{b}|^2+2|\vec{a}||\vec{b}||\cos\theta|}$

$10\sqrt{3}=\sqrt{49+121+2\times7\times11\cos\theta}$

$300=170+154\cos\theta$

$154\cos\theta=130$

And,

$|\vec{a}-\vec{b}|=\sqrt{|\vec{a}|^2+|\vec{b}|^2-2|\vec{a}||\vec{b}||\cos\theta|}$

$=\sqrt{170-154\cos\theta}$

$=\sqrt{170-130}$

$=\sqrt{40}=2\sqrt{10}$

Hence, option *(b)* is correct.

112. *(b)* Given that $\text{cov}(x,y)=30$ and $\text{var}(x)=25$,

$\text{var}(y)=144$

$\Rightarrow r(x,y)=\dfrac{\text{cov}(x,y)}{\sqrt{\text{var}(x)}\cdot\sqrt{\text{var}(y)}}$

$\Rightarrow r(x,y)=\dfrac{30}{\sqrt{25}\cdot\sqrt{144}}=\dfrac{30}{5\times 12}=0.5$

Hence, option *(b)* is correct.

113. *(a)* Given that $\vec{u}-\vec{v}=\vec{w}$

$\left(2x\vec{\alpha}+y\vec{\beta}\right)-\left(2y\vec{\alpha}+3x\vec{\beta}\right)=2\vec{\alpha}-5\vec{\beta}$

$(2x-2y)\vec{\alpha}+(y-3x)\vec{\beta}=2\vec{\alpha}-5\vec{\beta}$

$\therefore 2x-2y=2$ ....(i)

and, $3x-y=5$ ....(ii)

Solving equation (i) and (ii), we get

$x=2$ and $y=1$

Hence, option *(a)* is correct.

114. *(b)* We have $\sigma^2=\dfrac{1}{n}\sum_{i=1}^{n}x_i^2=\left(\dfrac{1}{n}\sum_{i=1}^{n}x_i\right)^2$

$=\dfrac{1}{n}\left(1^2+2^2+3^2+\ldots+n^2\right)-\left(\dfrac{1}{n}(1+2+3+\ldots+n)\right)^2$

$=\dfrac{1}{n}\cdot\dfrac{n(n+1)(2n+1)}{6}-\left(\dfrac{1}{n}\cdot\dfrac{n(n+1)}{2}\right)^2$

$=\dfrac{n^2-1}{12}$

$\therefore \sigma=\sqrt{\dfrac{n^2-1}{12}}$

Hence, option *(b)* is correct.

115. *(a)* Given that $|\vec{a}|=2,\ |\vec{b}|=5$ and $|\vec{a}\times\vec{b}|$

Also, $|\vec{a}\times\vec{b}|=|\vec{a}|\cdot|\vec{b}|\cdot|\sin\theta|$

$\Rightarrow |\sin\theta|=\dfrac{8}{2\times 5}=\dfrac{4}{5}\Rightarrow|\cos\theta|=\dfrac{3}{5}\ \Rightarrow\cos\theta=\pm\dfrac{3}{5}$

$\therefore \vec{a}\cdot\vec{b}=|\vec{a}|\cdot|\vec{b}|\cos\theta=2\times 5\times\dfrac{3}{5}=6$

Hence, option *(a)* is correct.

116. *(a)* Let E_1, E_2, E_3 & A be events defined as follows.

E_1 = person chosen is a scooter driver.

E_2 = person chosen is a car driver.

E_3 = person chosen is a truck driver, and

A = person meets with an accident.

$P(E_1)=\dfrac{2000}{12000}=\dfrac{1}{6},\ P(E_2)=\dfrac{1}{3}$ & $P(E_3)=\dfrac{1}{2}$

$\because$ Probability that a person meets with an accident given that he is a scooter driver $=P\left(\dfrac{A}{E_1}\right)=0.01$

$P\left(\dfrac{A}{E_2}\right)=0.03$ & $P\left(\dfrac{A}{E_3}\right)=0.15$

$\because$ the person meets with an accident.

$\therefore$ the probability that he was a scooter driver

$$P\left(\frac{E_1}{A}\right)=\frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)P\left(\frac{A}{E_2}\right)+P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$=\frac{\frac{1}{6}\times 0.01}{\left(\frac{1}{6}\times 0.01\right)+\left(\frac{1}{3}\times 0.03\right)+\left(\frac{1}{2}\times 0.15\right)}$$

$$=\frac{1}{52}$$

Hence, option *(a)* is correct.

117. *(b)* The standard derivation $=\sqrt{\dfrac{\left(x-\bar{x}\right)^2}{N}}$

Given data: $-\sqrt{6},-\sqrt{5},-\sqrt{4},-1,1,\sqrt{4},\sqrt{5},\sqrt{6}$

$\Rightarrow \bar{x}=\dfrac{\text{sum of data}}{\text{Number of data}}=0$

$$\left(x-\bar{x}\right)^2=\left(-\sqrt{6}-0\right)^2+\left(-\sqrt{5}-0\right)^2+\left(-\sqrt{4}-0\right)^2+\left(-1-0\right)^2+\left(1-0\right)^2+\left(\sqrt{4}-0\right)^2+\left(\sqrt{5}-0\right)^2+\left(\sqrt{6}-0\right)^2$$

$$=6+5+4+1+1+4+5+6$$

$$=32$$

$$\therefore \text{S.D.}=\sqrt{\frac{\left(x-\bar{x}\right)^2}{N}}=\sqrt{\frac{32}{8}}=\sqrt{4}=2$$

Hence, option *(b)* is correct.

118. *(a)* Let P denote the probability of getting head in a single toss of a coin.

$$\therefore p=\frac{1}{2}\Rightarrow q=\frac{1}{2}$$

Let X denote the no of heads in 5 tosses of a coin. Then, X is a binomial variate with parameters; $n=5$ and $p=\frac{1}{2}$

$$\therefore \text{Requird probability}=P\left(X>3\right)$$

$$=1-P\left(X\le 3\right)$$

$$=1-\left[P\left(x=0\right)+P\left(x=1\right)+P\left(x=2\right)+P\left(x=3\right)\right]$$

$$=1-\left[{}^5C_0+{}^5C_1+{}^5C_2+{}^5C_3\right]\frac{1}{2^5}$$

$$=1-\left[1+5+10+10\right]\frac{1}{32}$$

$$=\frac{32}{32}-\frac{26}{32}=\frac{6}{32}=\frac{3}{16}$$

Hence, option *(a)* is correct.

119. *(c)* The coefficient of correlation is independent of both change of scale and change of origin.
Hence, option *(c)* is correct.

120. *(b)* Here mean

$$=\frac{21+34+23+39+26+37+40+20+33+27}{10}$$

$$=\frac{300}{10}=30$$

Mean deviation

$$=\frac{|-9|+4+|-7|+9+|-4|+7+10+|-10|+3+|-3|}{10}$$

$$=\frac{66}{10}=6.6$$

$$\therefore \text{Coefficient of mean deviation}=\frac{\text{Mean Deviation}}{\text{Mean}}$$

$$=\frac{6.6}{30}=0.22$$

Hence, option *(b)* is correct.