HINTS & SOLUTION

1. *(c)* $(a,b) \in R \Leftrightarrow a^2 + b^2 = 25$, where $a, b \in Z$

For domain, we have to find 'a'

$\Rightarrow a^2 = 25 - b^2$

If $b = 0$, then $a^2 = 25 \Rightarrow a = \pm 5$

If $b = \pm 1$, then $a^2 = 24 \Rightarrow a = \pm\sqrt{24}$

(not possible as $a \in Z$)

If $b = \pm 2$, then $a^2 = 21 \Rightarrow a = \pm\sqrt{21}$

(not possible as $a \in Z$)

If $b = \pm 3$, then $a^2 = 16 \Rightarrow a = \pm 4$

If $b = \pm 4$, then $a^2 = 3 \Rightarrow a = \pm\sqrt{3}$

If $b = \pm 5$, then $a^2 = 0 \Rightarrow a = 0$

Thus, domain $= \{0, \pm 3, \pm 4, \pm 5\}$

Hence, option *(c)* is correct.

2. *(b)* Given, $a_1 = a_2 = a_3 = ... = a_{10} = 150$

Also, $a_{10}, a_{11}, a_{12}....$ are in AP and $d = -2$

Since, $a_{10} = 150$

AP is $150, 148, 146, ...$

For the first 10 min, he has counted $150 \times 10 = 1500$ notes

Time taken to count remaining 3000 notes

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$\Rightarrow 3000 = \frac{n}{2}\left[2 \times 148 + (n-1)(-2)\right]$$

$$\Rightarrow 3000 = \frac{n}{2} \times 2(148 - n + 1)$$

$$\Rightarrow 3000 = 148n - n^2 + n$$

$$\Rightarrow n^2 - 149n + 3000 = 0$$

$$\Rightarrow (n-24)(n-125) = 0$$

$$\Rightarrow n = 24, 125$$

Since, he has taken 10 min to count 1500 notes, he will not take 12 min to count 3000 notes.

So, $n = 24$

$\therefore$ Total time $= 10 + 24 = 34$ min

Hence, option *(b)* is correct.

3. *(d)* Given, $f(x) = \dfrac{\sin\left(e^{x-2} - 1\right)}{\ln(x-1)}$

Then, $\lim\limits_{x \to 2} f(x) = \lim\limits_{x \to 2} \dfrac{\sin\left(e^{x-2} - 1\right)}{\ln(x-1)}$

$$= \lim_{x \to 2} \frac{\cos\left(e^{x-2} - 1\right) \cdot e^{x-2}}{\frac{1}{x-1}}$$

This forms $\dfrac{0}{0}$ and apply L'Hopital rule, we get:

$$\Rightarrow \lim_{x \to 2} f(x) = \frac{\cos\left(e^0 - 1\right)e^0}{\frac{1}{2-1}} = \frac{1 \times 1}{1} = 1$$

Hence, option *(d)* is correct.

4. *(d)* Given, $f(x) = \dfrac{x^2 - x + 1}{x^2 + x + 1} = 1 - \dfrac{2x}{x^2 + x + 1}$

$$= 1 - \frac{2}{x + 1 + \frac{1}{x}}$$

$\Rightarrow f(x) = 1 - \dfrac{2}{A}$, where $A = x + 1 + \dfrac{1}{x}$

Then, $\dfrac{dA}{dx} = 1 - \dfrac{1}{x^2}$

Put $\dfrac{dA}{dx} = 0 \Rightarrow \dfrac{x^2 - 1}{x^2} = 0$

$\Rightarrow x = 1$ or $x = -1$

Now, $\dfrac{d^2A}{dx^2} = \dfrac{2}{x^3}$

$\because \left(\dfrac{d^2A}{dx^2}\right)_{(x=1)} = 2 > 0$

So, A is minimum at $x = 1$ and $\left(\dfrac{d^2A}{dx^2}\right)_{(x=-1)} = -2 < 0$

So, A is maximum at $x = -1$

So, $f(x)$ is maximum at $x=-1$ and $f(x)$ is minimum at $x=1$.
Hence, the maximum value of the function

$$= f(-1)$$

$$= \frac{(-1)^2-(-1)+1}{(1)^2+(1)+1} = 3$$

Hence, option *(d)* is correct.

5. *(c)* Let $I = \int \frac{xe^x dx}{(x+1)^2}$

$$\Rightarrow I = \int e^x \left\{ \frac{x+1-1}{(x+1)^2} \right\} dx$$

$$\Rightarrow I = \int e^x \left\{ \frac{1}{x+1} - \frac{1}{(x+1)^2} \right\} dx$$

Since, $\int e^x [f(x)+f'(x)]dx = e^x f(x) + C$

$$\therefore I = \frac{e^x}{x+1} + C$$

Hence, option *(c)* is correct.

6. *(b)* Given curve is $\sqrt{x}+\sqrt{y}=\sqrt{a}$ ....(i)

Let the point be (x_1, y_1) at the normal is parallel to *X*-axis.
On differentiating Eq.(i) w.r.t. *x*, we get

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\sqrt{\frac{y}{x}}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{(x_1,y_1)} = -\sqrt{\frac{y_1}{x_1}}$$

Slope of normal $= -\frac{1}{\left(\frac{dy}{dx}\right)_{(x_1,y_1)}} = \sqrt{\frac{x_1}{y_1}}$

Given, slope is parallel to *X*-axis,

$$\Rightarrow \sqrt{\frac{x_1}{y_1}} = 0$$

{$\because$ slope of *X*-axis $=0$, for parallel lines, slope are equal}

$$\Rightarrow x_1 = 0$$

Put $x_1 = 0$ in Eq. (i), $\sqrt{x_1}+\sqrt{y_1}=\sqrt{a}$

$$\Rightarrow y_1 = a$$

Thus, the required point is $(0, a)$

Hence, option (b) is correct.

7. *(a)* Let *n* missiles be fired and out of them μ missiles hit the target. Then, given

$$p = 0.3 \Rightarrow q = 1-p = 0.7$$

Target is hit when atleast one missile strikes the target.
i.e., $p(X>1) = 1-p(X=0)$
This must be greater than 80%. (given)

$$\Rightarrow 1-{}^nC_0(0.3)^0(0.7)^{n-0} \geq \frac{80}{100}$$

$$\Rightarrow 1-\left(\frac{7}{10}\right)^n \geq \frac{80}{100}$$

$$\Rightarrow \left(\frac{7}{10}\right)^n \leq \frac{20}{100}$$

$$\Rightarrow n \geq 5$$

Hence, option *(a)* is correct.

8. *(c)* Let vector $x\hat{i}+y\hat{j}+z\hat{k}$ be perpendicular to the vectors $4\hat{i}+2\hat{j}$ and $-3\hat{i}+2\hat{j}$
So, $4x+2y=0$ and $-3x+2y=0$
(for perpendicular vectors, dot product is zero)
On solving these equations, we get

$$x=0 \text{ and } y=0$$

So, the required vector is $\hat{k}$.
Hence, option *(c)* is correct.

9. *(a)* We have $\left[\det(kA)\right]^{-1}\det(A)$

$$=\frac{1}{\det(kA)}\det(A)$$

$$=\frac{1}{\mathrm{k}^n\det(A)}\cdot\det(A)$$

$$=\frac{1}{\mathrm{k}^n}=k^{-n}$$

Hence, option *(a)* is correct.

10. *(a)* Given, $x=\cos 2t$ and $y=\sin^2 t$

Differentiate w.r.t. t, we get

$$\frac{dx}{dt}=-2\sin t \text{ and } \frac{dy}{dt}=2\sin t\cos t$$

$$\therefore \frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{2\sin t\cos t}{-2\sin 2t}=-\frac{1}{2}$$

Now, $\dfrac{d^2y}{dx^2}=\dfrac{d}{dx}\left(\dfrac{dy}{dx}\right)=\dfrac{d}{dx}\left(-\dfrac{1}{2}\right)=0$

Hence, option *(a)* is correct.

11. *(c)* Consider $\left(1+x+x^2+x^3\right)^{11}=\left\{(1+x)\left(1+x^2\right)\right\}^{11}$

$$=(1+x)^{11}\left(1+x^2\right)^{11}$$

$$=\left\{{}^{11}C_0+{}^{11}C_1x+{}^{11}C_2x^2+{}^{11}C_3x^3+{}^{11}C_4x^4+\dots\right\}$$
$$\left\{{}^{11}C_0+{}^{11}C_1x^2+{}^{11}C_2x^4+\dots\right\}$$

So, coefficient of x^4

$$={}^{11}C_0\cdot{}^{11}C_2+{}^{11}C_2\cdot{}^{11}C_1+{}^{11}C_4\cdot{}^{11}C_0$$

$$=\frac{11\times10}{2}+\frac{11\times10}{2}\times11+\frac{11\times10\times9\times8}{4\times3\times2\times1}\times1$$

$$=55+605+330$$

$$=990$$

Hence, option *(c)* is correct.

12. *(a)* Let *A, B, C* and *D* be the set of the combatants lost one eye, an ear, an arm and a leg respectively.

Then $n(A)=70\%$, $n(B)=80\%$, $n(C)=75\%$

and $n(D)=85\%$

Combatants who lost one eye and one ear

$$n(A\cap B)\le 70\%+80\%-100\%=50\%$$

Combatants who lost one eye, one ear and one arm

$$n(A\cap B\cap C)\le 50\%+75\%-100\%=25\%$$

Combatants who lost one eye, one ear and one arm and one leg

$$n(A\cap B\cap C\cap D)\le 25\%+85\%-100\%=10\%$$

So, minimum 10% of the combatants have lost all the four limbs.

13. *(a)* Given, circle of radius b and centre $(0,b)$ touches line $y=x-\sqrt{2}$

So, radius of circle = perpendicular drawn from $(0,b)$ to line $x-y-\sqrt{2}=0$

$$\Rightarrow\ b=\left|\frac{0\times1+b\times(-1)-\sqrt{2}}{\sqrt{1+1}}\right|$$

$$\Rightarrow\ b\sqrt{2}=b+\sqrt{2}$$

$$\Rightarrow\ b=\frac{\sqrt{2}}{\sqrt{2}-1}=\sqrt{2}\left(\sqrt{2}+1\right)$$

$$\Rightarrow\ b=2+\sqrt{2}$$

Hence, option *(a)* is correct.

14. *(d)* Given, $\tan\theta+\sin\theta=m$ and $\tan\theta-\sin\theta=n$

Then, $m^2-n^2=(m+n)(m-n)$

$$=(2\tan\theta)(2\sin\theta)=4\tan\theta\sin\theta \qquad \dots\text{(i)}$$

Now, $mn=(\tan\theta+\sin\theta)(\tan\theta-\sin\theta)$

$$=\tan^2\theta-\sin^2\theta$$

$$=\sin^2\theta\left(\frac{1}{\cos^2\theta}-1\right)$$

$$=\sin^2\theta\left(\sec^2\theta-1\right)$$

$$=\sin^2\theta\tan^2\theta$$

$$\Rightarrow \sin\theta\tan\theta=\sqrt{mn}$$

So, from Eq. (i), we get

$$m^2-n^2=4\sqrt{mn}$$

Hence, option *(d)* is correct.

15. *(c)* Given, $f(x)$ is a continuous function and $f\left(\frac{9}{2}\right)=\frac{2}{9}$

So, $\lim_{x\to 0} f\left(\frac{1-\cos 3x}{x^2}\right)$ $\left[f\left(\frac{0}{0}\right)\text{ indeterminant form}\right]$

$=\lim_{x\to 0} f\left(\frac{2\sin^2\frac{3x}{2}}{x^2}\right)$ $\left[1-\cos 2\theta = 2\sin^2\theta\right]$

$=\lim_{x\to 0} f\left(2\left(\frac{\sin\frac{3x}{2}}{\frac{3x}{2}}\right)^2\times\frac{9}{4}\right)$

$=\lim_{x\to 0} f\left(\frac{9}{2}\right)$ $\left[\because \lim_{x\to 0}\frac{\sin x}{x}=1\right]$

$=\frac{2}{9}$

Hence, option *(c)* is correct.

