HINTS & SOLUTION

1. *(b)* $(10101)_2 = 2^4\times1+2^3\times0+2^2\times1+2^1\times0+2^0\times1$

$= 16+4+1 = 21$

$(1101)_2 = 2^3\times1+2^2\times1+2^1\times0+2^0\times1$
$= 8+4+1 = 13$

$(10101)_2\times(1101)_2 = 21\times13$
$= 273$
$= 256+16+1$
$= 2^8+2^4+2^0$

S0, there will be 1 at 9th, 5th, and first place from right and zero at other places.

So, $(273)_{10} = (100010001)_2$

`

2. *(b)* Given equation is $(\log_3 x)^2 + \log_3 x < 2$

$\Rightarrow (\log_3 x)^2 + (\log_3 x) - 2 < 0$

$\Rightarrow (\log_3 x + 2)(\log_3 x - 1) < 0$

$\Rightarrow \quad -2 < \log_3 x < 1$

$\Rightarrow \quad \log_3 3^{-2} < \log_3 x < \log_3 3$

$\Rightarrow \quad \frac{1}{9} < x < 3$

3. *(a)* The given series is $(1\times3)+(3\times5)+(5\times7)+\ldots$

Its general term is given by

$T_n = (2n-1)(2n+1) = 4n^2 - 1$

Sum of series

$$S_n = 4\sum n^2 - \sum 1$$
$$= \frac{4(n+1)(2n+1)}{6} - n$$
$$= n\left[\frac{2(2n^2+3n+1)}{3} - 1\right]$$
$$= n\left[\frac{4n^2+6n+2-3}{3}\right]$$
$$= \frac{n(4n^2+6n-1)}{3}$$

For sum of first 50 terms of the series,

$$S_{50} = \frac{50\left[4(50)^2+6(50)-1\right]}{3}$$
$$= \frac{50(10000+3000-1)}{3}$$
$$= \frac{50\times10299}{3}$$
$$= 171650$$

4. *(a)* Given, $|z_1| = |z_2| = \left|\frac{1}{z_1}+\frac{1}{z_2}\right| = 2$

$$\left|\frac{1}{z_1}+\frac{1}{z_2}\right| = 2 \Rightarrow \left|\frac{z_2+z_1}{z_1z_2}\right| = 2$$
$$\Rightarrow \frac{|z_2+z_1|}{|z_1z_2|} = 2$$
$$\Rightarrow \frac{|z_2+z_1|}{|z_1||z_2|} = 2$$
$$\Rightarrow |z_2+z_1| = 2|z_1||z_2|$$
$$\Rightarrow |z_2+z_1| = 2\cdot2\cdot2$$
$$\Rightarrow |z_2+z_1| = 8$$

5. *(b)* Given $C(n,12) = C(n,8)$

$$\Rightarrow \quad {}^nC_{12} = {}^nC_8$$
$$\Rightarrow \frac{n!}{(n-12)!12!} = \frac{n!}{(n-8)!8!}$$
$$\Rightarrow \frac{1}{(n-12)(12\times11\times10\times9\times8!)}$$
$$= \frac{1}{(n-8)(n-9)(n-10)(n-11)(n-12)!8!}$$
$$\Rightarrow \frac{1}{12\times11\times10\times9} = \frac{1}{(n-8)(n-9)(n-10)(n-11)}$$

This gives,

$$(n-8)(n-9)(n-10)(n-11)=12\times11\times10\times9$$

$$\Rightarrow n-8=12,\ n-9=11,\ n-10=10,\ n-11=9$$

$$\Rightarrow \quad n=20$$

So, we have

$$C(22,n)={}^{22}C_{20}$$

$$=\frac{22!}{2!20!}=\frac{22\times21}{2}=231$$

6. *(a)* Given lines are

$y=(2-\sqrt{3})x+5$ and $y=(2+\sqrt{3})x-7$

Therefore, the slope of the first line is $2-\sqrt{3}$ and the slope of the second line is $2+\sqrt{3}$.

$$\tan\theta=\left|\frac{m_2-m_1}{1+m_1m_2}\right|=\left|\frac{2+\sqrt{3}-2+\sqrt{3}}{1+(4-3)}\right|$$

$$=\left|\frac{2\sqrt{3}}{2}\right|$$

$$=\sqrt{3}=\tan\frac{\pi}{3}$$

$$\Rightarrow \theta=\frac{\pi}{3}=60^\circ$$

7. *(d)* $$\frac{\log_{27}9\times\log_{16}64}{\log_4\sqrt{2}}=\frac{\log 9}{\log 27}\times\frac{\log 64}{\log 16}\times\frac{\log 4}{\log\sqrt{2}}$$

$$\left(\because\ \log_a x=\frac{\log x}{\log a}\right)$$

$$=\frac{\log 3^2}{\log 3^3}\times\frac{\log 2^6}{\log 2^4}\times\frac{\log 2^2}{\log(2)^{1/2}}$$

$$=\frac{2\log 3}{3\log 3}\times\frac{6\log 2}{4\log 2}\times\frac{2\log 2}{\frac{1}{2}\log 2}$$

$$=\frac{2}{3}\times\frac{6}{4}\times4=4$$

8. *(b)* Let α and β be the roots of both the equation

$$x^2-(a-1)x+(a+b)=0$$

$$\Rightarrow \alpha+\beta=(a-1) \text{ and } \alpha\beta=(a+b)$$

and $ax^2-2x+b=0$

$$\Rightarrow \alpha+\beta=\frac{2}{a} \text{ and } \alpha\beta=\frac{b}{a}$$

Equating the sum of roots,

$$\therefore \quad (a-1)=\frac{2}{a}$$

$$\Rightarrow a^2-a-2=0$$

$$\Rightarrow \quad a=-1,\ 2$$

Equating the product of roots,

$$\therefore \quad a+b=\frac{b}{a}$$

If $a=-1,\ b=\frac{1}{2}$ and if $a=2,\ b=-4$

From the given options, $a=2$ and $b=-4$ matches.

9. *(b)* Since first term and common difference of an AP are u and v respectively,

pth term, $T_p=u+(p-1)v \quad \ldots(i)$

and qth term, $T_q=u+(q-1)v \quad \ldots(ii)$

According to the condition given in question,

$$T_p=T_q+15uv$$

$$\Rightarrow \quad u+(p-1)v=u+(q-1)v+15uv$$

$$\Rightarrow (p-1-q+1)v=15uv$$

$$\Rightarrow \quad (p-q)v=15uv$$

$$\Rightarrow \quad p-q=15u$$

$$\Rightarrow \quad p=q+15u$$

10. *(b)* Let $\sqrt{-5+12i}=x+iy$

$$\Rightarrow \quad (x+iy)^2=-5+12i$$

$$\Rightarrow x^2-y^2+i2xy=-5+12i$$

$$\Rightarrow x^2-y^2=-5 \text{ and } 2xy=12$$

$$\Rightarrow x^2-y^2=-5 \text{ and } \quad xy=6$$

$$(x^2-y^2)^2+4x^2y^2=(x^2+y^2)^2$$

$$\Rightarrow \ (-5)^2+4\times(6)^2=(x^2+y^2)^2$$

$$\Rightarrow \quad (x^2+y^2)^2=25+144=169$$

$$\Rightarrow \quad x^2+y^2=\pm13 \text{ and } x^2-y^2=-5$$

Adding both to get,

$$2x^2 = \pm 13 - 5$$
$$\Rightarrow 2x^2 = 8 \text{ or } -18$$
$$\Rightarrow \quad x^2 = 4 \ (-\text{ve discarded})$$
$$\Rightarrow \quad x = \pm 2$$

$$x^2 + y^2 = \pm 13 \Rightarrow y^2 = \pm 13 - 4$$
$$\Rightarrow y^2 = -17, 9$$
$$\Rightarrow y^2 = 9 \ (\text{discard} - \text{ve value})$$
$$\Rightarrow \quad y = \pm 3$$

Thus, $\sqrt{-5+12i} = x + iy = \pm(2+3i)$

11. *(b)* Let A, B, C be three sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$.

Let $x \in A \cup B \Rightarrow x \in A$ or $x \in B \quad \ldots(1)$

Since $A \cup B = A \cup C$, therefore

$x \in A$ or $x \in C \quad \ldots(2)$

Also, given $A \cap B = A \cap C$

$\therefore\ x \in A \cap B \Rightarrow x \in A$ and $x \in B \quad \ldots(3)$

and $x \in A$ and $x \in C \quad (\because A \cap B = A \cap C) \ldots(4)$

Thus, from (1), (2), (3), and (4), we have $B = C$ only

12. *(d)* Let the equation of the circle be

$(x-h)^2 + (y-k)^2 = r^2$

As the given circle passes through $(5,-8)$, $(-2,9)$ and $(2,1)$, we have

$$(5-h)^2 + (-8-k)^2 = r^2 \quad \ldots(\text{i})$$
$$(-2-h)^2 + (9-k)^2 = r^2 \quad \ldots(\text{ii})$$
$$(2-h)^2 + (1-k)^2 = r^2 \quad \ldots(\text{iii})$$

From (i) and (ii),

$$25 + h^2 - 10h + 64 + k^2 + 16k$$
$$= 4 + h^2 + 4h + 81 + k^2 - 18k$$
$$\Rightarrow -14h + 34k = -4$$
$$\Rightarrow -7h + 17k = -2 \quad \ldots(\text{iv})$$

From (ii) and (iii),

$$4 + h^2 + 4h + 81 + k^2 - 18k$$
$$= 4 + h^2 - 4h + 1 + k^2 - 2k$$
$$\Rightarrow 8h - 16k = -80$$
$$\Rightarrow \quad h - 2k = -10 \quad \ldots(\text{v})$$

Solving equations (iv) and (v), we get

$h = -58$ and $k = -24$

$\therefore$ centre is $(-58, -24)$

13. *(b)* Let two parts of an angle θ are ϕ and ψ. So, $\theta = \phi + \psi$. This gives,

$$\tan\theta = \tan(\phi + \psi)$$
$$= \frac{\tan\phi + \tan\psi}{1 - \tan\phi\tan\psi}$$
$$= \frac{\frac{1}{8} + \frac{7}{9}}{1 - \frac{1}{8}\cdot\frac{7}{9}}$$
$$= \frac{\frac{9+56}{72}}{\frac{72-7}{72}}$$
$$= 1$$

$$\Rightarrow \tan\theta = \tan\frac{\pi}{4}$$
$$\Rightarrow \quad \theta = \frac{\pi}{4}$$

14. *(a)* Given that $\sin A = \frac{4}{5}$ and $\cos B = -\frac{12}{13}$.

It is given that A and B are obtuse angle.

$$\Rightarrow \cos A = \pm\sqrt{1 - \sin^2 A} = \pm\sqrt{1 - \frac{16}{25}} = -\frac{3}{5}$$

Negative sign is taken from $\cos A$ as A is obtuse lies in the second quadrant.

$\sin B = \pm\sqrt{1-\cos^2 B} = \pm\sqrt{1-\left(\frac{-12}{13}\right)^2}$

$= \sqrt{\frac{169-144}{169}} = \frac{5}{13}$

Positive sign is taken from $\sin B$ as B is obtuse lies in the second quadrant.

$\Rightarrow \cos A = -\frac{3}{5}$ and $\sin B = \frac{5}{13}$

$\therefore \sin(A+B) = \sin A\cos B + \cos A\sin B$

$= \frac{4}{5}\times\left(\frac{-12}{13}\right)+\left(-\frac{3}{5}\right)\times\frac{5}{13}$

$= -\frac{48}{65}-\frac{15}{65}$

$= -\frac{63}{65}$

15. *(d)* If α and β are the roots of the equation

$x^2+x+1=0$,

$\Rightarrow \alpha = \omega$ and $\beta = \omega^2$

or, $\alpha = \omega^2$ and $\beta = \omega$

$\therefore \alpha^{19}+\beta^7 = \omega^{19}+\omega^{14} = \omega+\omega^2 = -1$

or, $\alpha^{19}+\beta^7 = \omega^{38}+\omega^7 = \omega^2+\omega = -1$

In either case, $\alpha^{19}+\beta^7 = -1$.

and $\alpha^{19}\cdot\beta^7 = \omega^{19}\omega^{14} = \omega^{33} = 1$

or, $\alpha^{19}\cdot\beta^7 = \omega^{38}\omega^7 = \omega^{45} = 1$

Thus, the required equation whose roots are α^{19} and β^7 is

$x^2-\left(\alpha^{19}+\beta^7\right)+\alpha^{19}\beta^7 = 0 \Rightarrow x^2+x+1=0$

16. *(d)* The equation of the ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

The point for which $\frac{x^2}{a^2}+\frac{y^2}{b^2}-1>0$ is outside ellipse.

