HINTS & SOLUTION

1. *(c)* We have $|\alpha|=1 \Rightarrow |\alpha|^2=1 \Rightarrow \alpha\cdot\bar{\alpha}=1$

Therefore,

$$\left|\frac{\alpha-\beta}{1-\alpha\bar{\beta}}\right| = \left|\frac{\alpha-\beta}{\alpha\bar{\alpha}-\alpha\bar{\beta}}\right|$$

$$= \left|\frac{\alpha-\beta}{\alpha\left(\bar{\alpha}-\bar{\beta}\right)}\right|$$

$$= \frac{1}{|\alpha|}\left|\frac{\alpha-\beta}{\bar{\alpha}-\bar{\beta}}\right|$$

$$= \frac{1}{|\alpha|}\left|\frac{\alpha-\beta}{\overline{\alpha-\beta}}\right|$$

$$= \frac{1}{|\alpha|}\left|\frac{\alpha-\beta}{\alpha-\beta}\right| \qquad \{\because |\bar{z}|=|z|\}$$

$$= \frac{1}{|\alpha|} = 1 \qquad \{\because |\alpha|=1\}$$

Hence, option *(c)* is correct.

2. *(a)* Here, we have

$$i^{1000}+i^{1001}+i^{1002}+i^{1003}$$

$$= i^{1000}\left(1+i+i^2+i^3\right)$$

$$= i^{1000}\left(1+i-1-i\right)$$

$$\{\because i^2=-1, i^3=-i\}$$

$$=0$$

Hence, option *(a)* is correct.

3. *(a)* We have

$$\frac{1}{\log_2 N}+\frac{1}{\log_3 N}+\frac{1}{\log_4 N}+\ldots+\frac{1}{\log_{100} N}$$

$$= \log_N 2+\log_N 3+\log_N 4+\ldots+\log_N 100$$

$$\left\{\because \log_b a=\frac{1}{\log_a b}\right\}$$

$$= \log_N 2\cdot3\cdot4\ldots100$$

$$\{\because \log a+\log b=\log ab\}$$

$$= \log_N 1\cdot2\cdot3\cdot4\ldots100$$

$$= \log_N (100!)$$

$$= \frac{1}{\log_{100!} N}$$

Hence, option *(a)* is correct.

4. *(a)* We have,

$$|1-x|+x^2=5$$

Case I: When $x<1$

$\Rightarrow 1-x+x^2=5$

$\Rightarrow x^2-x-4=0$

$\Rightarrow x=\dfrac{1\pm\sqrt{1-4(1)(-4)}}{2}$

$\Rightarrow x=\dfrac{1\pm\sqrt{17}}{2}$

$\Rightarrow x=\dfrac{1-\sqrt{17}}{2} \qquad \{\because x<1\}$

Case II: When $x\geq1$

$\Rightarrow -\left(1-x\right)+x^2=5$

$\Rightarrow -1+x+x^2=5$

$\Rightarrow x^2+x-6=0$

$\Rightarrow (x+3)(x-2)=0$

$\Rightarrow x=-3, 2$

$\Rightarrow x=2 \qquad \{\because x\geq1\}$

$\therefore$ The given equation has a rational root and an irrational root.

Hence, option *(a)* is correct.

5. *(b)* Let $z=\sqrt{3}+i$

$$\therefore |z|=\sqrt{\left(\sqrt{3}\right)^2+(1)^2}=\sqrt{4}=2$$

$$\left\{\because z=a+ib \Rightarrow |z|=\sqrt{a^2+b^2}\right\}$$

Now, $\text{amp}(z)=\tan^{-1}\left(\dfrac{b}{a}\right)$

$$=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)=\frac{\pi}{6}$$

$\therefore\ z = r(\cos\theta + i\sin\theta)$

$\Rightarrow z = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

$$\left\{\because r = |z| = 2 \text{ and } \theta = \text{amp}(z) = \frac{\pi}{6}\right\}$$

Hence, option *(b)* is correct.

6. *(c)* In $(a+b)^n + (a-b)^n$, the number of terms are:

$$= \begin{cases} \frac{n+2}{2}, & \text{if } n \text{ is even} \\ \frac{n+1}{2}, & \text{if } n \text{ is odd} \end{cases}$$

Therefore, the number of terms in

$(1+2\sqrt{3}x)^{11} + (1-2\sqrt{3}x)^{11} = \frac{11+1}{2} = 6$

$$\{\because n = 11, \text{ is odd}\}$$

Hence, option *(c)* is correct.