16. *(c)* Given equation is $2(x+\alpha)(x+\beta)-\gamma^2=0$

$\Rightarrow 2x^2+2(\alpha+\beta)x+2\alpha\beta-\gamma^2=0$

$\therefore$ Discriminant, $D=b^2-4ac$

For equation $ax^2+bx+c=0$

So, $D=4(\alpha+\beta)^2-4(2)(2\alpha\beta-\gamma^2)$

$\Rightarrow D=4\alpha^2+4\beta^2+8\alpha\beta-16\alpha\beta+8\gamma^2$

$\Rightarrow D=4(\alpha-\beta)^2+8\gamma^2$

$\Rightarrow D>0$

So, roots are real and thus statement I is correct.

Now, if $\alpha=\beta=\gamma=0$

Then, $D=0$

$\Rightarrow$Roots are real and equal. So, statement II is also correct.

Hence, option *(c)* is correct.

17. *(b)* We have $\left(\frac{-1+i\sqrt{3}}{2}\right)^{3n}+\left(\frac{-1-i\sqrt{3}}{2}\right)^{3n}$

$=\omega^{3n}+\left(\omega^2\right)^{3n}$

$=\left(\omega^3\right)^n+\left(\omega^3\right)^{2n}$

$=(1)^n+(1)^{2n}$

$=1+1=2$

Hence, option *(b)* is correct.

18. *(b)* We have $\cot\left(\frac{A}{2}\right)\cot\left(\frac{C}{2}\right)$

$=\sqrt{\frac{s(s-a)}{(s-b)(s-c)}}\cdot\sqrt{\frac{s(s-c)}{(s-a)(s-b)}}$

$=\frac{s}{s-b}=\frac{2s}{2s-2b}$

$=\frac{a+b+c}{a+b+c-2b}$

$=\frac{a+b+c}{a-b+c}$

$=\frac{3b}{b}=3$ $[\because 2b=a+c]$

Hence, option *(b)* is correct.

19. *(b)* Given, A and B are mutually exclusive events.

So, $P(A\cap B)=0$

Given, $P(A)=0.2$ and $P(\overline{A}\cap B)=0.3$

$\Rightarrow P(B)=0.3$ $\{\because P(A\cap B)=0\}$

Now, $P\left(\frac{A}{A\cup B}\right)=\frac{P(A\cap(A\cup B))}{P(A\cup B)}$

$=\frac{P(A)}{P(A)+P(B)}=\frac{0.2}{0.2+0.3}=\frac{2}{5}$

Hence, option *(b)* is correct.

20. *(c)* Let the plane cuts the axes at points $A(\alpha,0,0),B(0,\beta,0)$ and $C(0,0,\gamma)$ respectively.

Then, equation of plane is $\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=1$

Now, it passes through point (a,b,c)

$\Rightarrow \dfrac{a}{\alpha}+\dfrac{b}{\beta}+\dfrac{c}{\gamma}=1$ ....(i)

Let centre of sphere $OABC$ be $P(x, y, z)$

Then, $PA^2 = PB^2 = PC^2 = PO^2$

$\Rightarrow (x-\alpha)^2 + y^2 + z^2 = x^2 + y^2 + z^2$

$\Rightarrow \alpha = 2x$

Similarly, $\beta = 2y$ and $\gamma = 2z$

On putting Eq. (i), we get

$\dfrac{a}{2x}+\dfrac{b}{2y}+\dfrac{c}{2z}=1$

$\Rightarrow \dfrac{a}{x}+\dfrac{b}{y}+\dfrac{c}{z}=2$

Hence, option *(c)* is correct.

21. *(b)* We have, S_n = sum of first n terms of an AP

$\therefore S_n = \dfrac{n}{2}[2a+(n-1)d]$

Similarly, $S_{2n} = \dfrac{2n}{2}[2a+(2n-1)d]$

and $S_{3n} = \dfrac{3n}{2}[2a+(3n-1)d]$

Now, $3S_n = S_{2n}$

$\Rightarrow 3\left(\dfrac{n}{2}\right)[2a+(n-1)d] = 2\left(\dfrac{n}{2}\right)[2a+(2n-1)d]$

$\Rightarrow 2a = d(n+1)$

$\therefore S_n = \dfrac{n}{2}[d(n+1)+d(n-1)] = n^2 d$

$\Rightarrow S_{2n} = n[d(n+1+2n-1)] = 3n^2 d$

and $S_{3n} = \dfrac{3n}{2}[d(n+1+3n-1)] = 6n^2 d$

$\therefore \dfrac{S_{3n}}{S_n} = \dfrac{6n^2 d}{n^2 d} = 6:1$

Hence, option *(b)* is correct.

22. *(b)* We have $g[f(x)] = \log_e\left(\sqrt{3x^2-4x+5}\right)^2$

$= \log_e(3x^2-4x+5)$

Let $y = \log_e(3x^2-4x+5)$

$\Rightarrow e^y = 3x^2-4x+5$

$\Rightarrow 3x^2-4x+(5-e^y)=0$

For x to be real, discriminant ≥ 0

$\therefore 16-12(5-e^y) \geq 0$

$\Rightarrow 12e^y \geq 44$

$\Rightarrow e^y \geq \dfrac{11}{3}$

$\Rightarrow y \geq \log_e \dfrac{11}{3}$

$\therefore$ Range of $f = \left[\log_e \dfrac{11}{3}, \infty\right)$

Hence, option *(b)* is correct.

23. *(d)* Given, $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$\Rightarrow |A| = 1\times 4 - 2\times 3 = -2$

$\text{adj}(A) = \begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix}$

$\because A^{-1} = \dfrac{\text{adj}(A)}{|A|}$

$\Rightarrow A^{-1} = \dfrac{1}{-2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

$\Rightarrow A^{-1} = \begin{bmatrix} -2 & 1 \\ \dfrac{3}{2} & -\dfrac{1}{2} \end{bmatrix}$

Given, $A^{-1} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$

So, $b_{22} = -\dfrac{1}{2}$

Hence, option *(d)* is correct.

24. *(c)* Since, union is intersection, distribution over.

So, $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ and

$(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$ are correct.

Now, $(B' \cap A') \cup A = (B \cup A') \cup A$

$= (A' \cup B) \cup A$

$= A' \cup (A \cup B)$

This is also correct.

Now, $(C' \cap B') \cap A' = (C \cup B') \cap A' = A' \cup (B \cup C)$ is not correct.

Hence, option *(c)* is correct.

25. *(b)* Given equation is $2x^2 + 30|x| + 28 = 0$

$\because \quad x^2 \geq 0$ and $|x| \geq 0$

So, $2x^2 + 30|x| + 28 > 0$

for all real values of x.

Hence, there is no real root for which $2x^2 + 30|x| + 28 = 0$ exist.

Hence, option *(b)* is correct.

26. *(a)* Given, $f(\theta) = \dfrac{\cot\theta}{1+\cot\theta}$, then

$$f(\alpha)\cdot f(\beta) = \frac{\cot\alpha}{1+\cot\alpha}\cdot\frac{\cot\beta}{1+\cot\beta} \quad ....(i)$$

Given, $\alpha + \beta = \dfrac{5\pi}{4}$

$\Rightarrow f(\alpha+\beta) = \tan\dfrac{5\pi}{4}$

$\Rightarrow \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \tan\left(\pi + \dfrac{\pi}{4}\right)$

$\Rightarrow \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \tan\dfrac{\pi}{4} = 1$

$\Rightarrow \tan\alpha + \tan\beta = 1 - \tan\alpha\tan\beta$

Divide by $\tan\alpha\cdot\tan\beta$, we get

$\Rightarrow \cot\beta + \cot\alpha = \cot\alpha\cot\beta - 1$

$\Rightarrow 1 + \cot\alpha + \cot\beta = \cot\alpha\cot\beta \quad ....(ii)$

From Eq. (i), we get

$$f(\alpha)\cdot f(\beta) = \frac{\cot\alpha\cot\beta}{1+\cot\alpha+\cot\beta+\cot\alpha\cot\beta}$$

$$= \frac{\cot\alpha\cot\beta}{2\cot\alpha\cot\beta}$$

$$= \frac{1}{2}$$

Hence, option *(a)* is correct.

27. *(c)* Given, $f(x) = \left(\dfrac{1}{x}\right)^{2x^2} = x^{-2x^2} = e^{-2x^2\log x}$

$\Rightarrow f'(x) = e^{-2x^2\log x}\left[-2x^2\cdot\dfrac{1}{x} + \log x(-4x)\right]$

$\Rightarrow f'(x) = f(x)(-2x - 4x\log x)$

For maxima, $f'(x) = 0$

$\Rightarrow x^{-2x^2}(-2x - 4x\log x) = 0$

$\Rightarrow -2x(1 + 2\log x) = 0$

$\Rightarrow x = 0$ or $\log x = -\dfrac{1}{2}$

$\Rightarrow x = e^{-\frac{1}{2}} = \dfrac{1}{\sqrt{e}}$

Now, $f''(x) = f'(x)(-2x - 4x\log x)$

$+ f(x)\left(-2 - 4x\cdot\dfrac{1}{x} - 4\log x\right)$

$= -6 - 4\log x$

$f''(x)$ is negative at $x = \dfrac{1}{\sqrt{e}}$

Thus, $x = \dfrac{1}{\sqrt{e}}$ is the point of maxima.

Hence, option *(c)* is correct.

28. *(a)* Given, differential equation is

$$x^2(x^2-1)\frac{dy}{dx} + x(x^2+1)y = x^2 - 1 \quad ....(i)$$

On dividing Eq. (i) both sides by $x^2(x^2-1)$, we get

$$\frac{dy}{dx}+\frac{x(x^2+1)}{x^2(x^2-1)}y=\frac{x^2-1}{x^2(x^2-1)}$$

$$\Rightarrow \frac{dy}{dx}+\frac{x^2+1}{x(x^2-1)}y=\frac{1}{x^2} \quad ....(ii)$$

On comparing Eq. (ii) with $\frac{dy}{dx}+Py=Q$, we get

$$P=\frac{x^2+1}{x(x^2-1)},\ Q=\frac{1}{x^2}$$

$$\therefore \text{IF}=e^{\int Pdx}=e^{\int\frac{x^2+1}{x(x^2-1)}dx}=e^{\int\left(\frac{1}{x+1}+\frac{1}{x-1}-\frac{1}{x}\right)dx}$$

$$=e^{\log\left(\frac{x^2-1}{x}\right)}=\frac{x^2-1}{x}$$

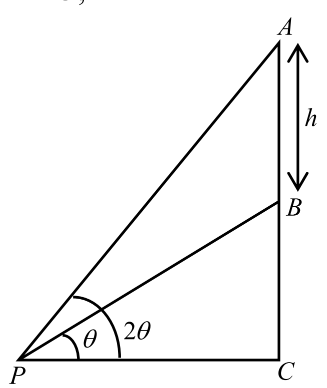
Hence, option *(a)* is correct.

29. *(c)* Let AB be a flagstaff of height h and BC be the tower.