Since at $(a,0)$, $1+0-1=0$, it lies on the ellipse.

At $(0,b)$, $0+1-1=0$, it lies on the ellipse

At $(-a,0)$, $1+0-1=0$, it lies on the ellipse

At (a,b), $1+1-1>0$.

So, the point (a,b) lies outside the ellipse.

17. *(b)* Let sets A and B have m and n elements respectively. The set made by subsets of finite sets A and B is known as power set i.e. $P(A)$ and $P(B)$.

We know, if set A has n elements, then $P(A)$ has 2^n elements.

Thus, the total number of subsets of a finite set $A = 2^m$ and set $B = 2^n$.

So, according to the question

$2^m-2^n = 56$

$\Rightarrow 2^n\left(2^{m-n}-1\right) = 56 = 8\times 7 = 2^3\times\left(2^3-1\right)$

On comparing both sides, we have

$n=3$ and $m-n=3$

$\Rightarrow m=6$ and $n=3$

18. *(d)* $\left(\frac{1+2i}{2+i}\right)^2 = \left(\frac{(1+2i)(2-i)}{(2+i)(2-i)}\right)^2$

$= \left(\frac{2+4i-i-2i^2}{4-i^2}\right)^2 = \left(\frac{4+3i}{5}\right)^2$

$= \frac{1}{25}\left(16+9i^2+24i\right)$

$= \frac{7}{25}+\frac{24}{25}i$

So, conjugate of $\left(\frac{1+2i}{2+i}\right)^2$ is $\frac{7}{25}-\frac{24}{25}i$

(Conjugate of $a+ib$ is $a-ib$)

19. *(c)* Let the coordinates of the point are (x,y).

$\therefore \sqrt{\{x-(a+b)\}^2+\{y-(a-b)\}^2}$

$= \sqrt{\{x-(b-a)\}^2+\{y-(a+b)\}^2}$

Squaring both sides,

$$\therefore x^2+(a+b)^2-2x(a+b)+y^2+(a-b)^2-2y(a-b)$$
$$=x^2+(b-a)^2-2x(b-a)+y^2+(a+b)^2-2y(a+b)$$
$$\Rightarrow 2x(a+b)+2y(a-b)=2x(b-a)+2y(a+b)$$
$$\Rightarrow x\{a+b-(b-a)\}+2y\{(a-b)-(a+b)\}=0$$
$$\Rightarrow 2ax-2by=0$$
$$\Rightarrow -ax+by=0$$

20. *(a)* Given expansion is $\left(3x-\frac{x^3}{6}\right)^9$, where

$a=3x,\ b=\frac{-x^3}{6},\ n=9$

Now, General term = $T_{r+1}={}^nC_r\,(a)^{n-r}\,b^r$

$$T_{r+1}={}^9C_r\,(3x)^{9-r}\left(\frac{-x^3}{6}\right)^r$$
$$={}^9C_r\,3^{9-r}\,x^{9-r}\,\frac{(-1)^r x^{3r}}{6^r}$$
$$={}^9C_r\,3^{9-r}\,(-1)^r\,\frac{x^{9+2r}}{6^r}$$

We can get coefficient of x^{17}

$9+2r=17$

$\Rightarrow 2r=17-9$

$\Rightarrow r=\frac{8}{2}=4$

Hence, required coefficient is

$$={}^9C_4\,\frac{3^5}{6^4}=\frac{126\times 3}{16}=\frac{189}{8}$$

21. *(b)* Given ellipse is $\frac{x^2}{169}+\frac{y^2}{25}=1$.

$$\therefore e=\sqrt{1-\frac{25}{169}}=\frac{12}{13}$$

Also, standard equation of ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

and eccentricity is $e=\sqrt{1-\frac{b^2}{a^2}}$.

$$\therefore \frac{12}{13}=\sqrt{1-\frac{b^2}{a^2}}\Rightarrow\frac{b^2}{a^2}=1-\frac{144}{169}=\frac{25}{169}$$
$$\Rightarrow\frac{b}{a}=\frac{5}{13}\Rightarrow\frac{a}{b}=\frac{13}{5}$$

22. *(b)* Given $f(x)=\log\left[\frac{1+x}{1-x}\right]$ and $g(x)=\frac{3x+x^3}{1+3x^2}$

$$f[g(x)]=\log\left[\frac{1+g(x)}{1-g(x)}\right]$$
$$=\log\left(\frac{1+3x^2+3x+x^3}{1+3x^2-3x-x^3}\right)$$
$$=\log\left(\frac{1+x}{1-x}\right)^3$$
$$=3\log\left(\frac{1+x}{1-x}\right)=3f(x)$$

23. *(a)* Given function is $f(x)=\begin{cases}ax^2+b & x<-1\\ bx^2+ax+4, & x\ge -1\end{cases}$

Derivative of $f(x)$ is given by,

$$f'(x)=\begin{cases}2ax & x<-1\\ 2bx+a, & x\ge -1\end{cases}$$

If $f'(x)$ is continuous everywhere, then it is also continuous at $x=-1$.

$$f'(x)\big|_{x=-1}=-2a=-2b+a$$
$$3a=2b\quad\ldots(\text{i})$$

From the given options, $a=2,\ b=3$ satisfied the equation.

24. *(b)* Given $\tan\alpha=m/(m+1),\ \tan\beta=1/(2m+1)$

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

$$=\frac{\frac{m}{m+1}+\frac{1}{2m+1}}{1-\frac{m}{m+1}\cdot\frac{1}{2m+1}}$$

$$=\frac{m(2m+1)+(m+1)}{(m+1)(2m+1)-m}$$

$$\tan(\alpha+\beta)=\frac{2m^2+2m+1}{2m^2+2m+1}=1$$

$$\Rightarrow \tan(\alpha+\beta)=\tan\frac{\pi}{4}$$

$$\Rightarrow \quad \alpha+\beta=\frac{\pi}{4}$$

25. *(a)* Total no. of students = 100

Let E denote the students who have passed in English.

Let M denote the students who have passed in Maths.

$\therefore n(E)=75,\ n(M)=60,\ n(E\cap M)=45$

We know $n(E\cup M)=n(E)+n(M)-n(E\cap M)$

$=75+60-45$

$=90$

Required no of students $90-45=45$

26. *(a)* Total no. of players = 12

No. of chosen players = 8

Number of ways to choose 8 players from 12 players

$$={}^{12}C_8=\frac{12!}{4!8!}$$

$$=\frac{12\times11\times10\times9}{4\times3\times2}$$

$$=13\times11\times5\times3$$

$$=495$$

Since, out of the 8 players, 1 is to be elected as captain and another vice captain,

$$={}^{8}C_1\times{}^{7}C_1=8\times7=56$$

Hence, the required number of ways is

$$=495\times56=27720$$

27. *(a)* Let $R=\{x\mid x\in N, x \text{ is a multiple of 3 and } x\le100\}$

and $S=\{x\mid x\in N, x \text{ is a multiple of 5 and } x\le100\}$

$\therefore R=\{3,6,9,12,15,\ldots,99\}$ and $S=\{5,10,15,\ldots,95,100\}$

$$(R\times S)\cap(S\times R)=(R\cap S)\times(S\cap R)$$

$$=\{15,30,45,60,75,90\}\times\{15,30,45,60,75,90\}$$

Therefore, number of elements in

$(R\times S)\cap(S\times R)=6\times6=36$

28. *(c)* Given $A(3,4)$ and $B(5,-2)$

Let the given point be $P(x,y)$.

Given that, $PA = PB$

$\Rightarrow PA^2 = PB^2$

$\Rightarrow (x-3)^2+(y-4)^2=(x-5)^2+(y+2)^2$

$\Rightarrow x^2-6x+9+y^2-8y+16$

$=x^2-10x+25+y^2+4y+4$

$\Rightarrow 4x-12y=4$

$\Rightarrow \quad x-3y=1 \quad \ldots(\text{i})$

Area of triangle PAB is given as 10.

$$\therefore \frac{1}{2}\begin{vmatrix}x & y & 1\\ 3 & 4 & 1\\ 5 & -2 & 1\end{vmatrix}=\pm10$$

$\Rightarrow x(4+2)-y(3-5)+1(-6-20)=\pm20$

$\Rightarrow \quad 6x+2y-26=\pm20$

This further gives,

$6x+2y-26=20$ or $6x+2y-26=-20$

$\Rightarrow \quad 6x+2y=46 \quad \ldots(\text{ii})$

or $\quad 6x+2y=6 \quad \ldots(\text{iii})$

From eqns (i) and (ii), we get $x=7,\ y=2$

Similarly, from eqns (i) and (iii), we get $x=1,\ y=0$

Hence coordinates of P are $(7,2)$ or $(1,0)$.

29. *(b)* Let $Z=x+iy=-2i$

Let square root of z be $a+ib$

Then, $\sqrt{x+iy}=a+ib$

$\Rightarrow x+iy=(a+ib)^2=(a^2-b^2)+i2ab$

Equating real and imaginary parts,

$a^2-b^2=0$ and $2ab=-2$

or $a^2-b^2=0$ and $ab=-1$

Since $ab<0$, therefore

$$\sqrt{x+iy}=\pm\left[\sqrt{\frac{\sqrt{x^2+y^2}+x}{2}}-i\sqrt{\frac{\sqrt{x^2+y^2}-x}{2}}\right]$$

$$=\pm\left[\sqrt{\frac{\sqrt{4}+0}{2}}-i\sqrt{\frac{\sqrt{4}-0}{2}}\right]$$

$$=\pm\left[\sqrt{\frac{2}{2}}-i\sqrt{\frac{2}{2}}\right]$$

$$=\pm(1-i)$$

30. *(c)* Let $A=\{a,b,c\}$ and

$R=\{(a,a),(a,b),(b,c),(b,b),(c,c),(c,a)\}$

Since, $(a,a),(b,b),(c,c)\in R$

$\therefore$ R is reflexive relation.

But $(a,b)\in R$ and $(b,a)\notin R$

$\therefore$ R is not symmetric relation.

Now $(a,b),(b,c)\in R$

$\Rightarrow (c,a)\in R$ but $(a,c)\notin R$

$\therefore$ R is not transitive relation.

31. *(d)* Given equation is $r^{1/3}+\frac{1}{r^{1/3}}=3$

Cubing both sides, we get

$$\left(r^{1/3}+\frac{1}{r^{1/3}}\right)^3=3^3$$

$$\Rightarrow r+\frac{1}{r}+3\left(r^{1/3}+\frac{1}{r^{1/3}}\right)=27$$

$$\left[\because\ (a+b)^3=a^3+b^3+3ab(a+b)\right]$$

$$\Rightarrow r+\frac{1}{r}+3\cdot 3=27$$

$$\Rightarrow \quad r+\frac{1}{r}=27-9$$

$$\Rightarrow \quad r+\frac{1}{r}=18$$

32. *(d)* Let the first term and common difference of an AP be a and d respectively.

As per given condition,

$$(p+1)\cdot(2p+1)=\left(\frac{2p+1}{2}\right)\{2a+(2p+1-1)d\}$$

$$\Rightarrow \quad (p+1)=\frac{1}{2}(2a+2pd)$$

$$\Rightarrow \quad (p+1)=a+pd$$

$$\Rightarrow \quad (p+1)=a+[(p+1)-1]d=T_{p+1}$$

This gives the $(p+1)^{\text{th}}$ of the AP is $(p+1)$.

33. *(b)* Given equation of ellipse is $25x^2+16y^2=400$ which can be re-written as

$$\frac{x^2}{16}+\frac{y^2}{25}=1$$

We know that the standard equation of ellipse is

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$$

On comparing both the equations, we get $a=4,\ b=5$.