7. *(a)* Here, we have

$x = 1 - y + y^2 - y^3 + \cdots\infty,\ |y| < 1$

$= \frac{1}{1-(-y)}$

$$\left\{\because a + ar + ar^2 + \ldots\infty = \frac{a}{1-r}, r < 1\right\}$$

$= \frac{1}{1+y}$

Hence, option *(a)* is correct.

8. *(a)* We have $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$

Therefore,

$A^2 = A \cdot A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1\cdot1+2\cdot2 & 1\cdot2+2\cdot3 \\ 2\cdot1+3\cdot2 & 2\cdot2+3\cdot3 \end{bmatrix}$

$= \begin{bmatrix} 5 & 8 \\ 8 & 13 \end{bmatrix}$

Now, it is given that,

$A^2 - kA - I_2 = 0$

$\Rightarrow \begin{bmatrix} 5 & 8 \\ 8 & 13 \end{bmatrix} - \begin{bmatrix} k & 2k \\ 2k & 3k \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} 4 & 8 \\ 8 & 12 \end{bmatrix} = \begin{bmatrix} k & 2k \\ 2k & 3k \end{bmatrix}$

$\Rightarrow k = 4$

Hence, option *(a)* is correct.

9. *(b)* Let A be the set of students who like music and B be the set of students who like dance.

$\because n(A) = 680, n(B) = 215$ and $n(U) = 850$

We know that,

$n(A\cap B) = n(A) + n(B) - n(A\cup B)$

Then,

$n(A\cap B)_{\min} = n(A) + n(B) - n(A\cup B)_{\max}$

$= 680 + 215 - 850$

$$\{\because n(A\cup B)_{\max} = n(U)\}$$

$= 45$

Hence, option *(b)* is correct.

10. *(b)* Here we have:

$x^2 - 4x + [x] = 0$

Case I: $x \in [0,1]$

$\therefore x^2 - 4x + 0 = 0 \quad \{\because x \in [0,1] \Rightarrow [x] = 0\}$

$\Rightarrow x^2 - 4x = 0$

$\Rightarrow x(x-4) = 0$

$\Rightarrow x = 0, 4$

$\Rightarrow x = 0 \qquad \{\because x \in [0,1]\}$

Case II: $x \in [1,2]$

$\Rightarrow x = 2 \pm \sqrt{3}$

$\Rightarrow x = 0.268, 3.732$

$\Rightarrow \text{no solution} \qquad \{\because x \in [1,2]\}$

$\therefore x^2 - 4x + 1 = 0 \quad \{\because x \in [1,2] \Rightarrow [x] = 1\}$

$\Rightarrow x = \frac{4 \pm \sqrt{16-4}}{2}$

This further gives,

$$x = \frac{4 \pm 2\sqrt{3}}{2}$$

$\Rightarrow x = 2 \pm \sqrt{3}$

$\Rightarrow x = 0.268, 3.732$

$\Rightarrow$ no solution $\quad \{\because x \in [1,2]\}$

Therefore, the given equation has only one solution i.e. $x = 0$.

Hence, option *(b)* is correct.

11. *(b)* We have,

$$121^n - 25^n + 190^n - (-4)^n$$

On putting $121^n - 25^n + 190^n - (-4)^n$, we get

$$121^1 - 25^1 + 190^1 - (-4)^1 = 121 - 25 + 190 + 4$$

$$= 2000$$

which is divisible by 2000.

Hence, option *(b)* is correct.

12. *(c)* Let a and b be two numbers

According to the question, we have:

$$\frac{\frac{a+b}{2}}{\sqrt{ab}} = \frac{5}{3}$$

$$\Rightarrow \frac{a+b}{\sqrt{ab}} = \frac{10}{3}$$

$$\Rightarrow \frac{(a+b)^2}{ab} = \frac{100}{9}$$

$$\Rightarrow \frac{a^2 + b^2 + 2ab}{ab} = \frac{100}{9}$$

$$\Rightarrow \frac{a}{b} + \frac{b}{a} + 2 = \frac{100}{9}$$

$$\Rightarrow t + \frac{1}{t} + 2 = \frac{100}{9}$$

$$\Rightarrow \frac{t^2 + 1 + 2t}{t} = \frac{100}{9}$$

$\Rightarrow 9t^2 - 82t + 9 = 0$

$\Rightarrow (t-9)(9t-1) = 0$

$\Rightarrow \quad t = 9, \frac{1}{9}$

$$\therefore \frac{a}{b} = 9 \text{ or } \frac{a}{b} = \frac{1}{9} \quad \left\{\because t = \frac{a}{b}\right\}$$

$\Rightarrow a : b = 9 : 1 \text{ or } 1 : 9$

Hence, option *(c)* is correct.