In ΔPBC, $\tan\theta=\frac{BC}{PC}$

$$\Rightarrow \quad PC=BC\cot\theta \quad ....(i)$$

In ΔPAC,



$$\tan 2\theta=\frac{h+BC}{PC}$$

$$\Rightarrow PC\tan 2\theta=h+BC$$

$$\Rightarrow BC\cot\theta\cdot\frac{2\tan\theta}{1-\tan^2\theta}=h+BC$$

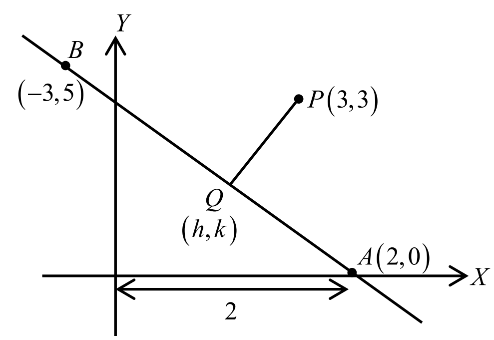
$$\Rightarrow \frac{2BC}{1-\tan^2\theta}-BC=h$$

$$\Rightarrow \frac{1+\tan^2\theta}{1-\tan^2\theta}BC=h$$

$$\Rightarrow BC=\frac{1-\tan^2\theta}{1+\tan^2\theta}h=h\cos 2\theta$$

Hence, option *(c)* is correct.

30. *(d)* Let $Q(h,k)$ be the foot of the perpendicular drawn from point $P(3,3)$.



Slope of line $AB=\frac{5-0}{-3-2}=-1$

Equation of line AB

$$y-0=-1(x-2)$$

$$\Rightarrow x+y-2=0 \quad ....(i)$$

Slope of line PQ perpendicular to $AB=\frac{-1}{-1}=1$

So, equation of line PQ is

$$y-3=1(x-3)$$

$$\Rightarrow x-y=0 \quad ....(ii)$$

From Eqs. (i) and (ii), we get

$x=1$ and $y=1$

Thus, $(h,k)=(1,1)$

Hence, option *(d)* is correct.

31. *(d)* Let the infinite geometric progression is $a, ar, ar^2, ...$

Given, $S_\infty = 6 \Rightarrow \frac{a}{1-r} = 6$ ....(i)

and $a + ar = \frac{9}{2}$

$\Rightarrow a(1+r) = \frac{9}{2}$ ....(ii)

On dividing Eqs. (i) and (ii), we get

$$\frac{1}{(1-r)(1+r)} = \frac{6\times 2}{9}$$

$\Rightarrow 1 - r^2 = \frac{3}{4} \Rightarrow r^2 = \frac{1}{4}$

$\Rightarrow r = \pm\frac{1}{2}$

From Eq. (i),

$$\frac{a}{\left(1-\frac{1}{2}\right)} = 6 \Rightarrow a = 3$$

or $\frac{a}{\left(1+\frac{1}{2}\right)} = 6 \Rightarrow a = 9$

Hence, option *(d)* is correct.

32. *(a)* Given equation $\log_4(x-1) = \log_2(x-3)$

$\Rightarrow \frac{1}{2}\log_2(x-1) = \log_2(x-3)$

$\Rightarrow \log_2(x-1) = 2\log_2(x-3)$

$\Rightarrow (x-1) = (x-3)^2$

$\Rightarrow x^2 - 6x + 9 - x + 1 = 0$

$\Rightarrow x^2 - 7x + 10 = 0$

$\Rightarrow (x-5)(x-2) = 0$

$\Rightarrow x = 5$ or $x = 2$ [not possible]

$\because \log(-1)$ is not defined.

$\therefore$ Number of solution $= 1$

Hence, option *(a)* is correct.

33. *(a)* Let the equation of ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Given, minor axis $= 1$ unit

$\Rightarrow 2a = 1 \Rightarrow a = \frac{1}{2}$

and foci are $(0,1)$ and $(0,-1)$

$\Rightarrow be = 1 \Rightarrow b^2e^2 = 1$

$\Rightarrow b^2\left(1 - \frac{a^2}{b^2}\right) = 1 \Rightarrow b^2 - a^2 = 1$

$\Rightarrow b^2 = 1 + a^2$

$\Rightarrow b^2 = 1 + \frac{1}{4} \Rightarrow b = \frac{\sqrt{5}}{2}$

Length of latus rectum

$$= \frac{2a^2}{b} = \frac{2\times\frac{1}{4}}{\frac{\sqrt{5}}{2}} = \frac{1}{\sqrt{5}}$$

Hence, option *(a)* is correct.

34. *(c)* Since, r_1, r_2 and r_3 are coplanar.

So, $[r_1, r_2, r_3] = 0$

$$\Rightarrow \begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$$

$\Rightarrow abc = a + b + c - 2$ ....(i)

Now, $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$

$$= \frac{(1-b)(1-c) + (1-a)(1-c) + (1-a)(1-b)}{(1-a)(1-b)(1-c)}$$

$$= \frac{3 - 2(a+b+c) + ab + bc + ca}{1 + (a+b+c) + ab + bc + ca - abc}$$

$$= \frac{3 - 2(a+b+c) + ab + bc + ca}{3 - 2(a+b+c) + ab + bc + ca} = 1$$

Hence, option *(c)* is correct.

35. *(a)* We have $g(x)=f(\sin x)+f(\cos x)$

$\Rightarrow g'(x)=f'(\sin x)\cos x-f'(\cos x)\sin x$

or $g''(x)=-f'(\sin x)\sin x+\cos^2 x f''(\sin x)$

$+f''(\cos x)\sin^2 x-f'(\cos x)\cos x>0\ \forall x\in\left(0,\frac{\pi}{2}\right)$

Since, it is given $f'(\sin x)=f'\left[\cos\left(\frac{\pi}{2}-x\right)\right]<0$ and

$f''(\sin x)=f''\left[\cos\left(\frac{\pi}{2}-x\right)\right]>0$

Thus, $g'(x)$ is increasing in $\left(0,\frac{\pi}{2}\right)$

Hence, option *(a)* is correct.

36. *(b)* Let $I=\int_0^{\pi/2}\frac{dx}{3+2\sin x+\cos x}$

$$=\int_0^{\pi/2}\frac{dx}{\frac{3}{1}+2\left(\frac{2\tan x/2}{1+\tan^2 x/2}\right)+\left(\frac{1-\tan^2 x/2}{1+\tan^2 x/2}\right)}$$

$$=\int_0^{\pi/2}\frac{\sec^2 x/2\,dx}{4+4\tan\frac{x}{2}+2\tan^2\frac{x}{2}}$$

Let $\tan\frac{x}{2}=t\Rightarrow\sec^2\frac{x}{2}dx=2dt$

$$\therefore I=\int_0^1\frac{2dt}{4+4t+2t^2}=\int_0^1\frac{dt}{t^2+2t+2}$$

$$=\int_0^1\frac{dt}{(t+1)^2+1}=\left[\tan^{-1}(t+1)\right]_0^1$$

$=\tan^{-1}(2)-\tan^{-1}(1)$

$=\tan^{-1}(2)-\frac{\pi}{4}$

Hence, option *(b)* is correct.

37. *(c)* Given, $x-h=p\sec\alpha$

$\Rightarrow \sec\alpha=\frac{x-h}{p}$

$\Rightarrow \cos\alpha=\frac{p}{x-h}$ ....(i)

Also given, $y=k+q\operatorname{cosec}\alpha$

$\Rightarrow \operatorname{cosec}\alpha=\frac{y-k}{q}$

$\Rightarrow \sin\alpha=\frac{q}{y-k}$ ....(ii)

$\because \sin^2\alpha+\cos^2\alpha=1$

$\Rightarrow \frac{p^2}{(x-h)^2}+\frac{q^2}{(y-k)^2}=1$

Hence, option *(c)* is correct.

38. *(a)* General term in the expansion of $\left(3x-\frac{x^3}{6}\right)^9$ is

$T_{r+1}={}^9C_r(3x)^{9-r}\left(\frac{x^3}{6}\right)^r$

$={}^9C_r\frac{(3)^{9-r}}{6^r}x^{9+2r}$

For coefficient of x^{17}, put $9+2r=17$

$\Rightarrow r=4$

So, required coefficient

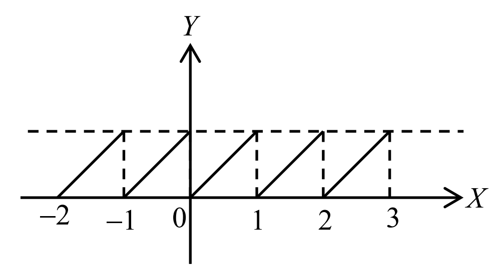
$={}^9C_4\frac{3^{9-4}}{6^4}=\frac{9!}{4!5!}\times\frac{3^5}{6^4}$,

$=\frac{189}{8}$

Hence, option *(a)* is correct.

39. *(d)* Given, $f:R\to R$ defined by $f(x)=x-[x]$

This is a fractional part function. Its graph is as follows:



From the graph it is clear that $f(x)$ is discontinuous at all integral values of x and continuous at all non-integral values of x. Moreover, $0\le f(x)<1,\ \forall\ x\in R$

Hence, all the three statements are correct here.
Hence, option *(d)* is correct.

40. *(c)* Given, $P(A_1)=P(A_2)=0.4,\ P(A_3)=0.2$

And $P\left(\frac{B}{A_1}\right)=0.25,\ P\left(\frac{B}{A_2}\right)=0.4,\ P\left(\frac{B}{A_3}\right)=0.125$

Then, required probability $=P\left(\frac{A_1}{B}\right)$

$$=\frac{P(A_1)\cdot P\left(\frac{B}{A_1}\right)}{P(A_1)\cdot P\left(\frac{B}{A_1}\right)+P(A_2)\cdot P\left(\frac{B}{A_2}\right)+P(A_3)\cdot P\left(\frac{B}{A_3}\right)}$$

$$=\frac{0.4\times0.25}{0.4\times0.25+0.4\times0.4+0.2\times0.125}$$

$$=\frac{0.1}{0.1+0.16+0.025}$$

$$=\frac{0.1}{0.285}=\frac{20}{57}$$

Hence, option *(c)* is correct.

41. *(d)* Give, equation is $|x-2|^2+|x-2|-2=0$

Let $|x-2|=y$ ....(i)

$\Rightarrow\ y^2+y-2=0$

$\Rightarrow\ (y+2)(y-1)=0$

$\Rightarrow\ y=-2$ and $y=1$

So, $|x-2|=1$

Since, $|x-2|=-2$ is not possible.

$\Rightarrow\ |x-2|=\pm1$

$\Rightarrow\ x=3$ or $x=1$

So, sum of real roots $=3+1=4$
Hence, option *(d)* is correct.

42. *(c)* Given function $f:[0,3]\to[1,29]$ defined by

$f(x)=2x^3-15x^2+36x+1$

$\Rightarrow\ f'(x)=6x^2-30x+36$

$=6(x-2)(x-3)$

$\because f'(x)>0\ \forall\ x<2$ and $x>3$ and $f'(x)<0\ \forall\ 2<x<3$

So, $f'(x)$ is increasing in $[0,2]$ and decreasing in $[2,3]$.

Moreover $f(x)$ is not strictly increasing or strictly decreasing in the entire domain. So, $f(x)$ is not one-one function.

Now, $f(x)$ has maximum value at $x=2$

i.e., $f(2)=29$ in the given domain $[0,3]$ and minimum value of $f(x)$ exist at $x=0$

So, $f(0)=1$

So, range of $f(x)=[1,29]$ which is equal to codomain.

Hence, f is onto.