So, the equation of the directrices are

$$y=\pm\frac{b}{e}=\pm\frac{b}{\sqrt{1-\frac{a^2}{b^2}}}=\pm\frac{5}{\sqrt{1-\frac{16}{25}}}=\pm\frac{25}{3}$$

$$\Rightarrow 3y\pm 25=0$$

34. *(a)* Let $x=\left(\sin 22\frac{1^\circ}{2}+\cos 22\frac{1^\circ}{2}\right)^4$

$$=\left(\left(\sin 22\frac{1°}{2}+\cos 22\frac{1°}{2}\right)^2\right)^2$$

$$=\left(\sin^2 22\frac{1°}{2}+\cos^2 22\frac{1°}{2}+2\sin 22\frac{1°}{2}\cos 22\frac{1°}{2}\right)^2$$

$$=(1+\sin 45°)^2$$

$$=\left(1+\frac{1}{\sqrt{2}}\right)^2$$

$$=\left(\frac{\sqrt{2}+1}{\sqrt{2}}\right)^2$$

$$\left(\sin 22\frac{1°}{2}+\cos 22\frac{1°}{2}\right)^4=\frac{2+1+2\sqrt{2}}{2}=\frac{3+2\sqrt{2}}{2}$$

35. *(c)* Let $z=\frac{\sqrt{2}+i}{\sqrt{2}-i}=\frac{\sqrt{2}+i}{\sqrt{2}-i}\times\frac{\sqrt{2}+i}{\sqrt{2}+i}$

$$=\frac{(\sqrt{2}+i)^2}{(\sqrt{2})^2-(i)^2}=\frac{2\sqrt{2}i+1}{3}$$

$$=\frac{1}{3}+\frac{2\sqrt{2}}{3}i$$

So,

$$|z|=\sqrt{\left(\frac{1}{3}\right)^2+\left(\frac{2\sqrt{2}}{3}\right)^2}$$

$$=\sqrt{\frac{1}{9}+\frac{8}{9}}$$

$$=\sqrt{\frac{9}{9}}=\sqrt{1}=1$$

36. *(c)* Given functions are $f(x)=x$ and $g(x)=|x|$.

$$(f+g)(x)=\begin{cases}x+x, & x\geq 0\\ x-x, & x<0\end{cases}$$

$$=\begin{cases}2x, & x\geq 0\\ 0, & x<0\end{cases}$$

37. *(b)* $3^x+3^y=3^{x+y}$

On differentiating both sides w.r.t. *x,* we get

$$3^x\log 3+3^y\log 3\frac{dy}{dx}=3^{x+y}\log 3\left(1+\frac{dy}{dx}\right)$$

$$\log 3\left(3^x+3^y 1\frac{dy}{dx}\right)=3^{x+y}\log 3\left(1+\frac{dy}{dx}\right)$$

$$\frac{dy}{dx}\left(-3^{x+y}+3^y\right)=3^{x+y}-3^x$$

$$\frac{dy}{dx}\left(3^y\left(1-3^x\right)\right)=3^x\left(3^y-1\right)$$

$$\frac{dy}{dx}=\frac{3^x\left(3^y-1\right)}{3^y\left(1-3^x\right)}=\frac{3^{x-y}\left(3^y-1\right)}{\left(1-3^x\right)}$$

38. *(c)* Equation of parabola is $y^2=4ax$.

Differentiate both sides w.r.t *x*

$$2y\frac{dy}{dx}=4a$$

$$\therefore\ \frac{dy}{dx}=\frac{2a}{y}$$

This represents the slope of the tangent.

Slope of normal is given by,

$$\left(\frac{dx}{dy}\right)_{(at^2,2at)}=\left(-\frac{y}{2a}\right)_{(at^2,2at)}$$

$$=-\frac{2at}{2a}$$

$$=-t$$

39. *(b)* Given $e^y+xy=e$.

On differentiating both sides w.r.t. *x*, we get

$$e^y\frac{dy}{dx}+y+x\frac{dy}{dx}=0\quad ...(i)$$

At $x=0$, we get $e^y+0=e\Rightarrow e^y=e\Rightarrow y=1$

By putting $y=1$ in equation (i), we get

$$e\frac{dy}{dx}+1+0=0$$

$$\Rightarrow\quad \frac{dy}{dx}=-\frac{1}{e}$$

Again, differentiating eq (i), we get

$$e^y \frac{d^2y}{dx^2} + e^y\left(\frac{dy}{dx}\right)^2 + \frac{dy}{dx} + x\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$$

$$\Rightarrow \quad \left(e^y + x\right)\frac{d^2y}{dx^2} + e^y\left(\frac{dy}{dx}\right)^2 + 2\frac{dy}{dx} = 0$$

Now, at $x = 0$, $y = 1$

$$(e+0)\frac{d^2y}{dx^2} + e\left(-\frac{1}{e}\right)^2 + 2\left(-\frac{1}{e}\right) = 0$$

$$\Rightarrow \quad e\frac{d^2y}{dx^2} - \frac{1}{e} = 0$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{1}{e^2} = e^{-2}$$

40. *(d)* Given $A = \{x \mid x \le 9, x \in N\} = \{1,2,3,4,5,6,7,8,9\}$

Total number of multiple of 3 are as follows: 3, 6, 9, 12, 15, 18, 21, 24, 27.
But 3 and 27 are not possible because 3 and 27 cannot be expressed as such that $a+b+c$ is multiple of 3.

$6 \to 1+2+3$

$9 \to 2+3+4, 5+3+1, 6+2+1$

$12 \to 9+2+1, 8+3+1, 7+1+4, 7+2+3$
$\quad 6+4+2, 6+5+1, 5+4+3$

$15 \to 9+4+2,\ 9+5+1, 8+6+1,\ 8+5+2,$
$\quad 8+4+3,\ 7+6+2, 7+5+3,\ 6+5+4$

$18 \to 9+8+1, 9+7+2, 9+6+3$

$21 \to 9+8+4, 9+7+5, 8+7+6$

$24 \to 9+8+7$

Hence, the total largest possible subsets are 30.

41. *(d)* Let $z = K(\cos\theta + i\sin\theta)$

$$|z| = \sqrt{K^2\left(\cos^2\theta + \sin^2\theta\right)} = 4(\text{given})$$

$$\Rightarrow K = 4$$

Again $\arg(z) = \frac{5\pi}{6}$, so $\theta = \frac{5\pi}{6}$

Now, $z = 4\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$

$$= 4\left(\frac{-\sqrt{3}}{2} + i\frac{1}{2}\right)$$

$$= -2\sqrt{3} + 2i$$

42. *(c)* As roots of the quadratic equation $(k+1)x^2 - 2(k-1)x + 1 = 0$ real and equal, we have

Its discriminant

$$\{-2(k-1)\}^2 - 4(k+1) = 0$$

$$\Rightarrow 4\left(k^2 - 2k + 1\right) - 4(k+1) = 0$$

$$\Rightarrow \quad k^2 - 2k + 1 - k - 1 = 0$$

$$\Rightarrow \quad k^2 - 3k = 0$$

$$\Rightarrow \quad k = 0,\ 3$$

43. *(b)* Given $\sin\theta = 3\sin(\theta + 2\alpha)$

$$\Rightarrow \frac{\sin(\theta+2\alpha)}{\sin\theta} = \frac{1}{3}$$

Applying componendo and dividendo rule,

$$\Rightarrow \frac{\sin(\theta+2\alpha) + \sin\theta}{\sin(\theta+2\alpha) - \sin\theta} = \frac{1+3}{1-3}$$

$$\Rightarrow \frac{2\sin(\theta+\alpha)\cos\alpha}{2\cos(\theta+\alpha)\sin\alpha} = \frac{4}{-2}$$

$$\Rightarrow \quad \frac{\tan(\theta+\alpha)}{\tan\alpha} = -2$$

$$\Rightarrow \quad \tan(\theta+\alpha) = -2\tan\alpha$$

$$\Rightarrow \tan(\theta+\alpha) + 2\tan\alpha = 0$$

44. *(c)* Let $2a$ and $2b$ be the length of major and minor axis respectively.

$$e = \frac{4}{5} \Rightarrow \sqrt{1 - \frac{b^2}{a^2}} = \frac{4}{5}$$

$$\Rightarrow \frac{b^2}{a^2} = \frac{9}{25} \quad ...(\text{i})$$

Length of latus rectum is 14.4, which gives

$$\frac{2b^2}{a} = 14.4 \Rightarrow \frac{b^2}{a} = 7.2$$

$$\Rightarrow b^2 = 7.2a$$

Put this value in (i)

$$\frac{7.2a}{a^2} = \frac{9}{25} \Rightarrow \frac{7.2}{a} = \frac{9}{25}$$

$$\Rightarrow a = 7.2 \times \frac{25}{9}$$

$$\Rightarrow a = 20$$

So,

$$b^2 = 7.2(20) = 144$$

$$b = 12$$

Sum of major and minor axes is,

$$2a + 2b = 2(a+b) = 2(20+12) = 64$$

45. *(b)* The given expression is $9^{1/3} \cdot 9^{1/9} \cdot 9^{1/27} \cdots \infty$.
This can be re-written as,

$$9^{\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\cdots\infty} = 9^{\frac{\frac{1}{3}}{1-\frac{1}{3}}} = 9^{\frac{1}{3}/\frac{2}{3}} = 9^{\frac{1}{2}} = 3$$

46. *(d)* LHL $= \lim_{h\to 0^-} e^{\frac{-1}{(0-h)}} = \lim_{h\to 0^-} e^{\frac{1}{h}} = e^{\infty} = \infty$.

RHL $= \lim_{h\to 0^+} e^{\frac{-1}{(0+h)}} = \lim_{h\to 0^+} e^{\frac{-1}{h}} = e^{-\infty} = 0$

As LHL is not equal to RHL, the limit at $x = 0$ does not exist.

So, $\lim_{x\to 0} e^{-1/x}$ does not exist.

47. *(d)* Given $f(x) = \begin{cases} 3x-4 & 0 \le x \le 2 \\ 2x+\lambda & 2 < x \le 3 \end{cases}$

Also, the given function is continuous at $x = 2$.

$$\therefore \lim_{x\to 2} f(x) = f(2)$$

Now, $f(2) = 3(2) - 4 = 6 - 4 = 2$

$$\Rightarrow \lim_{x\to 2}(2x+\lambda) = 2$$

$$\Rightarrow \quad 4 + \lambda = 2$$

$$\Rightarrow \quad \lambda = -2$$

48. *(c)* Let $y = \sin^{-1}\left(\frac{4x}{1+4x^2}\right) = \sin^{-1}\left(\frac{2\cdot 2x}{1+(2x)^2}\right)$

Put $2x = \tan\theta \Rightarrow \theta = \tan^{-1} 2x$

$$\therefore y = \sin^{-1}\left(\frac{2\cdot\tan\theta}{1+\tan^2\theta}\right)$$

$$= \sin^{-1}(\sin 2\theta)$$

$$= 2\theta$$

$$= 2\tan^{-1} 2x$$

On differentiating both sides, we get

$$\frac{dy}{dx} = \frac{2}{1+(2x)^2}\cdot 2 = \frac{4}{1+4x^2}$$

49. *(a)* To form a triangle, we need 3 points and a total of 12 points are given.

So, ${}^{12}C_3$ triangles can be formed in total.

But, given that 7 points are on a straight line. Now, selecting 3 points from this set will not form a triangle. So, the number of triangles formed is,

$${}^{12}C_3 - {}^{7}C_3 = \frac{12!}{3!9!} - \frac{7!}{3!4!}$$

$$= \frac{12\times 11\times 10}{3\times 2\times 1} - \frac{7\times 6\times 5}{3\times 2\times 1}$$

$$= 220 - 35$$

$$= 185$$

50. *(a)* Given that $xy = r^2$.

$$\Rightarrow y = \frac{r^2}{x}$$

Let $S = px + qy = px + \frac{qr^2}{x}$

$$\Rightarrow \frac{dS}{dx} = p - \frac{qr^2}{x^2}$$

Now, $\frac{dS}{dx}=0$ for maximum or minimum.

$$0=p-\frac{qr^2}{x^2}\Rightarrow x^2=\frac{qr^2}{p}$$

$$\Rightarrow x=\pm\sqrt{\frac{q}{p}}r$$

Now, $\frac{d^2S}{dx^2}=\frac{2qr^2}{x^3}$

At $x=\sqrt{\frac{q}{p}}r$, $\frac{d^2S}{dx^2}>0$.