13. *(c)* Let $z = -1 - i$

Now, $\tan\alpha = \frac{|b|}{|a|} = \frac{|-1|}{|-1|}$

$$\{\because a = -1, b = -1\}$$

$$\therefore \quad \alpha = \tan^{-1}(1) = \frac{\pi}{4}$$

Since a, b both are negative,

$$\therefore \arg(z) = \alpha - \pi = \frac{\pi}{4} - \pi = -\frac{3\pi}{4}$$

Hence, option *(c)* is correct.

14. *(b)* We have $(1+x)^{43}$

$\therefore$ General term, $T_{r+1} = {}^{43}C_r x^r$

Now, $T_{2r+1} = {}^{43}C_{2r} x^{2r}$

And $T_{r+2} = {}^{43}C_{r+1} x^{r+1}$

Now, according to the question, coefficients of $(2r+1)^{\text{th}}$ and $(r+2)^{\text{th}}$ teams are equal.

$$\therefore \quad {}^{43}C_{2r} = {}^{43}C_{r+1}$$

$\Rightarrow 2r + r + 1 = 43$

$$\{\text{if } {}^nC_x = {}^nC_y \Rightarrow x + y = n\}$$

$\Rightarrow 3r = 42$

$\Rightarrow r = 14$

Hence, option *(b)* is correct.

15. *(a)* Here, we have,

$|1 - 2i|^x = 5^x$

$$\Rightarrow \left(\sqrt{(1)^2 + (-2)^2}\right)^x = 5^x$$

$$\left\{\because |a + ib| = \sqrt{a^2 + b^2}\right\}$$

$$\Rightarrow \left(\sqrt{1+4}\right)^x = 5^x$$

$$\Rightarrow \quad \left(\sqrt{5}\right)^x = 5^x$$

This further gives,

$5^{x/2} = 5^x$

$\Rightarrow \frac{x}{2} = x \qquad \{\because a^m = a^n \Rightarrow m = n\}$

$\Rightarrow x - \frac{x}{2} = 0$

$\Rightarrow \frac{x}{2} = 0$

$\Rightarrow x = 0$

But x is non-zero integral

This means that the given equation has no solution.

Hence, option *(a)* is correct.

16. *(a)* Since, $\sin^{-1} x = \cot^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ and

$\cot x + \cot^{-1} y = \cot^{-1}\left(\frac{xy-1}{x+y}\right)$

Therefore,

$\cot\left[\sin^{-1}\left(\frac{3}{5}\right) + \cos^{-1}\left(\frac{3}{2}\right)\right]$

$\Rightarrow \cot\left[\cot^{-1}\left(\frac{\sqrt{1-\left(\frac{3}{5}\right)^2}}{\frac{3}{5}}\right) + \cot^{-1}\left(\frac{3}{2}\right)\right]$

$\Rightarrow \cot\left[\cot^{-1}\left(\frac{4}{3}\right) + \cot^{-1}\left(\frac{3}{2}\right)\right]$

$\Rightarrow \cot\left[\cot^{-1}\left(\frac{\frac{4}{3}\times\frac{3}{2}-1}{\frac{4}{3}+\frac{3}{2}}\right)\right]$

$\Rightarrow \frac{1}{\frac{17}{6}} = \frac{6}{17}$

Hence, option *(a)* is correct.

17. *(c)* Given that $4\sin^2 x = 3, 0 \le x \le \pi$

$\therefore \sin^2 x = \frac{3}{4}$

$\Rightarrow \sin x = \frac{\sqrt{3}}{2} = \sin\frac{\pi}{3} \text{ or } \sin\frac{2\pi}{3}$

$\therefore x = \frac{\pi}{3}, \frac{2\pi}{3}$

$\therefore \sin^2 x = \frac{3}{4}$

$\Rightarrow \sin x = \frac{\sqrt{3}}{2} = \sin\frac{\pi}{3} \text{ or } \sin\frac{2\pi}{3}$

$\therefore x = \frac{\pi}{3}, \frac{2\pi}{3}$

$\therefore \tan 3x = \tan\frac{3\pi}{3} = \tan\pi = 0$

Also, $\tan 3x = \tan 3\left(\frac{2\pi}{3}\right) = \tan 2\pi = 0$

Hence, option *(c)* is correct.

18. *(c)* Given the first term of $AP = p$

$\Rightarrow \quad a = p \qquad \text{....(i)}$

where, a denotes the first term.