43. *(c)* Given, $S_{2n}=3n+14n^2$

Replace $2n$ by $\frac{n}{2}$, we get

$S_{2n}=\frac{7n^2}{2}+\frac{3n}{2}=\frac{1}{2}(7n^2+3n)$

Now, $T_n=S_n-S_{n-1}$

$\Rightarrow T_n=\frac{1}{2}\{7n^2+3n-7(n-1)^2-3(n-1)\}$

$\Rightarrow T_n=\frac{1}{2}(14n-4)$

$\Rightarrow T_n=7n-2$

Common difference $=T_2-T_1=12-5=7$

Hence, option *(c)* is correct.

44. *(b)* We have, $f(x)=\begin{cases}\frac{\tan kx}{x}, & x<0\\ 3x+2k^2, & x\geq0\end{cases}$

$\because f(x)$ is continuous at $x=0$

$\therefore \lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$

$\Rightarrow \lim_{x\to 0^-} \dfrac{\tan kx}{kx}\times k = \lim_{x\to 0^+}\left(3x+2k^2\right)$

$\Rightarrow k = 2k^2 \Rightarrow k = \dfrac{1}{2}$

Hence, option *(b)* is correct.

45. *(c)* We have given that

$$\sin C+\cos C+\sin(2B+C)-\cos(2B+C)=2\sqrt{2}$$

$\Rightarrow \left[\sin C+\sin(2B+C)\right]+\left[\cos C-\cos(2B+C)\right]=2\sqrt{2}$

$\Rightarrow 2\sin(B+C)\cos B+2\sin(B+C)\sin B=2\sqrt{2}$

$\Rightarrow \sin(180°-A)\cos B+\sin(180°-A)\sin B=\sqrt{2}$

$\Rightarrow \sin A(\cos B+\sin B)=\sqrt{2}$

$\Rightarrow \sin A\left(\dfrac{1}{\sqrt{2}}\cos B+\dfrac{1}{\sqrt{2}}\sin B\right)=1$

$\Rightarrow \sin A\cdot\sin\left(\dfrac{\pi}{4}+B\right)=1$

It is possible only when $\sin A=1$ and $\sin\left(\dfrac{\pi}{4}+B\right)=1$

$\Rightarrow \angle A=90°$ and $\angle B=45°$, then $\angle C=45°$

Hence, ΔABC is an isosceles right angled-triangle.

Hence, option (c) is correct.

46. *(b)* Given, $f(x)=\begin{cases}\sin x & \text{if } x\neq n\pi, n=0,\pm1,\pm2,\ldots\\ 2 & \text{if } x=n\pi\end{cases}$

and $g(x)=\begin{cases}x^2+1 & \text{if } x\neq 2\\ 3 & \text{if } x=2\end{cases}$

$\text{LHL}=\lim_{x\to 0^-}(g\circ f)(x)=\lim_{x\to 0^-} g\{f(x)\}$

$=\lim_{x\to 0^-} g(\sin x)=\lim_{x\to 0^-}\left(\sin^2 x+1\right)$

$=1$

And,

$\text{RHL}=\lim_{x\to 0^+}(g\circ f)(x)=\lim_{x\to 0^+} g\{f(x)\}$

$=\lim_{x\to 0^-}\left(\sin^2 x+1\right)$

$=1$

$\Rightarrow \lim_{x\to 0}(g\circ f)(x)=1$

Hence, option *(b)* is correct.

47. *(c)* Given, $f(x)=k\sin x+\dfrac{1}{3}\sin 3x$ has maximum value at $x=\dfrac{\pi}{3}$.

So, $f'\left(\dfrac{\pi}{3}\right)=0$ ....(i)

$f'(x)=k\cos x+\cos 3x$

$f'\left(\dfrac{\pi}{3}\right)=k\cos\dfrac{\pi}{3}+\cos\pi=\dfrac{k}{2}-1$

From Eq.(i), $\dfrac{k}{2}-1=0 \Rightarrow k=2$

Hence, option *(c)* is correct.

48. *(d)* Let $x=\sqrt{8+2\sqrt{8+2\sqrt{8+2\sqrt{8+\ldots\infty}}}}$

$\Rightarrow x=\sqrt{8+2x}$

$\Rightarrow x^2=8+2x$ [squaring both sides]

$\Rightarrow x^2-2x-8=0$

$\Rightarrow (x-4)(x+2)=0$

$\Rightarrow x=4$ $[\because x>0]$

49. *(b)* Given, $A=\begin{bmatrix}\alpha & 0\\ 2 & 1\end{bmatrix}$ and $B=\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}$

$A^2=\begin{bmatrix}\alpha & 0\\ 2 & 1\end{bmatrix}\begin{bmatrix}\alpha & 0\\ 2 & 1\end{bmatrix}=\begin{bmatrix}\alpha^2 & 0\\ \alpha+1 & 1\end{bmatrix}$

Also given, $A^2=B$

$\Rightarrow \begin{bmatrix}\alpha^2 & 0\\ \alpha+1 & 1\end{bmatrix}=\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}$

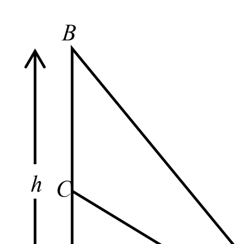
$\Rightarrow \alpha^2=1$ and $\alpha+1=2$

$\Rightarrow \alpha=1$

50. *(b)* Let AB be the tower of height h whose part BC is unfinished and P is a point at 120m away from its base.

Let $AC=x$

Then, $BC=h-x$



In ΔAPC, $\tan 45^\circ = \frac{x}{120}$

$\Rightarrow \quad x = 120\,\text{m}$

In ΔAPB, $\tan 60^\circ = \frac{h}{120}$

$\Rightarrow \quad h = 120\sqrt{3}$

So, tower must be raised to a height

$= h - x = 120\sqrt{3} - 120$

$= 120\left(\sqrt{3} - 1\right)\text{m}$

Hence, option *(b)* is correct.

51. *(a)* Given, $\left[\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{x-y}{x+y}\right)\right]$

$= \tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{\frac{x}{y} - 1}{1 + \frac{x}{y}}\right)$

$= \tan^{-1}\left(\frac{x}{y}\right) - \left\{\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}(1)\right\}$

$= \tan^{-1}(1) = \frac{\pi}{4}$

Hence, option *(a)* is correct.

52. *(b)* We have $\left(\frac{1-x}{1+x}\right)^2 = (1-x)^2(1+x)^{-2}$

$= \left(1 - 2x + x^2\right)\left(1 - 2x + 3x^2 - 4x^3 + 5x^4 + \ldots\infty\right)$

$\therefore$ Coefficient of x^4

$= (1)(5) + (-2)(-4) + (1)(3)$

$= 5 + 8 + 3$

$= 16$

Hence, option *(b)* is correct.

53. *(b)* 8 chairs are numbered 1 to 8. First 2 women choose the chairs from the chairs numbered 1 to 4. This is done in ${}^4C_2 \times 2! = \frac{4! \times 2}{2!2!} = 12$ ways

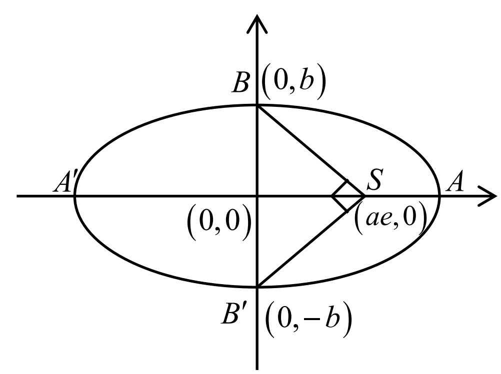
Now, 3 men choose the chairs from the remaining 6 chairs. This is done in $3! \times {}^6C_3 = \frac{6!}{3!3!} \times 3! = 120$ ways

Therefore, total number of ways

$= 12 \times 120 = 1440$ ways

Hence, option *(b)* is correct.

54. *(b)* For ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



Foci are $S(ae, 0)$ and $S'(-ae, 0)$

Coordinates of minor axis are $(0, b)$ and $(0, -b)$

Then, from diagram,

$\angle BSB' = 90^\circ \qquad \text{(given)}$

$\Rightarrow$ Slope of lines $BS \times$ Slope of line $B'S = -1$

$\Rightarrow -\frac{b}{ae} \times \frac{b}{ae} = -1$

$\Rightarrow b^2 = a^2e^2 \qquad \ldots\text{(i)}$

$\because \quad b^2 = a^2\left(1 - e^2\right)$

$\Rightarrow \quad a^2e^2 = a^2\left(1 - e^2\right) \qquad [\text{from Eq.(i)}]$

$\Rightarrow \quad 2e^2 = 1 \Rightarrow e = \frac{1}{\sqrt{2}}$

Hence, option *(b)* is correct.

55. *(c)* Relation R is defined on $N \times N$ as

$(a,b)R(c,d) \Leftrightarrow a+d=b+c$ ....(i)

Now, $(a,b)R(c,d) \Leftrightarrow a+b=b+a$

So, R is reflexive.

$(a,b)R(c,d) \Leftrightarrow a+d=b+c$

$\Rightarrow c+b=d+a$

$\Rightarrow (c,d)R(a,b)$

So, R is symmetric.

$(a,b)R(c,d) \Rightarrow a+d=b+c$ ....(ii)

$(c,d)R(e,f) \Rightarrow c+f=d+e$ ....(iii)

On adding Eqs. (ii) and (ii), we get

$a+d+c+f=b+c+d+e$

$\Rightarrow a+f=b+e$

$\Rightarrow (a,b)R(e,f)$

So, R is transitive.

Hence, R is an equivalence relation.

Hence, option *(c)* is correct.

56. *(d)* Given differential equation is

$\sin x\cos y dx+\cos x\sin y dy=0$

$\Rightarrow \sin x\cos y dx=-\cos x\sin y dy$

$\Rightarrow \tan x dx=-\tan y dy$

$\Rightarrow \tan x dx+\tan y dy=0$

On integrating both sides, we get

$\int \tan x dx+\int \tan y dy=0$

$\Rightarrow \log\sec x+\log\sec y=\log C^{-1}$

$\Rightarrow \log(\sec x\cdot\sec y)=\log\left(\frac{1}{C}\right)$

$\Rightarrow \frac{1}{\cos x\cdot\cos y}=\frac{1}{C}$

$\Rightarrow \cos x\cdot\cos y=C$

Since, it passes through the point $\left(0,\frac{\pi}{3}\right)$, then

$\cos 0\cdot\cos\frac{\pi}{3}=C \Rightarrow 1\cdot\frac{1}{2}=C \Rightarrow C=\frac{1}{2}$

From Eq.(i), we get $\cos x\cdot\cos y=\frac{1}{2}$

On comparing with given equation, we get

$k=\frac{1}{2}$

Hence, option *(d)* is correct.

57. *(c)* Let $I=\int\left(e^{\log x}+\sin x\right)\cos x dx$

$\Rightarrow I=\int(x+\sin x)\cos x dx \qquad \left[\because a^{\log_a x}=x\right]$

$\Rightarrow I=\int \underset{\text{I}}{x}\underset{\text{II}}{\cos x}\,dx+\int\sin x\cos x dx$

$\Rightarrow I=x(\sin x)-\int(1)\sin x dx+\int t dt$

$\left[\text{let} \sin x=t \Rightarrow \cos x dx=dt\right]$

$\Rightarrow I=x\sin x-\cos x+\frac{t^2}{2}+C$

$\Rightarrow I=x\sin x+\cos x+\frac{1}{2}\sin^2 x+C$

Hence, option *(c)* is correct.