So, S is minimum at $x=\sqrt{\frac{q}{p}}r$.

$$\Rightarrow y=\frac{r^2}{\sqrt{\frac{q}{p}}r}=\sqrt{\frac{p}{q}}r$$

Minimum value of $px+qy$ is,

$$p\sqrt{\frac{q}{p}}r+q\sqrt{\frac{p}{q}}r=\sqrt{pq}r+\sqrt{pq}r=2r\sqrt{pq}$$

51. *(d)* Since A is void set, the number of elements in power set of A is 1.

$\therefore P\{P(A)\}=2^1=2$

This is because if a set A has n elements, then $P(A)$ has 2^n elements.

$$\Rightarrow P\{P\{P(A)\}\}=2^2=4$$
$$\Rightarrow P\{P\{P\{P(A)\}\}\}=2^4=16$$

52. *(b)* $a=x+\sqrt{x^2+1}\Rightarrow a-x=\sqrt{x^2+1}$

Squaring both sides, we get

$$x^2+1=(a-x)^2$$
$$\Rightarrow x^2+1=a^2+x^2-2ax$$
$$\Rightarrow 2ax=a^2-1$$
$$\Rightarrow 2x=a-\frac{1}{a}$$
$$\Rightarrow x=\frac{1}{2}\left(a-a^{-1}\right)$$

53. *(d)* $\lim_{x\to\infty}\left(\frac{x+6}{x+1}\right)^{x+4}=\lim_{x\to\infty}\left(\frac{x+5+1}{x+1}\right)^{x+4}$.

$$=\lim_{x\to\infty}\left(\frac{x+1}{x+1}+\frac{5}{x+1}\right)^{x+4}$$

$$=\lim_{x\to\infty}\left(1+\frac{5}{x+1}\right)^{\frac{x+4}{5}\times\frac{5(x+1)}{x+1}}$$

$$=\left[\lim_{x\to\infty}\left(1+\frac{5}{x+1}\right)^{\frac{x+1}{5}}\right]^{5\left(\frac{x+4}{x+1}\right)}$$

$$=e^{5\lim_{x\to\infty}\left(\frac{x+4}{x+1}\right)}$$

$$=e^{5\lim_{x\to\infty}\left(\frac{1+\frac{4}{x}}{1+\frac{1}{x}}\right)}=e^5$$

54. *(a)* Let $x=k(\theta+\sin\theta)$ and $y=k(1+\cos\theta)$

Differentiate both the functions w.r.t θ

$$\frac{dx}{d\theta}=k(1+\cos\theta)\text{ and }\frac{dy}{d\theta}=-k\sin\theta$$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-k\sin\theta}{k(1+\cos\theta)}$$

$$= \frac{-2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}$$

$$= -\tan\frac{\theta}{2}$$

This gives,

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{2}} = -\tan\frac{\pi}{4} = -1$$

55. *(b)* Surface area of sphere, $S = 4\pi r^2$

Differentiate both sides w.r.t t.

$$\frac{dS}{dt} = 8\pi r\frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{1}{8\pi r}\frac{dS}{dt} \quad ...(\text{i})$$

Volume of sphere, $V = \frac{4}{3}\pi r^3$

$$\frac{dV}{dt} = \frac{4}{3}\pi\cdot 3r^2\frac{dr}{dt}$$

$$= 4\pi r^2\frac{dr}{dt}$$

$$= 4\pi r^2\left(\frac{1}{8\pi r}\frac{dS}{dt}\right) \ (\text{from (i)})$$

$$= \frac{1}{2}r\frac{dS}{dt}$$

56. *(c)* $f(x) = \frac{x^2}{e^x}$

$$f'(x) = \frac{2x\cdot e^x - e^x\cdot x^2}{(e^x)^2}$$

$$= \frac{2x - x^2}{e^x}$$

Since e^x is always positive, so for given function to be monotonically increasing, we have

$$2x - x^2 > 0 \Rightarrow x^2 - 2x < 0$$

$$\Rightarrow x(x-2) < 0$$

$$\Rightarrow x = (0, 2)$$

57. *(d)* Given expression $25\operatorname{cosec}^2 x + 36\sec^2 x$.

Least value $= \left(\sqrt{25}+\sqrt{36}\right)^2 = (5+6)^2 = 121$

58. *(a)* Let $y = \sin 2x\cdot\cos 2x$

$$\frac{dy}{dx} = \frac{d}{dx}(\sin 2x)\cdot\cos 2x + \sin 2x\cdot\frac{d}{dx}(\cos 2x)$$

$$= 2\left(\cos^2 2x - \sin^2 2x\right)$$

$$= 2\cos 4x$$

For maximum or minimum,

$$\frac{dy}{dx} = 0 \Rightarrow 4\cos 4x = 0$$

$$\Rightarrow \cos 4x = \cos\frac{\pi}{2}$$

$$\Rightarrow 4x = \frac{\pi}{2} \Rightarrow x = \frac{\pi}{8}$$

Now, $\frac{d^2y}{dx^2} = -4\sin 4x$

At $x = \frac{\pi}{8}, \frac{d^2y}{dx^2} = -4\sin\frac{\pi}{2} = -4$.

This implies that y is maximum at $x = \frac{\pi}{8}$.

$$y_{\max} = \sin 2\left(\frac{\pi}{8}\right)\cdot\cos 2\left(\frac{\pi}{8}\right)$$

$$= \sin\frac{\pi}{4}\cdot\cos\frac{\pi}{4}$$

$$= \frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}} = \frac{1}{2}$$

59. *(b)* Let $f(x) = \sin 4x + 2x$

$$f'(x) = 4\cos 4x + 2$$

For extreme values,

$f'(x)=0 \Rightarrow 4\cos 4x+2=0$

$\Rightarrow \quad \cos 4x=-\frac{1}{2}$

$\Rightarrow \quad \cos 4x=\cos\left(\pi-\frac{\pi}{3}\right)$

$\Rightarrow \quad \cos 4x=\cos\frac{2\pi}{3}$

This gives, $4x=2n\pi\pm\frac{2\pi}{3}$

$x=\frac{n\pi}{2}\pm\frac{\pi}{6}$

$x=\frac{\pi}{6},\frac{\pi}{3}\left[\because 0<x<\frac{\pi}{2}\right]$

60. *(b)* Let $I=\int \log(x+1)dx$

Let $x+1=t \Rightarrow dx=dt$

Integrating by parts, taking log t as first function.

$\Rightarrow I=t\log t-\int\frac{1}{t}\cdot t\,dt+c_1$

$=t\log t-\int 1\,dt+c_1$

$=t\log t-t+c_1$

Substitute the value of t

$I=(x+1)\log(x+1)-x-1+c_1$

$=(x+1)\log(x+1)-x+c\ [c=-1+c_1]$

61. *(b)* Given $\cos A=\frac{3}{4}$.

$\sin\left(\frac{A}{2}\right)\sin\left(\frac{3A}{2}\right)=\frac{1}{2}\left(2\sin\frac{A}{2}\sin\frac{3A}{2}\right)$

$=\frac{1}{2}\left(\cos\left(\frac{3A}{2}-\frac{A}{2}\right)-\cos\left(\frac{3A}{2}+\frac{A}{2}\right)\right)$

$=\frac{1}{2}(\cos A-\cos 2A)$

$=\frac{1}{2}(\cos A-\cos 2A)$

$=\frac{1}{2}(\cos A-2\cos^2 A+1)$

$=\frac{1}{2}\left(\frac{3}{4}-2\times\left(\frac{3}{4}\right)^2+1\right)$

$=\frac{1}{2}\left(\frac{3}{4}-\frac{18}{16}+1\right)$

$=\frac{1}{2}\left(\frac{10}{16}\right)$

$=\frac{5}{16}$

62. *(b)* Let $I=\int e^{\ln x}\sin x\,dx$.

Since $e^{\ln a}=a$, we get

$I=\int x\sin x\,dx$

$=-x\cos x+\int 1\cdot\cos x\,dx$

$=-x\cos x+\sin x+c$

$=\sin x-x\cos x+c$

63. *(a)* Let the given integral be $I=\int_0^1 x(1-x)^9\,dx$.

Put $1-x=t \Rightarrow x=1-t$ and $dx=-dt$

When $x=0,\ t=1$ and when $x=1,\ t=0$

$\Rightarrow I=\int_1^0 (1-t)t^9(-dt)$

$=\int_1^0\left(t^{10}-t^9\right)dt$

$=\left[\frac{t^{11}}{11}-\frac{t^{10}}{10}\right]_1^0$

$=0-\frac{1}{11}+\frac{1}{10}=\frac{1}{110}$

64. *(d)* $\int\frac{dx}{x\left(x^7+1\right)}=\int\frac{x^6dx}{x^7\left(x^7+1\right)}$

Let $x^7 = t \Rightarrow 7x^6 dx = dt$

Then, $\int \frac{dx}{x\left(x^7+1\right)} = \frac{1}{7}\int \frac{dt}{t(t+1)}$

$= \frac{1}{7}\int \left[\frac{1}{t} - \frac{1}{t+1}\right] dt$

$= \frac{1}{7}\left[\int \frac{1}{t}dt - \int \frac{1}{t+1}dt\right]$

$= \frac{1}{7}\left[\ln|t| - \ln|t+1|\right] + c$

$= \frac{1}{7}\ln\left|\frac{t}{t+1}\right| + c$

$= \frac{1}{7}\ln\left|\frac{x^7}{x^7+1}\right| + c$

65. *(b)* Given curve is $f(x) = xe^x$, $x = 0$ and $x = 1$

So, required area is $\int_0^1 f(x)\,dx = \int_0^1 xe^x dx$

Use integration by parts,

$$\int_0^1 f(x)\,dx = x\int_0^1 e^x dx - \int_0^1 \frac{d}{dx}(x)\left(\int e^x dx\right)dx$$

$$= \left[xe^x\right]_0^1 - \int_0^1 e^x\,dx$$

Integrate and substitute the limits

$$\int_0^1 f(x)\,dx = \left[xe^x\right]_0^1 - \left[e^x\right]_0^1$$

$$= e^1 - \left(e^1 - e^0\right)$$

$$= e^1 - \left(e^1 - 1\right)$$

$$= 1$$

66. *(c)* Let $I = \int_{-1}^{1} x|x|\,dx = \int_{-1}^{0} x(-x)\,dx + \int_0^1 x(x)\,dx$

$$\left[\because |x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}\right]$$

$$I = \int_{-1}^{1} x|x|\,dx = -\int_{-1}^{0} x^2\,dx + \int_0^1 x^2\,dx$$

$$= \left[-\frac{x^3}{3}\right]_{-1}^{0} + \left[\frac{x^3}{3}\right]_0^1$$

$$= -\left(0 + \frac{1}{3}\right) + \frac{1}{3}$$

$$= 0$$

67. *(d)* The given differential equation is,

$$\left[\frac{d^2y}{dx^2}\right] = \left[y + \left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{4}}$$

This can be re-written as by raising both the sides to the power 4 to make it a polynomial of derivative.

$$\left[\frac{d^2y}{dx^2}\right]^4 = y + \left(\frac{dy}{dx}\right)^2$$

Power of highest ordered derivative is 4.

So, the degree of the equation is 4.