And, $a_3 = q, a_5 = 3$

$\Rightarrow \quad a + 2d = q \qquad \text{....(ii)}$

$a + 4d = 3 \qquad \text{....(iii)}$

Therefore,

$pq = a(a+2d)$

$= (3-4d)(3-4d+2d)$

$= (3-4d)(3-2d)$

$= 9 - 18d + 8d^2$

Let $f = 9 - 18d + 8d^2$

Then $f' = 0 - 18 + 16d = -18 + 16d$

For maxima and minima $f' = 0$

$\Rightarrow -18 + 16d = 0$

$\Rightarrow \quad d = \frac{18}{16} = \frac{9}{8}$

Now, $f'' = 16$ (positive)

So, f will be maximum at $d = \frac{9}{8}$.

Hence, option *(c)* is correct.

19. *(a)* We have given that

$$x^3-8=0$$

$$\Rightarrow (x-2)(x^2+2x+4)=0$$

$$\Rightarrow \quad x=2,2\omega,2\omega^2$$

where $\omega=\frac{-1+\sqrt{3}i}{2}$

Hence, roots are non-collinear and will lie on a circle of radius 2 units.

Hence, option *(a)* is correct.

20. *(c)* We have given that,

$\sec x\cdot \operatorname{cos}ecx=p$

$$\Rightarrow \frac{1}{\sin x\cdot\cos x}=p$$

$$\Rightarrow \frac{2}{2\sin x\cos x}=p$$

$$\Rightarrow \frac{2}{\sin 2x}=p$$

where $\sin 2x\in[-1,1]$

if $\sin 2x=1$, then $p=2$ will be the smallest value.

Hence, option *(c)* is correct.

21. *(b)* Let $P=\sin\theta+\sin\theta\cos\theta$

$$\therefore \frac{dP}{d\theta}=\cos\theta+\cos^2\theta-\sin^2\theta$$

For maxima-minima $\frac{dP}{d\theta}=0$

$$\cos\theta+\cos^2\theta-\sin^2\theta=0$$

$$\cos\theta+\cos^2\theta-1+\cos^2\theta=0$$

$$2\cos^2\theta+\cos\theta-1=0$$

$$(2\cos\theta-1)(\cos\theta+1)=0$$

$$\Rightarrow \cos\theta=-1 \text{ or } \cos\theta=\frac{1}{2}$$

$$\Rightarrow \theta=\pi \text{ or } \theta=\frac{\pi}{3}$$

$\theta=\pi$ can be neglected as $\theta\in\left(0,\frac{\pi}{2}\right)$

Therefore, $\theta=\frac{\pi}{3}$

Hence, option *(b)* is correct.

22. *(c)* Given $n(X)=6, n(Y)=5, n(Z)=4$

$$S=(X-Y)\cup Z$$

Since, all 15 elements are different.

Hence, $n(X-Y)=6$

And $p(S)=6+4=10$

$\Rightarrow$ Number of proper subsets of S

$$=2^{10}-1$$

$$=1024-1$$

$$=1023$$

Hence, option *(c)* is correct.

23. *(b)* Since, we know that relations can be functions iff every element has unique image.

Hence, first statement is wrong.

If $R:A\to A$ then $R\subseteq A\times A$

And if $R:A\to B$ then $R\subseteq A\times B$

Hence, 2nd and 3rd statements are correct.

Hence, option *(b)* is correct.

24. *(b)* Given that $\log_{10}2\log_2 10+\log_{10}(10^x)=2$

$$\Rightarrow \log_{10}2\times\frac{1}{\log_{10}2}+x\log_{10}10=2$$

$$\Rightarrow 1+x=2$$

$$\Rightarrow x=1$$

Hence, option *(b)* is correct.

25. *(d)* Given that ABC is a triangle and

$\cos 2A+\cos 2B+\cos 2C=-1$

$$\Rightarrow 1+\cos 2A+\cos 2B+\cos 2C=0$$

$$\Rightarrow 2\cos^2 A+2\cos\left(\frac{2B+2C}{2}\right)\cdot\cos\left(\frac{2B-2C}{2}\right)=0$$

$$\Rightarrow 2\cos^2 A+2\cos(B+C)\cdot\cos(B-C)=0$$

$$\{\because A+B+C=180\}$$

$$\Rightarrow 2\cos^2 A+2\cos(180^\circ-A)\cdot\cos(B-C)=0$$

$$\Rightarrow 2\cos^2 A-2\cos A\cdot\cos(B-C)=0$$

$$\Rightarrow 2\cos A[\cos A-\cos(B-C)]=0$$

$$\Rightarrow 2\cos A[\cos(180^\circ-(B+C))-\cos(B-C)]=0$$

$$\Rightarrow 2\cos A[\cos(B+C)+\cos(B-C)]=0$$

$$\Rightarrow -2\cos A\left[2\cos\frac{B+C+B-C}{2}\cdot\cos\frac{B+C-B+C}{2}\right]=0$$
$$\Rightarrow -4\cos A\cdot\cos B\cdot\cos C=0$$
$$\Rightarrow \cos A\cdot\cos B\cdot\cos C=0$$
Hence, option *(d)* is correct.