58. *(c)* We have $\begin{vmatrix} x & y & z \\ y & z & x \\ z & x & y \end{vmatrix}=(x+y+z)\begin{vmatrix} 1 & 1 & 1 \\ y & z & x \\ z & x & y \end{vmatrix}$

$\left(\text{apply } R_1\to R_1+R_2+R_3\right)$

$=(x+y+z)\left(-x^2-y^2-z^2+xy+yz+zx\right)$

$=-(x+y+z)\left(x^2+y^2+z^2-xy-yz-zx\right)$

According to the question,

$-(x+y+z)\left(x^2+y^2+z^2-xy-yz-zx\right)$

$=-(x+y+z)\left(x+yk+zk^2\right)\left(x+yk^2+zk\right)$

$\Rightarrow x^2+y^2+z^2-xy-yz-zx$

$=x^2+y^2k^3+z^2k^3+xy\left(k+k^2\right)$

$+yz\left(1+k^2\right)k^2+zx\left(k+k^2\right)$

This condition is true only when $k=\omega$

$\left[\because k+k^2=\omega+\omega^2=-1\right]$

Hence, option *(c)* is correct.

59. *(c)* Given, $y=\left(x+\sqrt{1+x^2}\right)^n$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx}=n\left(x+\sqrt{1+x^2}\right)^{n-1}\cdot\left(1+\frac{x}{\sqrt{x^2+1}}\right)$$

$$=\frac{n\left(x+\sqrt{1+x^2}\right)^n}{\sqrt{1+x^2}}$$

$$=\frac{ny}{\sqrt{1+x^2}}$$

60. *(c)* From the previous solution, $\frac{dy}{dx}=\frac{ny}{\sqrt{1+x^2}}$

$$\Rightarrow \left(\frac{dy}{dx}\right)^2\left(1+x^2\right)=n^2y^2$$

Again, differentiating w.r.t. x, we get

$$2\frac{dy}{dx}\cdot\frac{d^2y}{dx^2}\left(1+x^2\right)+2x\left(\frac{dy}{dx}\right)^2=2n^2y\frac{dy}{dx}$$

$$\Rightarrow \frac{d^2y}{dx^2}\left(1+x^2\right)+x\frac{dy}{dx}=n^2y$$

Hence, option *(c)* is correct.

61. *(a)* Since, $\sec(\theta-\alpha), \sec\theta$ and $\sec(\theta+\alpha)$ are in AP

$$\Rightarrow 2\sec\theta=\sec(\theta-\alpha)+\sec(\theta+\alpha)$$

$$\Rightarrow \frac{2}{\cos\theta}=\frac{1}{\cos(\theta-\alpha)}+\frac{1}{\cos(\theta+\alpha)}$$

$$\Rightarrow 2\cos(\theta-\alpha)\cos(\theta+\alpha)=\left[\cos(\theta-\alpha)+\cos(\theta+\alpha)\right]\cos\theta$$

$$\Rightarrow \cos2\theta+\cos2\alpha=2\cos\theta\cos\alpha\cos\theta$$

$$\Rightarrow 1-2\sin^2\theta+2\cos^2\alpha-1=2\cos^2\theta\cos\alpha$$

$$\Rightarrow -\sin^2\theta+\cos^2\alpha-\cos^2\theta\cos\alpha=0$$

$$\Rightarrow -\sin^2\theta+\cos^2\alpha-\left(1-\sin^2\theta\right)\cos\alpha=0$$

$$\Rightarrow -\sin^2\theta(1-\cos\alpha)-(1-\cos\alpha)\cos\alpha=0$$

$$\Rightarrow (1-\cos\alpha)\left(\sin^2\theta+\cos\alpha\right)=0$$

$$\Rightarrow \sin^2\theta+\cos\alpha=0$$

Hence, option *(a)* is correct.

62. *(c)* Let the three points are $A(k,1,3)$, $B(1,-2,k+1)$ and $C(15,2,-4)$.

According to the question, these three points A, B and C are collinear.

So, $\begin{vmatrix} k & 1 & 3 \\ 1 & -2 & k+1 \\ 15 & 2 & -4 \end{vmatrix}=0$

$$\Rightarrow k(8-2k-2)-1(-4-15k-15)+3(2+30)=0$$

$$\Rightarrow 2k^2-21k-115=0$$

So, k has two values as it is the quadratic equation.

Hence, option *(c)* is correct.

63. *(b)* Let the GP of 200 terms be $a\,ar, ar^2, \ldots. ar^{199}$

Given, sum of odd terms $=m$

$$a+ar^2+ar^4+\ldots.+ar^{198}=m \quad \ldots.(i)$$

And sum of even terms $=n$

$$ar+ar^3+ar^5+\ldots.+ar^{199}=n \quad \ldots.(ii)$$

On dividing Eqs. (i) and (ii), we get

$$r=\frac{n}{m}$$

Hence, option *(b)* is correct.

64. *(d)* Common roots of equations $z^3+2z^2+2z+1=0$ and $z^{2017}+z^{2018}+1=0$ are ω, ω^2

Since, these values satisfy both the equations.

65. *(b)* Since, $|z_1-z_2|\ge|z_1|-|z_2|$

$\Rightarrow \left|z-\frac{4}{z}\right| \geq |z|-\left|\frac{4}{z}\right|$

$\Rightarrow 2 \geq |z|-\frac{4}{|z|} \qquad \left[\because \left|z-\frac{4}{z}\right| = 2(\text{given})\right]$

$\Rightarrow 2|z| \geq |z|^2-4$

$\Rightarrow |z|^2-2|z|-4 \leq 0 \qquad ....(\text{i})$

On solving quadratic equation, $x^2-2x-4=0$, we get

$x=\frac{2\pm\sqrt{4-4(1)(-4)}}{2(1)}=1\pm\sqrt{5}$

So, if $|z|^2-2|z|-4=0$

$\Rightarrow |z|=1\pm\sqrt{5}$

$\because |z|$ is always (+)ve.

So, $|z|=1+\sqrt{5}$ is possible.

Moreover, if $|z|^2-2|z|-4\leq 0$

$\Rightarrow 1-\sqrt{5}\leq |z| \leq 1+\sqrt{5}$

So, maximum value of $|z|=1+\sqrt{5}$

Hence, option *(b)* is correct.

66. *(a)* If $z_1=x+iy$ and $z_2=x+iy$,

then $z_1-z_2=0=z_2-z_1$

So, Statement I is correct.

Let $z=x+iy$

Then, its conjugate is $\bar{z}=x-iy$

So, conjugate depends on imaginary part and thus Statement II is correct.

$\because \quad z+0=0+z=z$

So, additive identity of complex number is 0 and thus Statement III is not correct.

Hence, option *(a)* is correct.

67. *(b)* We have $\left|i^{2n+1}(-i)^{2n-1}\right|=\left|i^{2n+1}\cdot i^{2n-1}\right|$

$=\left|i^{4n}\right|=1$

Hence, option *(b)* is correct.

68. *(b)* Given, $f: R-\{-1,-1\}\rightarrow A$ defined by

$f(x)=\frac{x^2}{1-x^2}$, is surjective

As, f is surjective, so every element in codomain must have a pre-image in domain

Let $y=\frac{x^2}{1-x^2}$

$\Rightarrow y-yx^2=x^2$

$\Rightarrow x=\pm\sqrt{\frac{y}{1+y}}$

For x to be defined,

$1+y\neq 0 \Rightarrow y\neq -1 \qquad ....(\text{i})$

and $\frac{y}{1+y}\geq 0$

$\Rightarrow y\in(-\infty,-1)\cup[0,\infty] \qquad ....(\text{ii})$

From Eqs. (i) and (ii), we get

$y\in R-[-1,0]$

Hence, option *(b)* is correct.

69. *(b)* Let $I=\int(\sin x)^{-1/2}(\cos x)^{-3/2}\,dx$

$\Rightarrow I=\int\frac{1}{\sqrt{\sin x\cos x\cos x}}dx$

$\Rightarrow I=\int\frac{\sec^2 x}{\sqrt{\frac{\sin x}{\cos x}}}dx$

$\Rightarrow I=\int\frac{\sec^2 x}{\sqrt{\tan x}}dx$

$\Rightarrow I=\int\frac{dt}{\sqrt{t}} \qquad \left[\text{let } t=\tan x \Rightarrow dt=\sec^2 x dx\right]$

$\Rightarrow I=2\sqrt{t}+C$

$\Rightarrow I=2\sqrt{\tan x}+C$

Hence, option *(b)* is correct.

70. *(b)* We have $f(x)=\begin{cases}\dfrac{\tan kx}{x}, & x<0\\ 3x+2k^2, & x\ge 0\end{cases}$

$\because f(x)$ is continuous at $x=0$

$\because \lim_{x\to 0^-} f(x)=\lim_{x\to 0^+} f(x)=f(0)$

$\Rightarrow \lim_{x\to 0^-}\dfrac{\tan kx}{kx}\times k=\lim_{x\to 0^-}\left(3x+2k^2\right)$

$\Rightarrow k=2k^2 \Rightarrow k=\dfrac{1}{2}$

Hence, option *(b)* is correct.

71. *(a)* Given, $\sin x+\sin y=\cos x+\cos y$

$\Rightarrow 2\sin\left(\dfrac{x+y}{2}\right)\cos\left(\dfrac{x-y}{2}\right)=2\cos\left(\dfrac{x+y}{2}\right)\cos\left(\dfrac{x-y}{2}\right)$

$\Rightarrow \sin\left(\dfrac{x+y}{2}\right)=\cos\left(\dfrac{x+y}{2}\right)$

$\Rightarrow \tan\left(\dfrac{x+y}{2}\right)=1$

$\Rightarrow \tan\left(\dfrac{x}{2}+\dfrac{y}{2}\right)=1$

Hence, option *(a)* is correct.

72. *(b)* Given in Binomial distribution, $p=q=\dfrac{1}{2}$ and mean $=6$

$\Rightarrow np=6 \Rightarrow n\left(\dfrac{1}{2}\right)=6 \Rightarrow n=12$

So, required number of trials $=12$

Hence, option *(b)* is correct.

73. *(c)* Let $F=\hat{i}+3\hat{j}+2\hat{k}$, $A\left(\hat{i}+2\hat{j}-3\hat{k}\right)$ and $B\left(3\hat{i}-\hat{j}+5\hat{k}\right)$

$\therefore$ Displacement $=AB=OB-OA$

$=2\hat{i}-3\hat{j}+8\hat{k}$

Work done $W=F\cdot S$

$W=\left(\hat{i}+3\hat{j}+2\hat{k}\right)\cdot\left(2\hat{i}-3\hat{j}+8\hat{k}\right)$

$=(1)(2)+(3)(-3)+(2)(8)$

$=2-9+19=9$ units

Hence, option *(c)* is correct.

74. *(b)* Given $f:N\to N$ defined by $f(x)=x+1$

$\because f(1)=2,\ f(2)=3,\ f(3)=4,\ f(4)=5,....$

Clearly, f is one-one because not two values of x gives the same value for $f(x)$.

Moreover it is clear that,

Range $R=N-\{1\}$

$\Rightarrow$ Range $\subset$ Codomain

So, f is not onto.

Hence option *(b)* is correct.

75. *(b)* Total hand-shakes possible among n married couples i.e., $2n$ persons $={}^{2n}C_2$

But there will be n hand-shakes in which person shakes hand with his or her spouse.

So, required number of shake hands is

$={}^{2n}C_2-n$

$=2n(n-1)$

Hence, option *(b)* is correct.

76. *(d)* Given, $AB=A$ and $BA=B$ ....(i)

Then, $A^2=A\cdot A=(AB)(A)$

$=A(BA)=AB$ [from Eq.(i)]

$=A$

Similarly, $B^2=B\cdot B=(BA)B$ [from Eq.(i)]

$=B(AB)=BA$

$=B$

Now, $(AB)^2=(AB)(AB)=A(BA)B$

$=(AB)B$

$=A$ [from Eq.(i)]

Hence, all the three statements are correct here.
Hence, option *(d)* is correct.