68. *(a)* Given equation is

$$y = ax^2 + bx \quad ...(\text{i})$$

Differentiating both sides w.r.t x

$$\frac{dy}{dx} = 2ax + b \quad ...(\text{ii})$$

Again, differentiating w.r.t. x

$$\frac{d^2y}{dx^2} = 2a$$

$$a = \frac{1}{2}\frac{d^2y}{dx^2} \quad ...(\text{iii})$$

From (ii) and (iii),

$$b = \frac{dy}{dx} - x\frac{d^2y}{dx^2}$$

Substitute the values of a and b in (1)

$$y = \frac{1}{2}\frac{d^2y}{dx^2}\cdot x^2 + x\left(\frac{dy}{dx} - x\frac{d^2y}{dx^2}\right)$$

$$\Rightarrow 2y = x^2 \frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 2x^2 \frac{d^2y}{dx^2}$$

$$\Rightarrow 2y = -x^2 \frac{d^2y}{dx^2} + 2x\frac{dy}{dx}$$

This gives,

$$x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$$

69. *(a)* $\int \frac{dx}{x\left(x^n+1\right)} = \int \frac{x^{n-1}dx}{x^n\left(x^n+1\right)}$

Let $x^n = z \Rightarrow nx^{n-1}dx = dz$

Then, $\int \frac{dx}{x\left(x^n+1\right)} = \frac{1}{n}\int \frac{dz}{z(z+1)}$

$$= \frac{1}{n}\int\left[\frac{1}{z} - \frac{1}{z+1}\right]dz$$

$$= \frac{1}{n}\left[\int \frac{1}{z}dz - \int \frac{1}{z+1}dz\right]$$

$$= \frac{1}{n}\left[\ln|z| - \ln|z+1|\right] + c$$

$$= \frac{1}{n}\ln\left|\frac{z}{z+1}\right| + c = \frac{1}{n}\ln\left|\frac{x^n}{x^n+1}\right| + c$$

70. *(b)* Given equation of ellipse is $x^2 + 2y^2 = 1$

It can be re-written as $\frac{x^2}{1} + \frac{y^2}{1/2} = 1$

So, we have $a^2 = 1$ and $b^2 = \frac{1}{2}$.

$$\because\ b^2 = a^2 - c^2$$

$$\Rightarrow c^2 = a^2 - b^2$$

$$= 1 - \frac{1}{2}$$

$$= \frac{1}{2}$$

$$\Rightarrow c = \frac{1}{\sqrt{2}}$$

The distance between the foci of ellipse is

$$2c = \frac{2}{\sqrt{2}} = \sqrt{2}$$

71. *(b)* Harmonic mean of three number x_1, x_2, x_3 is,

$$\frac{3}{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3}}$$

$$\therefore \text{H.M.} = \frac{3}{\frac{1}{3} + \left(\frac{-1}{6}\right) + \left(\frac{-1}{6}\right)} = \frac{3}{\frac{1}{3} - \frac{1}{3}} = \frac{3}{0} = \infty$$

72. *(a)* Given limit $\lim_{x\to\infty}\left(\sqrt{a^2x^2+ax+1} - \sqrt{a^2x^2+1}\right)$.

$$= \lim_{x\to\infty} \frac{\left(\sqrt{a^2x^2+ax+1} - \sqrt{a^2x^2+1}\right) \times \left(\sqrt{a^2x^2+ax+1} + \sqrt{a^2x^2+1}\right)}{\sqrt{a^2x^2+ax+1} + \sqrt{a^2x^2+1}}$$

$$= \lim_{x\to\infty} \frac{ax}{x\sqrt{a^2 + \frac{a}{x} + \frac{1}{x^2}} + \sqrt{a^2 + \frac{1}{x^2}}}$$

$$= \lim_{x\to\infty} \frac{a}{\sqrt{a^2 + \frac{a}{x} + \frac{1}{x^2}} + \sqrt{a^2 + \frac{1}{x^2}}}$$

$$= \frac{a}{\sqrt{a^2} + \sqrt{a^2}} = \frac{a}{2a} = \frac{1}{2}$$

73. *(a)* The given differential equation is,

$$\left(1+e^x\right)y\,dy = e^x\,dx$$

By separating the variables, we get

$$y\,dy = \frac{e^x}{1+e^x}dx$$

Integrating both sides,

$$\int y\,dy = \int \frac{e^x}{1+e^x}\,dx$$

$$\Rightarrow \frac{y^2}{2} = \log\left(1+e^x\right) + \log c$$

$$\Rightarrow y^2 = 2\log\left[c\left(1+e^x\right)\right]$$

$$[\because \log m + \log n = \log mn]$$

$$\Rightarrow y^2 = 2\log\left[c^2\left(1+e^x\right)^2\right]$$

$$\left[\because a\log m = \log m^a\right]$$

74. *(c)* Let the AP be

$a, a+d, a+2d, \ldots, a+(2n-1)d,\ a+2nd$

Series of even terms,

$a+d, a+3d, \ldots, a+(2n-1)d$ has n terms.

Sum of even numbers

$$= \frac{n}{2}\left[(a+d)+\{a+(2n-1)d\}\right]$$

$$= \frac{n}{2}[2a+2nd] = n(a+nd)$$

Series of odd terms,

$a,\ a+2d, a+4d, \ldots, a+2nd$ has n + 1 terms.

Sum of even numbers

$$= \frac{n+1}{2}\left[a+(a+2nd)\right]$$

$$= \frac{n+1}{2}[2a+2nd] = (n+1)(a+nd)$$

So, the required ratio is $\frac{n+1}{n}$.

75. *(b)* Given AB = 6 cm, BC = 8 cm and CA = 10 cm.

So, $a = 8$ cm, $b = 10$ cm, $c = 6$ cm

and $s = \frac{a+b+c}{2} = \frac{24}{2} = 12$

$$\therefore \tan\frac{A}{2} = \sqrt{\frac{(12-10)(12-6)}{12(12-8)}}$$

$$\left[\because \tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}\right]$$

$$= \sqrt{\frac{1}{4}} = \frac{1}{2}$$

$$\therefore \cot\frac{A}{2} = 2$$

Now, $\cot\left(\frac{A}{4}+\frac{A}{4}\right) = \frac{\cot^2\frac{A}{4}-1}{2\cot\frac{A}{4}}$

$$\cot\left(\frac{A}{2}\right) = \frac{\cot^2\frac{A}{4}-1}{2\cot\frac{A}{4}}$$

Let $\cot\frac{A}{4} = x$

$$\therefore\ 2 = \frac{x^2-1}{2x}$$

$$\Rightarrow x^2 - 4x - 1 = 0$$

$$\Rightarrow \quad x = \frac{4\pm\sqrt{16+4}}{2}$$

$$\Rightarrow \quad x = \frac{4\pm 2\sqrt{5}}{2} = 2\pm\sqrt{5}$$

Let $\cot\frac{A}{4} = 2+\sqrt{5}$ or $2-\sqrt{5}$

76. *(a)* The given differential equation is

$$\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$$

$$\Rightarrow \frac{dy}{\sqrt{1-y^2}} = \frac{-dx}{\sqrt{1-x^2}}$$

This gives,

$$\int \frac{dy}{\sqrt{1-y^2}} + \int \frac{dx}{\sqrt{1-x^2}} = 0$$

$$\Rightarrow \quad \sin^{-1} y + \sin^{-1} x = C$$

77. *(a)* $f: R \to R,\ f(x) = \begin{cases} x^2, & x \ge 0 \\ -x, & x < 0 \end{cases}$

For continuity at $x = 0$

$$f(0-0) = \lim_{h\to 0} f(0-h)$$

$$= \lim_{h\to 0}[0-h] = \lim_{h\to 0} h = 0$$

$$f(0+0) = \lim_{h\to 0} f(0+h)$$

$$= \lim_{h\to 0}(0+h)^2$$

$$= 0$$

Also, $f(0) = 0$

So, f is continuous at $x = 0$.

For differentiability,

$$\text{LHD} = \lim_{h\to 0} \frac{f(0-h) - f(0)}{-h} = \lim_{h\to 0} \frac{-(-h)-0}{-h}$$

$$= \lim_{h\to 0} \frac{h}{-h} = -1$$

$$\text{RHD} = \lim_{h\to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h\to 0} \frac{h^2 - 0}{h}$$

$$= \lim_{h\to 0} h = 0$$

So, f is non differentiable at $x = 0$.

78. *(c)* $\frac{dy}{dx} = \sqrt{1 - x^2 - y^2 + x^2y^2}$

This can be re-written as

$$\frac{dy}{dx} = \sqrt{(1-x^2)(1-y^2)}$$

Separate the variables,

$$\frac{dy}{\sqrt{1-y^2}} = \sqrt{1-x^2}\,dx$$

Integrating both sides,

$$\sin^{-1}\left(\frac{y}{1}\right) = \frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}\left(\frac{x}{1}\right) + c$$

$$2\sin^{-1} y = x\sqrt{1-x^2} + \sin^{-1} x + c$$

79. *(d)* Let $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix},\ B = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix},\ C = \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix}$

Now, $A^2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ ...(i)

$B^2 = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}\begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ ...(ii)

From (i) and (ii), we have

$$A^2 = B^2$$

Now, $C^2 = \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix}\begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ ..(iii)

From (ii) and (iii), we have

$$B^2 = C^2$$

Now, $AB = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix} = \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix} = C$

Next, $BA = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix} \ne C$

Hence, $AB \ne BA$

80. *(b)* Let $c = 2,\ \angle A = 120°,\ a = \sqrt{6}$

By Sine rule, we have

$$\frac{a}{\sin A} = \frac{c}{\sin C} \Rightarrow \frac{\sqrt{6}}{\sin 120°} = \frac{2}{\sin C}$$

$$\Rightarrow \sin C = \frac{2\times\sqrt{3}}{\sqrt{6}\times 2} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \sin C = \sin 45°$$

$$\Rightarrow \quad C = 45°$$

81. *(c)* From the given conditions, we get

$$P(T) = \frac{1}{3}, P\left(\frac{H}{T}\right) = 1,\ P(F) = \frac{1}{3}, P\left(\frac{H}{F}\right) = \frac{1}{2}$$

$$P(B) = \frac{1}{3}, P\left(\frac{H}{B}\right) = \frac{3}{4}$$

Using Bayes theorem,

$$P\left(\frac{T}{H}\right)=\frac{P(T)P\left(\frac{H}{T}\right)}{P(T)P\left(\frac{H}{T}\right)+P(F)P\left(\frac{H}{F}\right)+P(B)P\left(\frac{H}{B}\right)}$$

$$=\frac{\frac{1}{3}}{\frac{1}{3}+\frac{1}{6}+\frac{1}{4}}=\frac{\frac{1}{9}}{\frac{9}{4}}=\frac{4}{9}$$

82. *(c)* Let X and Y be two matrices of order 2×2 each.
Given

$$2X-3Y=\begin{bmatrix}-7 & 0\\ 7 & -13\end{bmatrix} \quad ...(i)$$

$$3X+2Y=\begin{bmatrix}9 & 13\\ 4 & 13\end{bmatrix} \quad ...(ii)$$

Eq.(i)×3 and Eq.(ii)×2, we get

$$6X-9Y=\begin{bmatrix}-21 & 0\\ 21 & -39\end{bmatrix} \quad ...(iii)$$

$$6X+4Y=\begin{bmatrix}18 & 26\\ 8 & 26\end{bmatrix} \quad ...(iv)$$

Subtract eq. (iii) from eq. (iv)

$$13Y=\begin{bmatrix}39 & 26\\ -13 & 65\end{bmatrix}\Rightarrow Y=\begin{bmatrix}3 & 2\\ -1 & 5\end{bmatrix}$$

83. *(a)* Since $\lambda\vec{a}+\vec{b}$ and $\vec{a}-\lambda\vec{b}$ are perpendicular to each other, we have $\left(\lambda\vec{a}+\vec{b}\right)\cdot\left(\vec{a}-\lambda\vec{b}\right)=0$

$$\Rightarrow \lambda\vec{a}\cdot\vec{a}-\lambda^2\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{a}-\lambda\vec{b}\cdot\vec{b}=0$$

$$\Rightarrow \lambda\left|\vec{a}\right|^2+\left(1-\lambda^2\right)\vec{a}\cdot\vec{b}-\lambda\left|\vec{b}\right|^2=0$$

$$\Rightarrow \lambda\left|\vec{a}\right|^2+\left(1-\lambda^2\right)\vec{a}\cdot\vec{b}-\lambda\left|\vec{a}\right|^2=0 \quad \left[\because \left|\vec{a}\right|=\left|\vec{b}\right|\right]$$

$$\Rightarrow \left(1-\lambda^2\right)\vec{a}\cdot\vec{b}=0 \quad ...(i)$$

Since $\cos 60^\circ=\frac{\vec{a}\cdot\vec{b}}{\left|\vec{a}\right|\cdot\left|\vec{b}\right|}=\frac{\vec{a}\cdot\vec{b}}{\left|\vec{a}\right|^2}\Rightarrow \vec{a}\cdot\vec{b}=\left|\vec{a}\right|^2\cos 60^\circ$