77. *(b)* Since, in $(1+x)^n$

$^nC_1 + {}^nC_3 + {}^nC_5 + ... = 2^{n-1}$

So, in expansion of $(1+x)^{50}$, sum of coefficients of odd power i.e. $^{50}C_1 + {}^{50}C_3 + {}^{50}C_5 + ... + {}^{50}C_{49} = 2^{49}$

Hence, option *(b)* is correct.

78. *(d)* In word MORADABAD, there are *3A, 2D, 1M, 1R, 1O, 1B* viz there are 6 different alphabets. We have to make a word of 4 letters at a time. For this first we have to select 4 alphabets and then arrange them. This could be done as follows:

Case I: When all 4 letters are different.

Number of wards $= {}^6C_4 \times 4! = 360$

Case II: When 3 are same and 1 is different.

Number of wards $= {}^6C_1 \times {}^5C_1 \times \frac{4!}{3!} = 20$

Case III: When 2 are same and 2 are different.

Number of words $= {}^2C_1 \times {}^5C_2 \times \frac{4!}{2!} = 240$

Case IV: When 2 are same of one type and 2 are same of another type.

Number of ways $= {}^2C_2 \times \frac{4!}{2!2!} = 6$

Thus, total words $= 360 + 20 + 240 + 6 = 626$

Hence, option *(d)* is correct.

79. *(c)* Given points are $A(1,8,4)$, $B(0,-11,4)$ and $C(2,-3,1)$

Equation of BC is

$$\frac{x-0}{2-0} = \frac{y+11}{-3+11} = \frac{z-4}{1-4}$$

$$\Rightarrow \frac{x}{2} = \frac{y+11}{8} = \frac{z-4}{-3} = k \quad \text{(say)}$$

$\Rightarrow$Point $D(2k, 8k-11, -3k+4)$ be the foot of perpendicular drawn from A on BC.

Direction ratios of AD

$= \langle 2k-1, 8k-11-8, -3k+4-4 \rangle$

$= \langle 2k-1, 8k-19, -3k \rangle$

$\because AD \perp BC$

$\Rightarrow 2(2k-1) + 8(8k-19) - 3(-3k) = 0$

$\Rightarrow k = 2$

So, coordinates of $D(p,q,r) = (4,5,-2)$

$\therefore p+q+r = 4+5-2 = 7$

Hence, option *(c)* is correct.

80. *(a)* Given, $\left|\frac{z-4}{z-8}\right| = 1$

$\Rightarrow |z-4| = |z-8| \qquad \{\text{let } z = x+iy\}$

$\Rightarrow |(x-4)+iy| = |(x-8)+iy|$

$\Rightarrow \sqrt{(x-4)^2 + y^2} = \sqrt{(x-8)^2 + y^2}$

$\Rightarrow x^2 + 16 - 8x + y^2 = x^2 + 64 - 16x + y^2$

$\Rightarrow 8x = 48$

$\Rightarrow x = 6$

Also, given $\left|\frac{z}{z-2}\right| = \frac{3}{2}$

$\Rightarrow 2|z| = 3|z-2|$

$\Rightarrow 2|x+iy| = 3|(x-2)+iy|$

$\Rightarrow 2\sqrt{x^2+y^2} = 3\sqrt{(x-2)^2 + y^2}$

$\Rightarrow 4x^2 + 4y^2 = 9x^2 + 36 - 36x + 9y^2$

$\Rightarrow 5x^2 + 5y^2 - 36x + 36 = 0$

Put $x = 6$ from Eq. (i), we get

$5(36) + 5y^2 - 36(6) + 36 = 0 \Rightarrow y = 0$

So, required number $|z| = x + iy = 6 + i0 = 6$

Hence, option *(a)* is correct.

81. *(c)* Given circle is $x^2 + y^2 = 5$ and parabola is $y^2 = 4x$

Solving both equations, $x^2 + 4x = 5$

$\Rightarrow \quad x^2+4x-5=0$

$\Rightarrow \quad (x+5)(x-1)=0$

$\Rightarrow \quad x=-5$ or $x=1$

It is clear that parabola lies in the first and fourth quadrant where x coordinates is positive.

So, from the equation of the given curve, when $x=1$

$\Rightarrow \quad y=\pm 2$

Therefore, intersection points are $P(1,2)$ and $Q(1,-2)$

Then, $PQ=\sqrt{(1-1)^2+(2+2)^2}=4$ units

Hence, option (c) is correct.

82. *(b)* Let $A=\{1,2\} \Rightarrow P(A)=[Q,\{1\},\{2\},\{1,2\}]$

$\therefore \quad A\cup P(A)=[1,2,Q,\{1\},\{2\},\{1,2\}]$

$\Rightarrow A\cup P(A)\neq P(A)$

Now, $\{A\}\cap P(A)=A$ and $P(A)-\{A\}=\{Q,\{1\},\{2\}\}$

$\Rightarrow P(A)-\{A\}\neq P(A)$

Thus, only statement II is correct.

Hence, option *(b)* is correct.

83. *(c)* Let $\Delta=\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

$\Rightarrow \Delta=(a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ b & c & a \\ c & a & b \end{vmatrix}$

[apply $R_1 \to R_1+R_2+R_3$]

$\Rightarrow \Delta=(a+b+c)\{bc-a^2-b^2+ac+ab-c^2\}$

$\Rightarrow \Delta=-\frac{1}{2}(a+b+c)\{2a^2+2b^2+2c^2-2ab-2bc-2ca\}$

$\Rightarrow \Delta=-\frac{1}{2}(a+b+c)\left[(a-b)^2+(b-c)^2+(c-a)^2\right]$

Given, $a\neq b\neq c$ and a, b and c all are positive.

So, $\Delta<0$ i.e., Δ is negative.

Hence, option *(c)* is correct.

84. *(b)* Let $u=\sqrt{x^2+16}$ and $v=x^2$

Now, $\frac{du}{dx}=\frac{1}{2\sqrt{x^2+16}}\cdot 2x=\frac{x}{\sqrt{x^2+16}}$

And $\frac{dv}{dx}=2x$

Now, rate of change of u w.r.t. v

$$\frac{du}{dx}=\frac{\frac{du}{dx}}{\frac{dv}{dx}}=\frac{x}{\sqrt{x^2+16}}\times\frac{1}{2x}=\frac{1}{2\sqrt{x^2+16}}$$

$$\Rightarrow \left(\frac{du}{dx}\right)_{(x=3)}=\frac{1}{2\sqrt{9+16}}=\frac{1}{2\sqrt{25}}=\frac{1}{10}$$

Hence, option *(b)* is correct.

85. *(a)* Let $I=\int_0^{2\pi}\sqrt{1+\sin\frac{x}{2}dx}$

Since, $\sin\frac{x}{2}=\sin 2\left(\frac{x}{4}\right)=2\sin\frac{x}{4}\cos\frac{x}{4}$

$$\therefore I=\int_0^{2\pi}\sqrt{\sin^2\frac{x}{4}+\cos^2\frac{x}{4}+2\sin\frac{x}{4}\cos\frac{x}{4}}\,dx$$

$$=\int_0^{2\pi}\sqrt{\left(\sin\frac{x}{4}+\cos\frac{x}{4}\right)^2}\,dx$$

$$=\int_0^{2\pi}\left(\sin\frac{x}{4}+\cos\frac{x}{4}\right)dx$$

$$=4\left[-\cos\frac{x}{4}+\sin\frac{x}{4}\right]_0^{2\pi}$$

$$=4\left[-\left(\cos\frac{2\pi}{4}-\cos 0\right)+\left(\sin\frac{2\pi}{4}-\sin 0\right)\right]$$

$$=4[-(-1)+(1)]$$

$$=4\times 2=8$$

Hence, option *(a)* is correct.

86. *(d)* Given, $e^x\tan y dx+(1-e^x)\sec^2 y dy=0$

$$\Rightarrow \frac{e^x dx}{1-e^x}+\frac{\sec^2 y}{\tan y}dy=0$$

On integrating, we get

$$\int \frac{e^x}{1-e^x}dx + \int \frac{\sec^2 y}{\tan y}dy = 0$$

$$\Rightarrow -\log\left(1-e^x\right) + \log\tan y = \log C$$

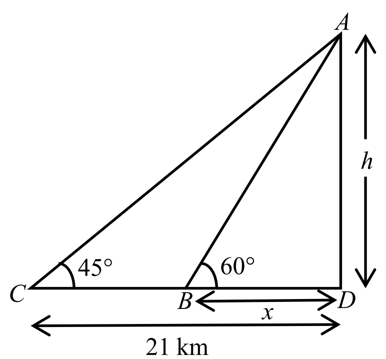
$$\Rightarrow \log\tan y = \log C + \log\left(1-e^x\right)$$

$$\Rightarrow \log\tan y = \log C\left(1-e^x\right)$$

$$\Rightarrow \tan y = C\left(1-e^x\right)$$

Hence, option *(d)* is correct.

87. *(a)* Let h be the height of the tower.



In ΔACD, $\tan 45° = \frac{h}{21} \Rightarrow 1 = \frac{h}{21}$

$\therefore \quad h = 21\text{m} \qquad ....(\text{i})$

Again, in ΔABD,

$$\tan 60° = \frac{h}{x} \Rightarrow \sqrt{3} = \frac{h}{x}$$

$\therefore x = \frac{21}{\sqrt{3}} = 7\sqrt{3}\text{ m} \qquad [\text{from Eq.(i)}]$

Hence, option *(a)* is correct.

88. *(a)* Let $3^x = \tan t \Rightarrow t = \tan^{-1}\left(3^x\right)$

Then, $\sin^{-1}\left(\frac{2-3^x}{1+9^x}\right) = \sin^{-1}\left(\frac{2\tan t}{1+\tan^2 t}\right)$

$$= \sin^{-1}\left(\sin 2t\right) = 2t = 2\tan^{-1}\left(3^x\right)$$

Therefore, $f\left(x\right) = 2\tan^{-1}\left(3^x\right)$

On differentiating w.r.t. x, we get

$$f'\left(x\right) = 2\cdot\frac{1}{1+3^{2x}}\cdot 3^x \log_e 3$$

Put $x = -\frac{1}{2}$, we get

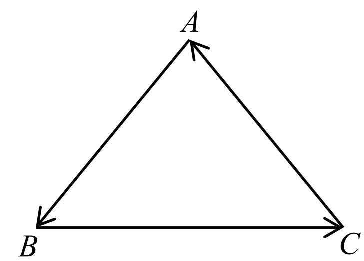
$$f'\left(-\frac{1}{2}\right) = 2\cdot\frac{1}{1+3^{-1}}\cdot 3^{-\frac{1}{2}}\log_e 3$$

$$= 2\cdot\frac{3}{4}\frac{1}{\sqrt{3}}\log_e 3$$

$$= \sqrt{3}\log_e\sqrt{3}$$

Hence, option *(a)* is correct.

89. *(a)* Given, in ΔABC, sides are taken in order. From triangular law of addition of vectors.



$$AB + BC + CA = 0$$

or $\quad -BA + BC + CA = 0$

$\Rightarrow \quad BA - BC - CA = 0$

Hence, only statement I is correct.

Hence, option *(a)* is correct.

90. *(b)* Given, $n \in N$

So, let $n = 1$

$$121^n - 25^n + 1900^n - \left(-4\right)^n = 121 - 25 + 1900 - \left(-4\right)$$

$$= 125 - 25 + 1900$$

$$= 2000$$

Hence, option *(b)* is correct.

91. *(c)* In the expansion of $\left(1+x\right)^{2n}$, there are total $\left(2n+1\right)$ terms.