Thus, from (i), we have

$$\left(1-\lambda^2\right)\left|\vec{a}\right|^2\cos 60^\circ=0$$

$$\left(1-\lambda^2\right)\left|\vec{a}\right|^2\cdot\frac{1}{2}=0$$

$$1-\lambda^2=0$$

$$\lambda=\pm 1$$

84. *(a)* Let $\Delta=\begin{vmatrix}x^2 & 1 & y^2+z^2\\ y^2 & 1 & z^2+x^2\\ z^2 & 1 & x^2+y^2\end{vmatrix}$

Applying $C_1\to C_1+C_3$

$$\Delta=\begin{vmatrix}x^2+y^2+z^2 & 1 & y^2+z^2\\ x^2+y^2+z^2 & 1 & z^2+x^2\\ x^2+y^2+z^2 & 1 & x^2+y^2\end{vmatrix}$$

$$=\left(x^2+y^2+z^2\right)\begin{vmatrix}1 & 1 & y^2+z^2\\ 1 & 1 & z^2+x^2\\ 1 & 1 & x^2+y^2\end{vmatrix}$$

$$=0 \text{ (Since } C_1 \text{ and } C_2 \text{ are identical)}$$

85. *(a)* Let $A=(0,2,2), B=(2,0,-1)$ and $C=(3,4,0)$

$\overrightarrow{AB}=(2-0,0-2,-1-2)$ and $\overrightarrow{AC}=(3-0,4-2,0-2)$

$\Rightarrow \overrightarrow{AB}=(2-,-2,-3)$ and $\overrightarrow{AC}=(3-,2,-2)$

$\therefore$ Area of triangle $=\frac{1}{2}\times$ magnitude of $\overrightarrow{AB}\times\overrightarrow{AC}$

$$=\frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|$$

$$=\frac{1}{2}\left\|\begin{matrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -2 & -3\\ 3 & 2 & -2\end{matrix}\right\|$$

$$=\frac{1}{2}\left|\hat{i}(4+6)-\hat{j}(-4+9)+\hat{k}(4+6)\right|$$

$$=\frac{1}{2}\left|10\hat{i}-5\hat{j}+10\hat{k}\right|$$

$$=\frac{1}{2}\sqrt{100+25+100}$$

$$=\frac{15}{2}$$

86. *(c)* Let $\vec{a}$ and $\vec{b}$ be two unit vectors.

$\therefore\ |\vec{a}|=1$ and $|\vec{b}|=1$

Since α be the angle between $\vec{a}$ and $\vec{b}$,

$$\cos\alpha=\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}=\frac{\vec{a}\cdot\vec{b}}{1\cdot 1}=\vec{a}\cdot\vec{b}$$

As, $\vec{a}+\vec{b}$ is also a unit vector, we have

$$|\vec{a}+\vec{b}|=1$$

Squaring both sides,

$$|\vec{a}+\vec{b}|^2=1$$

$$\Rightarrow |\vec{a}|^2+|\vec{b}|^2+2\vec{a}\cdot\vec{b}=1$$

$$\Rightarrow 1+1+2\cos\alpha=1$$

$$\Rightarrow \cos\alpha=-\frac{1}{2}=\cos\frac{2\pi}{3}$$

$$\Rightarrow \alpha=\frac{2\pi}{3}$$

87. *(b)* Given $\overline{A}$ and $\overline{B}$ are mutually exclusive.

So, we have $P(\overline{A}\cap\overline{B})=0$

Given, $P(A)=0.5$ and $P(B)=0.6$

So, we have $P(\overline{A})=0.5$ and $P(\overline{B})=0.4$

This implies that,

$$P(A/B)=\frac{P(A\cap B)}{P(B)}$$

$$=\frac{1-P(\overline{A}\cup\overline{B})}{P(B)}$$

$$=\frac{1-\left[P(\overline{A})+P(\overline{B})\right]}{P(B)}$$

$$=\frac{1-(0.5+0.4)}{0.6}$$

$$=\frac{1-0.9}{0.6}=\frac{0.1}{0.6}=\frac{1}{6}$$

88. *(c)* Let ABC be a triangle with sides

$a=1+\sqrt{3},\ b=2$ and $c=\sqrt{6}$

So, $\cos B=\dfrac{a^2+c^2-b^2}{2ac}=\dfrac{(1+\sqrt{3})^2+(\sqrt{6})^2-4}{2(1+\sqrt{3})(\sqrt{6})}$

$$=\frac{2\sqrt{3}+6}{2(\sqrt{6}+\sqrt{18})}=\frac{\sqrt{3}+3}{\sqrt{6}+3\sqrt{2}}$$

$$=\frac{\sqrt{3}+\sqrt{3}\sqrt{3}}{(\sqrt{3}+\sqrt{3}\sqrt{3})\sqrt{2}}=\frac{1}{\sqrt{2}}$$

This implies that $B=45°$ is the smallest angle as smallest side is b = 2.

89. *(a)* Let $\overrightarrow{PQ}=3\hat{i}+2\hat{j}-m\hat{k}$, $\overrightarrow{PS}=\hat{i}+3\hat{j}+\hat{k}$, where PQRS is a parallelogram.

$$\text{Area of parallelogram}=\left\|\begin{matrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 2 & -m\\ 1 & 3 & 1\end{matrix}\right\|$$

$$=\left|\hat{i}(2+3m)-\hat{j}(3+m)+\hat{k}(9-2)\right|$$

$$=\sqrt{(2+3m)^2+(3+m)^2+7^2}$$

Given area is $\sqrt{90}$, which gives

$$(2+3m)^2+(3+m)^2+7^2=90$$

$$\Rightarrow 4+9m^2+12m+9+m^2+6m+49=90$$

$$\Rightarrow 10m^2+18m-28=0$$

$$\Rightarrow 5m^2+9m-14=0$$

$$\Rightarrow (5m+14)(m-1)=0$$

$$\Rightarrow m=-\frac{14}{5},1$$

90. *(c)* Let position vectors of the two points be

$$\overrightarrow{OA}=(\sqrt{3}-1)\hat{i}-(\sqrt{3}+1)\hat{j}+4\hat{k}$$

$$\overrightarrow{OB}=-(\sqrt{3}+1)\hat{i}+(\sqrt{3}-1)\hat{j}+4\hat{k}$$

$$\left|\overrightarrow{OA}\right| = \sqrt{\left(\sqrt{3}-1\right)^2 + \left(\sqrt{3}+1\right)^2 + 4^2}$$
$$= \sqrt{3+1-2\sqrt{3}+3+1+2\sqrt{3}+16}$$
$$= \sqrt{24}$$
$$\left|\overrightarrow{OB}\right| = \sqrt{\left(\sqrt{3}+1\right)^2 + \left(\sqrt{3}-1\right)^2 + 4^2}$$
$$= \sqrt{3+1+2\sqrt{3}+3+1-2\sqrt{3}+16}$$
$$= \sqrt{24}$$
$$\overrightarrow{OA}\cdot\overrightarrow{OB} = -\left(\sqrt{3}-1\right)\left(\sqrt{3}+1\right) - \left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right) + 4\times 4$$
$$= -(3-1)-(3-1)+16$$
$$= -3+1-3+1+16$$
$$= 12$$

Required angle is,

$$\cos\theta = \frac{\overrightarrow{OA}\cdot\overrightarrow{OB}}{\left|\overrightarrow{OA}\right|\left|\overrightarrow{OB}\right|}$$
$$= \frac{12}{\sqrt{24}\sqrt{24}}$$
$$= \frac{1}{2}$$
$$\Rightarrow \cos\theta = \cos\frac{\pi}{3}$$
$$\Rightarrow \quad \theta = \frac{\pi}{3}$$

91. *(c)* Mean, $\bar{x} = np = 12 \quad ...(\text{i})$.

and standard deviation $\sqrt{npq} = 2$, which gives

$npq = 4 \quad ...(\text{ii})$

Dividing (ii) by (i),

$$\frac{npq}{np} = \frac{4}{12} \Rightarrow q = \frac{1}{3}$$

This gives, $p = 1 - \frac{1}{3} = \frac{2}{3}$

From (i), $np = 12 \Rightarrow n\left(\frac{2}{3}\right) = 12 \Rightarrow n = 18$

92. *(c)* Let the direction ratio of the line be a, b, c.

The line is contained by both the planes

$3x + y + 2z = 7,\ x + 2y + 3z = 5$

$\Rightarrow \quad 3a + b + 2c = 7 \quad ...(\text{i})$

and $\ a + 2b + 3c = 5 \quad ...(\text{ii})$

Solving these two equations,

$$\frac{a}{\begin{vmatrix}1 & 2\\ 2 & 3\end{vmatrix}} = \frac{-b}{\begin{vmatrix}3 & 2\\ 1 & 3\end{vmatrix}} = \frac{c}{\begin{vmatrix}3 & 1\\ 1 & 2\end{vmatrix}} = k$$
$$\Rightarrow \frac{a}{-1} = \frac{-b}{7} = \frac{c}{5} = k$$
$$\Rightarrow a = -k,\ b = -7k,\ c = 5k$$

So, the direction cosines are

$$\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}$$

Now, $\sqrt{a^2+b^2+c^2} = \sqrt{k^2+49k^2+25k^2}$

$$= \sqrt{75k^2}$$
$$= \sqrt{75}\,k$$

So, we have direction cosines as

$$\left(\frac{-k}{\sqrt{75}k}, \frac{-7k}{\sqrt{75}k}, \frac{5k}{\sqrt{75}k}\right) = \left(\frac{-1}{\sqrt{75}}, \frac{-7}{\sqrt{75}}, \frac{5}{\sqrt{75}}\right)$$

93. *(a)* Let A and B are two matrices such that

$AB = A$ and $BA = B$,.

Now consider $AB = A$

Take transpose on both sides

$$(AB)^T = A^T$$
$$\Rightarrow A^T = B^T A^T \quad ...(\text{i})$$

Now, $BA = B$

Take transpose on both sides

$$(BA)^T = B^T$$
$$\Rightarrow B^T = A^T B^T \quad ...(\text{ii})$$

Now, from equations (i) and (ii), we have

$$A^T = \left(A^T B^T\right) A^T$$
$$= A^T \left(B^T A^T\right)$$
$$= A^T (AB)^T$$
$$= A^T A^T$$

Thus, $A^T = \left(A^T\right)^2$

94. *(b)* We know that if,

$a_1 x + b_1 y + c_1 z = d_1$ and

$a_2 x + b_2 y + c_2 z = d_2$

are two planes, then angle between them is

$$\cos\theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

Here, given equation of planes are $2x - y + z = 6$ and $x + y + 2z = 3$

So, we have

$a_1 = 2,\ b_1 = -1,\ c_1 = 1,\ d_1 = 6$ and

$a_2 = 1,\ b_2 = 1,\ c_2 = 2,\ d_2 = 3$

$$\therefore\ \cos\theta = \frac{2\times1 + 1\times(-1) + 1\times2}{\sqrt{4+1+1}\sqrt{1+1+4}} = \frac{3}{6} = \frac{1}{2}$$

$$\therefore\ \cos\theta = \cos\frac{\pi}{3} \Rightarrow \theta = \frac{\pi}{3}$$

95. *(b)* We know that the angle between the vectors $a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$ and $a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$ is given by,

$$\cos\theta = \left|\frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}\right|$$

So, the angle between the vectors $\hat{i} + 2\hat{j} + 3\hat{k}$ and $-\hat{i} + 2\hat{j} + 3\hat{k}$ is

$$\cos\theta = \left|\frac{1\times(-1) + 2\times2 + 3\times3}{\sqrt{1+4+9}\sqrt{1+4+9}}\right|$$

$$\Rightarrow \cos\theta = \left|\frac{-1+4+9}{\sqrt{14}\sqrt{14}}\right| = \frac{12}{14} = \frac{6}{7}$$

Now, $\sin\theta = \sqrt{1 - \frac{36}{49}} = \sqrt{\frac{13}{49}} = \frac{\sqrt{13}}{7}$

96. *(d)* Direction ratios are $\langle a+b, b+c, c+a\rangle$

Then, direction cosines are

$$l = \frac{a+b}{\sqrt{(a+b)^2 + (b+c)^2 + (c+a)^2}}$$

$$m = \frac{b+c}{\sqrt{(a+b)^2 + (b+c)^2 + (c+a)^2}}$$

$$n = \frac{c+a}{\sqrt{(a+b)^2 + (b+c)^2 + (c+a)^2}}$$

Sum of the squares of the direction cosines

$$l^2 + m^2 + n^2 = \frac{(a+b)^2 + (b+c)^2 + (c+a)^2}{(a+b)^2 + (b+c)^2 + (c+a)^2} = 1$$

97. *(d)* Given vectors $-\hat{i} - 2x\hat{j} - 3y\hat{k}$ and $\hat{i} - 3x\hat{j} - 2y\hat{k}$ are orthogonal to each other.

$$\therefore\quad \left(-\hat{i} - 2x\hat{j} - 3y\hat{k}\right)\cdot\left(\hat{i} - 3x\hat{j} - 2y\hat{k}\right) = 0$$

$$\Rightarrow (-1)(1) + (-2x)(-3x) + (-3y)(-2y) = 0$$

$$\Rightarrow\quad -1 + 6x^2 + 6y^2 = 0$$

$$\Rightarrow\quad 6x^2 + 6y^2 = 1$$

$$\Rightarrow\quad x^2 + y^2 = \left(\frac{1}{\sqrt{6}}\right)^2$$

Hence, the locus of (x, y) is a circle.