So, middle term $= \frac{2n+1+1}{2}$ th term

$= \left(n+1\right)$ th term $= T_{n+1} = {}^{2n}C_n x^n$

$\therefore$ Coefficients of middle term, $\alpha = {}^{2n}C_n \qquad ....(\text{i})$

In the expansion of $\left(1+x\right)^{2n-1}$, there are total $2n$ terms.

So, there are two middle terms $= \frac{2n}{2}$ th term and $\left(\frac{2n}{2}+1\right)$ th term

nth term $= T_{(n-1)+1}$

$= {}^{2n-1}C_{n-1}x^{n-1}$

$(n+1)$ th term $= T_{n+1} = {}^{2n-1}C_n x^n$

$\therefore$ Coefficient of middle terms

$\beta = {}^{2n-1}C_{n-1}$ and $\gamma = {}^{2n-1}C_n$ ....(ii)

$\because {}^{2n-1}C_{n-1} + {}^{2n-1}C_n = {}^{2n}C_n$

$\Rightarrow \beta + \gamma = \alpha$ [from Eqs.(i) and (ii)]

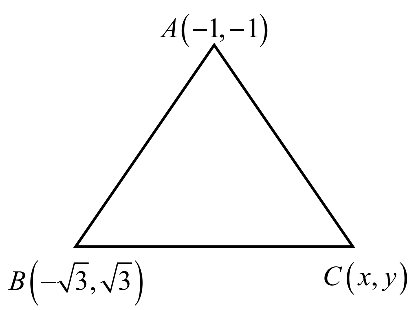
Hence, option *(c)* is correct.

92. *(d)* $\because a^n + b^n$ is divisible by $(a+b)$, if n is odd.

So, $5^5 + 7^5$ is divisible by $5+7=12$

Hence, option *(d)* is correct.

93. *(c)* Equilateral ΔABC has vertices $A(-1,-1)$, $B(-\sqrt{3},\sqrt{3})$ and $C(x,y)$



$AB = \sqrt{\left(-1+\sqrt{3}\right)^2 + \left(-1-\sqrt{3}\right)^2} = \sqrt{8}$

By option method, distance between $(-1,-1)$ and $(1,1)$ is $\sqrt{2^2+2^2} = \sqrt{8}$

So, third vertex is $C(1,1)$

Hence, option *(c)* is correct.

94. *(c)* Let $I = \int_{\pi/12}^{\pi/4} \frac{8\cos 2x}{(\tan x + \cot x)^3} dx$

$= \int_{\pi/12}^{\pi/4} \frac{8\cos 2x (\sin x \cos x)^3}{\left(\sin^2 x + \cos^2 x\right)^3} dx$

$= \int_{\pi/12}^{\pi/4} 8\cos 2x \left(\frac{\sin 2x}{2}\right)^3 dx$

$= \int_{\pi/12}^{\pi/4} \cos 2x \sin^3 2x dx$

Let $= \int_{\pi/12}^{\pi/4} 8\cos 2x \left(\frac{\sin 2x}{2}\right)^3 dx$

When $x = \frac{\pi}{12}$, then $t = \frac{1}{2}$

At $x = \frac{\pi}{4}$, then $t = 1$

So, $I = \int_{1/2}^{1} \frac{t^3}{2} dt = \frac{1}{2}\left[\frac{t^4}{4}\right]_{1/2}^{1} = \frac{1}{2}\left[\frac{1}{4} - \frac{1}{64}\right] = \frac{15}{128}$

Hence, option *(c)* is correct.

95. *(a)* Given, $A = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$

So, $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}\begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$

$= \begin{bmatrix} \alpha^2+\beta^2 & 2\alpha\beta \\ 2\alpha\beta & \alpha^2+\beta^2 \end{bmatrix}$

Given, $A^2 = I$

$\Rightarrow \begin{bmatrix} \alpha^2+\beta^2 & 2\alpha\beta \\ 2\alpha\beta & \alpha^2+\beta^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \alpha^2 + \beta^2 = 1$ and $2\alpha\beta = 0$

$\Rightarrow \alpha = 0, \beta = 1$ or $\alpha = 1, \beta = 0$

Hence, option *(a)* is correct.

96. *(c)* Given $x\log_e(\log_e x) - x^2 + y^2 = 4$

Differentiate w.r.t. x, we get

$x\left(\frac{1}{\log x}\right)\left(\frac{1}{x}\right) + \log(\log x) - 2x + 2y\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{2x - \log(\log x) - (\log x)^{-1}}{2y}$ ....(i)

Given, $x = e$

So, from given equation

$e(\log(\log e)) - e^2 + y^2 = 4$

$\Rightarrow y = \sqrt{4+e^2}$

Put the values of x and y in Eq. (i), we get

$$\frac{dy}{dx} = \frac{2e - \log(\log e) - (\log e)^{-1}}{2\sqrt{4+e^2}}$$

$$= \frac{2e-1}{2\sqrt{4+e^2}} \qquad [\because \log e = 1 \text{ and } \log 1 = 0]$$

Hence, option *(c)* is correct.

97. *(c)* Maximum or minimum value of function exist at the critical points where $\left(\frac{dy}{dx}\right)$ derivative is zero. So, statement I is correct.

If function is differentiable at a point, then it must be continuous at that point while continuous function is not always differentiable. So, statement II is also correct.

Hence, option *(c)* is correct.

98. *(b)* The word 'TIGER' contains 2 vowels and 3 consonants. If the vowels should not occupy the even positions means even places should be occupied by the consonants only.

To fill the even places by the consonants, select any 2 consonants out of 3 and arrange them.

Selection of any 2 consonants out of $3 = {}^3C_2$

The arrangement of these two consonants is given by $2!$

Number of ways to fill even places $= {}^3C_2 \times 2! = 6$

Number of ways to fill odd places $= 3! = 6$

The total number of ways $= 6 \times 6 = 36$

Hence, option *(b)* is correct.

99. *(b)* We have $6x + 8y + 15 = 0$ and $6x + 8y + 18 = 0$

$\therefore$ Perpendicular distance between them is

$$\left|\frac{18-15}{\sqrt{36+64}}\right| = \frac{3}{10} \text{ units}$$

Hence, option *(b)* is correct.

100. *(d)* Since, $99° = 99° \times \frac{\pi}{180°}$ radian $= \frac{11\pi}{20°}$

Now, in ΔABC, let $\angle A = \frac{1}{2}$, $\angle B = \frac{11\pi}{20°}$

$\because \angle A + \angle B + \angle C = \pi$

$\Rightarrow \angle C = \pi - (\angle A + \angle B)$

$$= \pi - \left(\frac{1}{2} + \frac{11\pi}{20}\right)$$

$$= \frac{9\pi - 10}{20}$$

Hence, option *(d)* is correct.

101. *(d)* We have $f(\theta) \times f(\phi)$

$$= \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta\cos\phi - \sin\theta\sin\phi & -\sin\phi\cos\theta - \sin\theta\cos\phi & 0 \\ \sin\theta\cos\phi + \cos\theta\sin\phi & -\sin\theta\sin\phi + \cos\theta\cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\theta+\phi) & -\sin(\theta+\phi) & 0 \\ \sin(\theta+\phi) & \cos(\theta+\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$= f(\theta + \phi)$

So, statement I is correct.

$$|f(\theta) \times f(\phi)| = \begin{bmatrix} \cos(\theta+\phi) & -\sin(\theta+\phi) & 0 \\ \sin(\theta+\phi) & \cos(\theta+\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \cos^2(\theta+\phi) + \sin^2(\theta+\phi)$$

$$= 1$$

So, statement II is correct.

Now, $|f(x)| = 1$

Let $g(x) = |f(x)|$

$\Rightarrow g(-x) = |f(-x)| = |f(x)| = g(x)$

$\Rightarrow g(x)$ is an even function.

So, all the three statements are correct.

Hence, option *(d)* is correct.

102. *(b)* Given curve is $x = e^x y \Rightarrow y = xe^{-x}$ ....(i)

$\Rightarrow \frac{dy}{dx} = x(-e^{-x}) + e^{-x}(1) = e^{-x}(1-x)$

Put $\frac{dy}{dx} = 0 \Rightarrow e^{-x}(1-x) = 0$

$\Rightarrow 1 - x = 0$ $[\because e^{-x} \neq 0]$

$\Rightarrow x = 1$

Now, $\frac{d^2y}{dx^2} = \frac{d}{dx}\left[e^{-x}(1-x)\right]$

$\Rightarrow \frac{d^2y}{dx^2} = e^{-x}(0-1) + (1-x)(-e^{-x})$

$\Rightarrow \frac{d^2y}{dx^2} = e^{-x}(-1-1+x)$

$\therefore \left(\frac{d^2y}{dx^2}\right)_{x=1} = -e^{-1} = -\frac{1}{e} = -\text{ve}$

$\Rightarrow y$ is maximum at $x = 1$

Put $x = 1$ in Eq. (i) $\Rightarrow y = e^{-1}$

Thus, maximum point on the curve $x = e^x y$ is $(1, e^{-1})$.

Hence, option *(b)* is correct.

103. *(b)* Let the radius of the spherical balloon be r.

Then, volume $V = \frac{4}{3}\pi r^3$ and surface area $S = 4\pi r^2$

Given, $\frac{dV}{dt} = 4\,\text{cm}^3/\text{sec}$

$\Rightarrow \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) = 4$

$\Rightarrow \frac{4}{3}\pi(3r^2)\frac{dr}{dt} = 4 \Rightarrow \frac{dr}{dt} = \frac{1}{\pi r^2}$ ....(i)

Rate of increase of surface area,

$\frac{dS}{dt} = \frac{d}{dt}(4\pi r^2) = 4\pi(2r)\frac{dr}{dt}$ [from Eq.(i)]

$= 8\pi r\left(\frac{1}{\pi r^2}\right) = \frac{8}{r}$

$\therefore \frac{dS}{dt} = \frac{8}{4} = 2\,\text{cm}^2/\text{s}$

Hence, option *(b)* is correct.

104. *(c)* We have $f(x) = (x-1)^2(x+1)(x-2)^3$

On taking $\log$ both sides, we get

$\log f(x) = 2\log(x-1) + \log(x+1) + 3\log(x-2)$

Differentiate both sides w.r.t. x, we get

$\Rightarrow \frac{1}{f(x)} f'(x) = \frac{2}{x-1} + \frac{1}{x+1} + \frac{3}{x-2}$

$\Rightarrow f'(x) = f(x)\left\{\frac{2(x+1)(x-2) + (x-1)(x-2) + 3(x-1)(x+1)}{(x+1)(x-1)(x-2)}\right\}$

$\Rightarrow f'(x) = (x-1)^2(x+1)(x-2)^3 \left\{\frac{2x^2 - 2x - 4 + x^2 - 3x + 2 + 3x^2 - 3}{(x+1)(x-1)(x-2)}\right\}$

$\Rightarrow f'(x) = (x-1)(x-2)^2(6x^2 - 5x - 5)$

Put $f'(x) = 0$

$\Rightarrow x = 1,\ x = 2,\ x = \frac{5 \pm \sqrt{145}}{12}$

Now,

$f''(x) = 2(x-2)(3x-5)(5x^2 - 5x - 1)$

For $x = 2,\ f''(x) = 0$

For $x = 1,\ f''(x) < 0$

For $x = \frac{5 \pm \sqrt{145}}{2},\ f''(x) > 0$

So, at $x = \frac{5 \pm \sqrt{145}}{2}$, there is local minima.

Thus, there are two local minima of function $f(x)$

Hence, option (c) is correct.

105. *(b)* From the previous part, at $x = 1$, there is local maxima.

Thus, there is one local maxima of function $f(x)$.