98. *(d)* Given observations are: 7, 9, 11, 13, 15

$$\bar{x} = \frac{7+9+11+13+15}{5} = \frac{55}{5} = 11$$

Now, variance = $\frac{1}{n}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2$

$$=\frac{1}{5}\left\{\begin{matrix}(7-11)^2+(9-11)^2+(11-11)^2\\+(13-11)^2+(15-11)^2\end{matrix}\right\}$$

$$=\frac{1}{5}(16+4+4+16)$$

$$=\frac{1}{5}\times 40=8$$

$\therefore$ S.D. $=\sqrt{\text{variance}}=\sqrt{8}=2\sqrt{2}=2.8$

99. *(c)* Let $\vec{a}=(2,1,-1),\vec{b}=(1,-1,0),\vec{c}=(5,-1,1)$

$\therefore\ \vec{a}+\vec{b}-\vec{c}=(-2,1,-2)$

Let $\vec{n}=x\hat{i}+y\hat{j}+z\hat{k}$ be the unit vector which is parallel to $(-2,1,-2)$ in the opposite direction.

$$\therefore\ x^2+y^2+z^2=1 \text{ and } \begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\x&y&z\\-2&1&-2\end{vmatrix}=0$$

$\Rightarrow x=-2y,\ y=y,\ z=-2y$

This gives,

$$x^2+y^2+z^2=1$$
$$\Rightarrow 4y^2+y^2+4y^2=1$$
$$\Rightarrow 9y^2=1$$
$$\Rightarrow y=\pm\frac{1}{3}$$

Hence, the required vector is

$$\vec{n}=\frac{2}{3}\hat{i}-\frac{1}{3}\hat{j}+\frac{2}{3}z\hat{k}$$

100. *(c)* $x=4\tan^{-1}\left(\frac{1}{5}\right)=2\left(2\tan^{-1}\left(\frac{1}{5}\right)\right)$

$$x=2\left(\tan^{-1}\frac{\frac{2}{5}}{1-\frac{1}{25}}\right)=2\tan^{-1}\left(\frac{\frac{2}{5}\times 25}{24}\right)$$

$$=2\tan^{-1}\frac{10}{24}=2\tan^{-1}\frac{5}{12}$$

$$=\tan^{-1}\frac{2\times\frac{5}{12}}{1-\frac{25}{144}}=\tan^{-1}\frac{120}{119}$$

So, $x-y=\tan^{-1}\left(\frac{120}{119}\right)-\tan^{-1}\left(\frac{1}{70}\right)$

$$=\tan^{-1}\left(\frac{\frac{120}{119}-\frac{1}{70}}{1+\frac{120}{119}\cdot\frac{1}{70}}\right)=\tan^{-1}\left(\frac{\frac{8400-119}{8330}}{\frac{8330+120}{8330}}\right)$$

$$=\tan^{-1}\left(\frac{\frac{8281}{8330}}{\frac{8450}{8330}}\right)$$

$$=\tan^{-1}\left(\frac{8281}{8450}\right)$$

101. *(c)* In ΔABC, $\tan 45^\circ=\frac{AB}{BC}$

$$\Rightarrow 1=\frac{h}{x}$$

$$\Rightarrow x=h \quad \ldots(\text{i})$$

Now, in ΔABD, $\tan 30^\circ=\frac{AB}{BD}$

$$\Rightarrow \frac{1}{\sqrt{3}}=\frac{h}{x+50}$$

$$\Rightarrow x+50=\sqrt{3}h$$

$$\Rightarrow h+50=\sqrt{3}h \quad (\text{from (i)})$$

$$\Rightarrow h=\frac{50}{\sqrt{3}-1}$$

102. *(d)* $\lim\limits_{x\to 0}\frac{x}{\sqrt{1-\cos x}}=\lim\limits_{x\to 0}\frac{x}{\sqrt{1-\left(1-2\sin^2\frac{x}{2}\right)}}$

$$=\lim_{x\to 0}\frac{x}{\sqrt{2\sin^2\frac{x}{2}}}=\frac{1}{\sqrt{2}}\lim_{x\to 0}\frac{x}{\left|\sin\frac{x}{2}\right|}$$

$$\text{LHL}=f(0-0)=\lim_{h\to 0}f(0-h)=\frac{1}{\sqrt{2}}\lim_{h\to 0}\frac{h}{\sin\frac{h}{2}}$$

$$=-\frac{1}{\sqrt{2}}\lim_{h\to 0}\frac{2\left(\frac{h}{2}\right)}{\sin\frac{h}{2}}$$

$$=-\frac{1}{\sqrt{2}}\times 2\times 1\ \left[\because \lim_{x\to 0}\frac{x}{\sin x}=1\right]$$

$$=-\sqrt{2}$$

$$\text{RHL}=f(0+0)=\lim_{h\to 0}f(0+h)=\frac{1}{\sqrt{2}}\lim_{h\to 0}\frac{2\left(\frac{h}{2}\right)}{\sin\frac{h}{2}}$$

$$=\frac{1}{\sqrt{2}}\times 2\times 1=\sqrt{2}$$

Since LHL and RHL are not the same, the limit does not exist.

103. *(a)* $f(x)=\begin{cases}\dfrac{\alpha\cos x}{\pi-2x} & \text{if } x\neq\dfrac{\pi}{2}\\ 3 & \text{if } x=\dfrac{\pi}{2}\end{cases}$.

For continuity at $x=\frac{\pi}{2}$

$$\text{LHL}=\lim_{x\to\frac{\pi}{2}^-}f(x)=\lim_{x\to\frac{\pi}{2}^-}\frac{\alpha\cos x}{\pi-2x}$$

Put $x=\frac{\pi}{2}-h$, where $x\to\frac{\pi}{2}$, then $h\to 0$

$$\therefore \text{LHL}=\lim_{h\to 0}\frac{\alpha\cos\left(\frac{\pi}{2}-h\right)}{\pi-2\left(\frac{\pi}{2}-h\right)}=\lim_{h\to 0}\frac{\alpha\sin h}{\pi-\pi+2h}$$

$$=\lim_{h\to 0}\frac{\alpha\sin h}{2h}=\frac{\alpha}{2}\lim_{h\to 0}\frac{\sin h}{h}$$

$$=\frac{\alpha}{2}\left[\because \lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1\right]$$

$$\text{RHL}=\lim_{x\to\frac{\pi}{2}^+}f(x)=\lim_{x\to\frac{\pi}{2}^+}\frac{\alpha\cos x}{\pi-2x}$$

Put $x=\frac{\pi}{2}+h$, where $x\to\frac{\pi}{2}$, then $h\to 0$

$$\therefore \text{RHL}=\lim_{h\to 0}\frac{\alpha\cos\left(\frac{\pi}{2}+h\right)}{\pi-2\left(\frac{\pi}{2}+h\right)}=\lim_{h\to 0}\frac{\alpha(-\sin h)}{\pi-\pi-2h}$$

$$=\lim_{h\to 0}\frac{-\alpha\sin h}{-2h}=\frac{\alpha}{2}\lim_{h\to 0}\frac{\sin h}{h}$$

$$=\frac{\alpha}{2}\left[\because \lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1\right]$$

Also, $f\left(\frac{\pi}{2}\right)=3$

Since, $f(x)$ is continuous at $x=\frac{\pi}{2}$

$$\text{LHL}=\text{RHL}=f\left(\frac{\pi}{2}\right)=3$$

$$\Rightarrow\frac{\alpha}{2}=3 \text{ or } \alpha=6$$

104. *(a)* Given, $P(A)=\frac{3}{5}$, $P(B)=\frac{3}{10}$

$$\Rightarrow P(\overline{B})=1-P(B)=1-\frac{3}{10}=\frac{7}{10}$$

Now

$$P(A/B)=\frac{2}{3}\Rightarrow\frac{P(A\cap B)}{P(B)}=\frac{2}{3}$$

$$\Rightarrow P(A\cap B)=\frac{2}{3}\times\frac{3}{10}=\frac{1}{5}$$

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

$$=\frac{3}{5}+\frac{3}{10}-\frac{1}{5}=\frac{7}{10}$$

This implies that

$$P(\overline{A}\cap\overline{B})=P(\overline{A\cup B})$$

$$=1-P(A\cup B)$$

$$=1-\frac{7}{10}=\frac{3}{10}$$

$$P\left(\frac{\overline{A}}{\overline{B}}\right)=\frac{P(\overline{A}\cap\overline{B})}{P(\overline{B})}=\frac{\frac{3}{10}}{\frac{7}{10}}=\frac{3}{7}$$

105. *(c)* Three vectors $x_1\hat{i}+y_1\hat{j}+z_1\hat{k},\ x_2\hat{i}+y_2\hat{j}+z_2\hat{k}$ and $x_3\hat{i}+y_3\hat{j}+z_3\hat{k}$ will be coplaner iff

$$\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} = 0$$

Here $x_1=2, y_1=-1, z_1=1, x_2=1, y_2=2, z_2=-3$, $x_3=3, y_3=m, z_3=5$.

$$\therefore \quad \begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & m & 5 \end{vmatrix} = 0$$

$$\Rightarrow 2(10+3m)+1(5+9)+1(m-6)=0$$

$$\Rightarrow \quad 7m+28=0$$

$$\Rightarrow \quad m=-4$$

106. *(b)* $\begin{vmatrix} bc & a & a^2 \\ ca & b & b^2 \\ ab & c & c^2 \end{vmatrix}$

Applying $R_1 \to aR_1, R_2 \to bR_2, R_3 \to cR_3$ and divide whole determinant by abc

$$\begin{vmatrix} bc & a & a^2 \\ ca & b & b^2 \\ ab & c & c^2 \end{vmatrix} = \frac{1}{abc}\begin{vmatrix} abc & a^2 & a^3 \\ abc & b^2 & b^3 \\ abc & c^2 & c^3 \end{vmatrix}$$

$$= \frac{abc}{abc}\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$$

107. *(c)* The geometric mean G_1 of the observations $x_1, x_2, x_3, \ldots, x_n$ is

$$G_1 = [x_1 \times x_2 \times x_3 \times \cdots \times x_n]^{1/n}$$

The geometric mean of the observations G_2 $y_1, y_2, y_3, \ldots, y_n$ is

$$G_2 = [y_1 \times y_2 \times y_3 \times \cdots \times y_n]^{1/n}$$

$$\therefore \frac{G_1}{G_2} = \frac{[x_1 \times x_2 \times x_3 \times \cdots \times x_n]^{1/n}}{[y_1 \times y_2 \times y_3 \times \cdots \times y_n]^{1/n}}$$

$$= \left[\frac{x_1}{y_2} \times \frac{x_2}{y_2} \times \frac{x_3}{y_3} \times \cdots \times \frac{x_n}{y_n}\right]^{1/n}$$

Thus, $\frac{G_1}{G_2}$ is the geometric mean of $\frac{x_1}{y_2}, \frac{x_2}{y_2}, \frac{x_3}{y_3}, \ldots, \frac{x_n}{y_n}$.

108. *(b)* Let the equation of the plane through z-axis be $ax+by=0$

It is given that this plane is parallel to the line

$$\frac{x-1}{\cos\theta} = \frac{y+2}{\sin\theta} = \frac{z-3}{0}$$

Since the plane is parallel to the line,

$$\therefore a\cos\theta + b\sin\theta = 0$$

$$\Rightarrow \quad a\cos\theta = -b\sin\theta$$

$$\Rightarrow \quad a = -b\tan\theta$$

This gives,

$$(-b\tan\theta)x + by = 0$$

$$\Rightarrow \quad x\tan\theta - y = 0 \quad (\because\ b \neq 0)$$

109. *(b)* The given situation is a case of binomial distribution.

Number of r successes in n trials is

$$P_r = {}^nC_r p^r q^{n-r}$$

Where p is probability of success and q is probability of failure.

Given, $n=10,\ p=\frac{2}{3},\ q=\frac{1}{3}$

For $r=10$, $P_{10} = {}^{10}C_{10}\left(\frac{2}{3}\right)^{10}\left(\frac{1}{3}\right)^{0} = \frac{2^{10}}{3^{10}}$

For $r=7$, $P_7 = {}^{10}C_7\left(\frac{2}{3}\right)^{7}\left(\frac{1}{3}\right)^{3} = 120 \cdot \frac{2^7}{3^{10}}$

For $r=5$, $P_5 = {}^{10}C_5\left(\frac{2}{3}\right)^{5}\left(\frac{1}{3}\right)^{5} = 252 \cdot \frac{2^5}{3^{10}}$

For $r=4$, $P_4 = {}^{10}C_4\left(\frac{2}{3}\right)^{4}\left(\frac{1}{3}\right)^{6} = 210 \cdot \frac{2^4}{3^{10}}$

It is maximum for $r = 7$.

110. *(d)* Given planes are

$$x - 2y + z - 1 = 0$$

$$x - 2y + z = 1 \quad \ldots \text{(i)}$$

and $-3x + 6y - 3z + 2 = 0$

$$-3x + 6y - 3z = -2$$

$$x - 2y + z = \frac{2}{3} \quad \ldots \text{(ii)}$$

From equations (i), and (ii)

$$a = 1,\ b = -2,\ c = 1,\ d_1 = -1,\ d_2 = -\frac{2}{3}$$

Since both the planes are parallel,

$$\therefore \text{Distance} = \left| \frac{d_2 - d_1}{\sqrt{a^2 + b^2 + c^2}} \right|$$

$$= \left| \frac{-\frac{2}{3} + 1}{\sqrt{1 + 4 + 1}} \right|$$

$$= \left| \frac{\frac{1}{3}}{\sqrt{6}} \right| = \frac{1}{3\sqrt{6}}$$

111. *(a)* Let the three students be represented by A, B, and C respectively.

Odds in favour for student (A) = $\dfrac{5}{2} = \dfrac{P(A)}{P(A')}$

Odds in favour for student (B) = $\dfrac{4}{3} = \dfrac{P(B)}{P(B')}$

Odds in favour for student (C) = $\dfrac{3}{4} = \dfrac{P(C)}{P(C')}$

$$\Rightarrow P(A') = \frac{2}{5} P(A),\ P(B') = \frac{4}{3} P(B),$$

and $P(C') = \dfrac{4}{3} P(C)$

Now,

$$P(A) + P(A') = 1 \Rightarrow P(A) + \frac{2}{5} P(A) = 1 \Rightarrow P(A) = \frac{5}{7}$$

$$P(B) + P(B') = 1 \Rightarrow P(B) + \frac{4}{3} P(B) = 1 \Rightarrow P(B) = \frac{4}{7}$$

$$P(C) + P(C') = 1 \Rightarrow P(C) + \frac{4}{3} P(C) = 1 \Rightarrow P(C) = \frac{3}{7}$$

This further gives,

$$P(A') = \frac{2}{5} \times \frac{5}{7} = \frac{2}{7},\ P(B') = \frac{3}{4} \times \frac{4}{7} = \frac{3}{7}$$

$$P(C') = \frac{4}{3} \times \frac{3}{7} = \frac{4}{7}$$

Required probability is,

$$= P(A) \times P(B) \times P(C') + P(A) \times P(B') \times P(C)$$
$$+ P(A') \times P(B) \times P(C) + P(A) \times P(B) \times P(C)$$
$$= \frac{5}{7} \times \frac{4}{7} \times \frac{4}{7} + \frac{5}{7} \times \frac{3}{7} \times \frac{3}{7}$$
$$+ \frac{2}{7} \times \frac{4}{7} \times \frac{3}{7} + \frac{5}{7} \times \frac{4}{7} \times \frac{3}{7}$$
$$= \frac{209}{343}$$

112. *(c)* $A = \begin{bmatrix} x & x^2 & 1 + x^2 \\ y & y^2 & 1 + y^2 \\ z & z^2 & 1 + z^2 \end{bmatrix}$

$$|A| = \begin{vmatrix} x & x^2 & 1 + x^2 \\ y & y^2 & 1 + y^2 \\ z & z^2 & 1 + z^2 \end{vmatrix}$$

Applying $R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$

$$|A| = \begin{vmatrix} x - y & x^2 - y^2 & x^2 - y^2 \\ y - z & y^2 - z^2 & y^2 - z^2 \\ z & z^2 & 1 + z^2 \end{vmatrix}$$

$$= (x - y)(y - z) \begin{vmatrix} 1 & x + y & x + y \\ 1 & y + z & y + z \\ z & z^2 & 1 + z^2 \end{vmatrix}$$

Applying $C_3 \to C_3 - C_2$

$$|A| = (x-y)(y-z)\begin{vmatrix} 1 & x+y & 0 \\ 1 & y+z & 0 \\ z & z^2 & 1 \end{vmatrix}$$

$$= (x-y)(y-z)\left[(y+z)-(x+y)\right]$$

$$= (x-y)(y-z)(z-x)$$

113. *(c)* Let $u = x+y$ and $v = x-y$.

$\therefore \bar{u} = \bar{x}+\bar{y}$ and $\bar{v} = \bar{x}-\bar{y}$

$$\text{cov}(u,v) = E\{(u-\bar{u})(v-\bar{v})\}$$

$$= E\left[\{(x-\bar{x})+(y-\bar{y})\}\cdot\{(x-\bar{x})-(y-\bar{y})\}\right]$$

$$= E\left\{(x-\bar{x})^2-(y-\bar{y})^2\right\}$$

$$= E(x-\bar{x})^2 - E(y-\bar{y})^2$$

$$= {\sigma_x}^2 - {\sigma_y}^2$$

$$\text{var}(u) = E(u-\bar{u})^2$$

$$= E\{(x-\bar{x})+(y-\bar{y})\}^2$$

$$= E\left\{(x-\bar{x})^2+(y-\bar{y})^2+2(x-\bar{x})(y-\bar{y})\right\}$$

$$= E(x-\bar{x})^2 + E(y-\bar{y})^2 + 2E(x-\bar{x})(y-\bar{y})$$

$$= {\sigma_x}^2 + {\sigma_y}^2 \quad \left[\because E(x-\bar{x})(y-\bar{y}) = 0\right]$$

Similarly, we get

$$\text{var}(v) = {\sigma_x}^2 + {\sigma_y}^2$$

Thus, $\text{r}(u,v) = \dfrac{\text{cov}(u,v)}{\sqrt{\text{var}(u)\,\text{var}(v)}} = \dfrac{{\sigma_x}^2 - {\sigma_y}^2}{{\sigma_x}^2 + {\sigma_y}^2}$

114. *(b)* Given, Mean = 5, Standard deviation = 2
If 5 is added to each value, mean = 5 + 5 =10
Standard deviation will no change.

$$\text{Coefficient of variation} = \frac{\text{Standard deviation}}{\text{Mean}} \times 100$$

$$= \frac{2}{10}\times 100$$

$$= 20$$

115. *(a)* The equation of the line joining the points $(-3,4,-8)$ and $(5,-6,4)$ is

$$\frac{x+3}{8} = \frac{y-4}{-10} = \frac{z+8}{12} = k(\text{say})$$

$$\Rightarrow x = 8k-3,\ y = -10k+4,\ z = 12k-8$$

Given that the line intersects with xy plane. So, $z = 0$

$\therefore\ 12k-8 = 0$

$$\Rightarrow \quad k = \frac{8}{12} = \frac{2}{3}$$

$$\therefore x = 8\left(\frac{2}{3}\right)-3 = \frac{16}{3}-3 = \frac{7}{3}$$

$$y = -10\left(\frac{2}{3}\right)+4 = -\frac{20}{3}+4 = -\frac{8}{3}$$

So, required point is $\left(\dfrac{7}{3}, -\dfrac{8}{3}, 0\right)$

116. *(d)* The given system of equations are

$x+y+z = 2 \quad ...(\text{i})$

$2x+y+z = 3 \quad ...(\text{ii})$

$3x+2y+kz = 4 \quad ..(\text{iii})$

The system of equations has unique solution if

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix} \neq 0$$

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

$$\begin{vmatrix} 1 & 0 & 0 \\ -2 & -1 & -3 \\ 3 & -1 & k-3 \end{vmatrix} \neq 0$$

$\Rightarrow -1(k-3)-3 \neq 0$

$\Rightarrow \quad -k+3-3 \neq 0$

$\Rightarrow \quad k \neq 0$

117. *(a)* Given $X+Y=15$.

The total numbers of ordered pairs which satisfies $X+Y=15$ is

$(5,10),(6,9),(7,8),(8,7),(9,6),(10,5)$

$\therefore n(S)=6$

In each above pairs exactly one of the two numbers is even number.

$\therefore E=\{(5,10),(6,9),(7,8),(8,7),(9,6),(10,5)\}$

and $n(E)=6$

Thus, required probability

$$P(E)=\frac{n(E)}{n(S)}=\frac{6}{6}=1$$

118. *(a)* The equation of the place passing through the intersection of the planes $2x+y+2z=9$, $4x-5y-4z=1$ is given by,

$(2x+y+2z-9)+\lambda(4x-5y-4z-1)=0$

Given that this plane passes through the point $(3,2,1)$, so we have

$[2(3)+2+2(1)-9]+\lambda[4(3)-5(2)-4(1)-1]=0$

$(6+2+2-9)+\lambda(12-10-4-1)=0$

$1-3\lambda=0$

$\lambda=\frac{1}{3}$

So, the required equation is,

$(2x+y+2z-9)+\frac{1}{3}(4x-5y-4z-1)=0$

$6x+3y+6z-27+4x-5y-4z-1=0$

$10x-2y+2z-28=0$

119. *(b)* $f(x)=\frac{1}{\sqrt{|x|-x}}$

For a rational function to be defined, the denominator should be non-zero.

$|x|-x \neq 0$

$\Rightarrow x<0 \quad ...(i)$

Also, square root function is defined for positive values.

$|x|-x>0$

$|x|>x$

This is possible only when x is negative.

$x=(-\infty,0) \quad \ldots(ii)$

From equations (i) and (ii),

domain $=(-\infty,0)$

120. *(b)* Given $P(A)=1/3$, $P(B)=1/4$, $P(A/B)=1/6$

But $P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}$

$\Rightarrow \quad \frac{1}{6}=\frac{P(A\cap B)}{\frac{1}{4}}$

$\Rightarrow P(A\cap B)=\frac{1}{24}$

Thus, $P\left(\frac{B}{A}\right)=\frac{P(A\cap B)}{P(A)}=\frac{\frac{1}{24}}{\frac{1}{3}}=\frac{1}{8}$