Hence, option *(b)* is correct.

106. *(b)* Only II and III statements are correct.

107. *(b)* Let $A = \dfrac{\sin\theta + \cos\theta - \tan\theta}{\sec\theta + \operatorname{cosec}\theta - \cot\theta}$

Put $\theta = \dfrac{3\pi}{4}$

$$\therefore\ A = \frac{\sin\frac{3\pi}{4} + \cos\frac{3\pi}{4} - \tan\frac{3\pi}{4}}{\sec\frac{3\pi}{4} + \operatorname{cosec}\frac{3\pi}{4} - \cot\frac{3\pi}{4}}$$

$\because \dfrac{3\pi}{4} = \pi - \dfrac{\pi}{4}$ and $\sin(\pi - \theta) = \sin\theta$, similarly,

$\cos(\pi - \theta) = -\cos\theta$

$\tan(\pi - \theta) = -\tan\theta$

$\cot(\pi - \theta) = -\cot\theta$

$\sec(\pi - \theta) = -\sec\theta$ and

$\operatorname{cosec}(\pi - \theta) = \operatorname{cosec}\theta$

$$\text{So, } A = \frac{\sin\frac{\pi}{4} - \cos\frac{\pi}{4} + \tan\frac{\pi}{4}}{-\sec\frac{\pi}{4} + \operatorname{cosec}\frac{\pi}{4} + \cot\frac{\pi}{4}}$$

$$= \frac{\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} + 1}{-\sqrt{2} + \sqrt{2} + 1} = 1$$

Hence, option *(b)* is correct.

108. *(b)* Here, $n = 5$ we shall use the change of variables.

$u_i = x_i - 8$ and $v_i = y_i - 7$

We obtain,

x_i	y_i	u_i	v_i	u_1^2	v_1^2	$u_i v_i$
2	4	-6	-3	36	9	18
10	6	2	-2	4	1	-2
8	7	0	0	0	0	0
6	10	-2	3	4	9	-6
8	6	0	-1	0	1	0

We have $\sum x_i = 34$, $\sum y_i = 33$

$\sum u_i = -6$, $\sum v_i = -2$

$\sum u_i^2 = 44$, $\sum v_i^2 = 20$ and $\sum u_i v_i = 10$

Hence, we obtain,

$$\rho(x, y) = \frac{n\sum u_i v_i - \left[\sum u_i\right]\left[\sum v_i\right]}{\sqrt{\left[n\sum u_i^2 - \left(\sum u_i\right)^2\right]\left[n\sum v_i^2 - \left(\sum v_i\right)^2\right]}}$$

$$= \frac{5(10) - (-6)(-2)}{\sqrt{\left[5(44) - (-6)^2\right]\left[5(20) - (-2)^2\right]}}$$

$$= \frac{50 - 12}{\sqrt{(220 - 36)(100 - 4)}}$$

$$= \frac{38}{\sqrt{184 \times 96}} = \frac{38}{16\sqrt{69}} = 0.286$$

Hence, option *(b)* is correct.

109. *(c)* Let the radius of spherical soap bubble $= r$.

Then, its volume, $V = \dfrac{4}{3}\pi r^3$ and surface area, $S = 4\pi r^2$

Given, $\dfrac{dV}{dt} = k$ (constant)

$$\Rightarrow \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) = k \Rightarrow \frac{4}{3}\pi \cdot 3r^2 \frac{dr}{dt} = k$$

$$\Rightarrow \frac{dr}{dt} = \frac{k}{4\pi r^2} \qquad \text{....(i)}$$

$$\because\ S = 4\pi r^2 \Rightarrow \frac{dS}{dt} = 8\pi r \frac{dr}{dt}$$

$$\Rightarrow \frac{dS}{dt} = 8\pi r \frac{k}{4\pi r^2} \qquad \left(\text{from Eq.(i), put value of } \frac{dr}{dt}\right)$$

$$\Rightarrow \frac{dS}{dt} = \frac{2k}{r} \Rightarrow \frac{dS}{dt} \propto \frac{1}{r}$$

Hence, option *(c)* is correct.

110. *(b)* Given, $\displaystyle\int_{-2}^{5} f(x)\,dx = 4$ and $\displaystyle\int_{0}^{5} \{1 + f(x)\}\,dx = 7$

$$\Rightarrow \int_0^5 f(x)\,dx = 7 - \int_0^5 1\,dx = 7 - 5 = 2$$

Since, $\int_{-2}^{5} f(x)\,dx = 4$

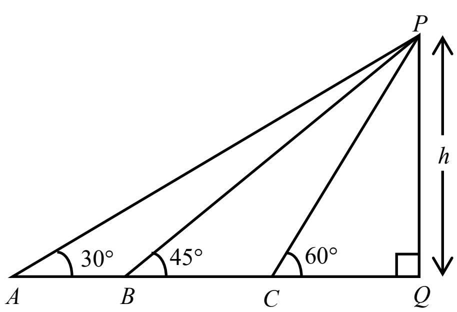
$$\Rightarrow \int_{-2}^{0} f(x)\,dx + \int_0^5 f(x)\,dx = 4$$

$$\Rightarrow \int_{-2}^{0} f(x)\,dx = 4 - \int_0^5 f(x)\,dx$$

$$\Rightarrow \int_{-2}^{0} f(x)\,dx = 4 - 2 = 2$$

Hence, option *(b)* is correct.

111. *(c)* Consider the following figure:



In ΔPQA,

$$\tan 30° = \frac{h}{AQ}$$

$$\Rightarrow \quad AQ = \sqrt{3}h \qquad ....(i)$$

In ΔPQB,

$$\tan 45° = \frac{h}{BQ}$$

$$\Rightarrow \quad BQ = h \qquad ....(ii)$$

In ΔPQC,

$$\tan 60° = \frac{h}{CQ}$$

$$\Rightarrow \quad CQ = \frac{h}{\sqrt{3}} \qquad ....(iii)$$

Now, $\dfrac{AB}{BC} = \dfrac{AQ - BQ}{BQ - CQ}$

$$= \frac{\sqrt{3} - 1}{1 - \frac{1}{\sqrt{3}}} = \frac{\sqrt{3}}{1}$$

$\therefore\ AB : BC = \sqrt{3} : 1$

Hence, option *(c)* is correct.

112. *(d)* Given, 5-digits are 0, 1, 2, 3 and 4

Sum of digits $= 0 + 1 + 2 + 3 + 4 = 10$

$\because$ Sum of digits is divisible by 3.

So, five-digit number divisible by 3 is not possible.

Hence, option *(d)* is correct.

113. *(b)* Given, $A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$

Here, $|A| = \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = 1 - 2 = -1$

$$\text{adj}(A) = \begin{bmatrix} 1 & -1 \\ -2 & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & -2 \\ -1 & 1 \end{bmatrix}$$

$$\therefore\ A^{-1} = \frac{\text{adj}(A)}{|A|} = -1\begin{bmatrix} 1 & -2 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$$

Here, $|B| = \begin{vmatrix} 0 & -1 \\ 1 & 2 \end{vmatrix} = 0 - (-1) = 1$

$$\text{adj}(B) = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}^T = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}$$

$$\therefore\ B^{-1} = \frac{\text{adj}(B)}{|B|} = 1 \cdot \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}$$

$$\therefore\ B^{-1}A^{-1} = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -2+1 & 4-1 \\ 1+0 & -2+0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 3 \\ 1 & -2 \end{bmatrix}$$

Hence, option *(b)* is correct.

114. *(b)* Given $X = 65, Y = 67, \sigma_x = 2.5, \sigma_y = 3.5$ and $r(x, y) = 0.8$

Since, $b_{yx} = r\dfrac{\sigma_y}{\sigma_x}$

$$= 0.8 \times \frac{3.5}{2.5} = 1.12$$

So, regression line of Y on X is

$$Y - \overline{Y} = b_{yx}\left(X - \overline{X}\right)$$

$\Rightarrow Y - 67 = 1.12(X - 65)$

$\Rightarrow Y = 1.12X - 5.8$

Hence, option *(b)* is correct.

115. *(b)* Equation of line perpendicular to the line $6x + 8y + 15 = 0$ is $6x + 8y + 15 = 0$

Since, it passes through point $(2, 2)$

$\therefore\ 2 - 6 + k = 0$

$\Rightarrow \quad k = 4$

$\therefore$ Required equation of the line is

$\Rightarrow\ x - 3y + 4 = 0$

$\Rightarrow\ \dfrac{x}{-4} - \dfrac{y}{4/3} = 1$

So, y-intercept is $\dfrac{4}{3}$

Hence, option *(b)* is correct.

116. *(a)* Two chits are drawn one by one from ten chits number 0 to 9.

If second chit drawn is to be 9, then it is clear that first chit drawn is other than 9.

So, required probability is $= \dfrac{{}^{9}C_1}{{}^{10}C_1} \times \dfrac{1}{{}^{9}C_1} = \dfrac{1}{10}$

Hence, option *(a)* is correct.

117. *(d)* Given line $\dfrac{x-1}{2} = \dfrac{y-2}{3} = \dfrac{z-3}{4} = k$ (say)

$\Rightarrow$ Point $P(2k+1, 3k+2, 4k+3)$ lies on the line.

Now checking through option, this point satisfies the planes $3x + 2y - 3z = -2$ and $3x - 6y + 3z = 0$

Hence, option *(d)* is correct.

118. *(d)* Let $z = \left(\dfrac{1+2i}{2+i}\right)^2$

$$z = \left(\frac{i(2-i)}{(2+i)}\right)^2 = \frac{i^2\left(4 + i^2 - 4i\right)}{4 + i^2 + 4i}$$

$$= \frac{-1(3-4i)}{3+4i} = \frac{-1(3-4i)^2}{3^2 - 16i^2}$$

$$= \frac{-\left(9 + 16i^2 - 24i\right)}{9 + 16}$$

$$= \frac{-(9 - 16 - 24i)}{25}$$

$$= \frac{7 + 24i}{25}$$

So, conjugate of z is $\bar{z} = \dfrac{7 - 24i}{25}$

Hence, option *(d)* is correct.

119. *(a)* Let, $\Delta = \begin{vmatrix} 1-a & a-b-c & b+c \\ 1-b & b-c-a & c+a \\ 1-c & c-a-b & a+b \end{vmatrix}$

On applying $C_2 \to C_2 + C_3$, we get

$$\Delta = \begin{vmatrix} 1-a & a & b+c \\ 1-b & b & c+a \\ 1-c & c & a+b \end{vmatrix}$$

Now applying operations $C_1 \to C_1 + C_2$ and $C_3 \to C_3 + C_2$ and taking common $a + b + c$ from C_3, we get

$$\Delta = (a+b+c)\begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix} \qquad [\because C_1 \text{ and } C_3 \text{ are identical}]$$

Hence, the value of determinant is 0.

Hence, option *(a)* is correct.

120. *(a)* Given observations are $-1, 0, 1$ and k.

Also, standard deviation of these four observations $= \sqrt{5}$.

$$\therefore \sqrt{\frac{(-1)^2 + (0)^2 + (1)^2 + k^2}{4} - \left(\frac{-1+0+1+k}{4}\right)} = \sqrt{5}$$

$$\Rightarrow \frac{2+k^2}{4} - \frac{k^2}{16} = 5 \qquad [\text{squaring both sides}]$$

$$\Rightarrow \frac{8+4k^2-k^2}{16} = 5$$

$$\Rightarrow 3k^2 = 72$$

$$\Rightarrow k^2 = 24$$

$$\Rightarrow k = 2\sqrt{6} \text{ or } 2\sqrt{6}$$

$$\Rightarrow k = 2\sqrt{6} \qquad [\because k > 0]$$