26. *(b)* Let $\Delta=\begin{vmatrix}\cos C & \tan A & 0\\ \sin B & 0 & -\tan A\\ 0 & \sin B & \cos C\end{vmatrix}$

Therefore,
$$\Delta=\cos C[0+\sin B\tan A]-\tan A[\sin B\cos C-0]$$
$$=\tan A\sin B\cos C-\tan A\sin B\cos C$$
$$=0$$
Hence, option *(b)* is correct.

27. *(c)* Given that, A consists of first 250 natural numbers that are multiple of 3

Therefore, $A=\{3,6,9,\ldots,750\}$
$$n(A)=250$$
Set B consists of first 200 even natural numbers.

Therefore, $B=\{2,4,6,8,\ldots,400\}$

So, $A\cap B=\{6,12,\ldots,750\}$
$$n(A\cup B)=n(A)+n(B)-n(A\cap B)$$
$$=250+200-66$$
$$n(A\cup B)=384$$
Hence, option *(c)* is correct.

28. *(c)* Let's take the first k terms are first k natural numbers.
$$\therefore\quad S_k=\frac{k(k+1)}{2}$$
Consider $\dfrac{S_{30}}{S_{20}-S_{10}}=\dfrac{\frac{30(31)}{2}}{\frac{20(21)}{2}-\frac{10(11)}{2}}=\dfrac{930}{310}=3$

Hence, option *(c)* is correct.

29. *(c)* Given equation, $4x^2-(5k+1)x+5k=0$

Let the roots are α and β.

Then,
$$\alpha+\beta=-\frac{(-(5k+1))}{4}=\frac{5k+1}{4}$$
And,
$$\alpha\cdot\beta=\frac{5k}{4}$$
Given that, $\alpha-\beta=1$
$$\sqrt{(\alpha+\beta)^2-4\alpha\beta}=1$$
$$\left(\frac{5k+1}{4}\right)^2-4\left(\frac{5k}{4}\right)=1$$
$$\frac{25k^2+1+10k}{16}=1+5k$$
$$25k^2+10k+1=80k+16$$
$$\Rightarrow 25k^2-70k-15=0$$
$$\Rightarrow 5k^2-14k-3=0$$
$$\Rightarrow (k-3)(5k+1)=0$$
$$\Rightarrow k=3 \text{ or } -\frac{1}{5}$$
Hence, option *(c)* is correct.

30. *(a)* Given digits are 3, 5, 7, 9.

Since, the number of ways to find 5-digit numbers is $5!$

But using 3, 5, 7, 9 every time one-digit will be repeated.

So, number of 5-digit numbers with digit 3 repeated $=\dfrac{5!}{2!}$

Number of 5-digit numbers with digit 5 repeated $=\dfrac{5!}{2!}$

Number of 5-digit numbers with digit 7 repeated $=\dfrac{5!}{2!}$

Number of 5-digit numbers with digit 9 repeated $=\dfrac{5!}{2!}$

$\therefore$ Total 5-digit numbers
$$=\frac{5!}{2!}+\frac{5!}{2!}+\frac{5!}{2!}+\frac{5!}{2!}$$
$$=4\left(\frac{5\times4\times3\times2!}{2!}\right)$$
$$=240$$
Hence, option *(a)* is correct.

31. *(a)* The given digits are 1 and 2.

Let the matrix be $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Since, each entry can be filled with 2 ways.
Therefore, number of distinct matrices

$$= 2\times2\times2\times2 = 16$$

Hence, option *(a)* is correct.

32. *(c)* Let $a, ar, ar^2, ...$ is in GP.

Then $ak, akr, akr^2, ...$ will also be in GP, where k is non-zero number.

If $k = \frac{1}{m}, m \neq 0$

$\Rightarrow \frac{a}{m}, \frac{a}{m}r, \frac{a}{m}r^2, ...$ will also be in GP.

Therefore, both statements are correct.
Hence, option *(c)* is correct.

33. *(d)* Given digit are 1, 2, 3, 4, 5
Since, the sum of digits $= 1+2+3+4+5 = 15$ is divisible by the number 3.
$\Rightarrow$ Every 5 digit number formed by the given digits will be divisible by 3.
$\Rightarrow$ There is no prime number.
Hence, option *(d)* is correct.

34. *(b)* If $f(x+1) = x^2 - 3x + 2$

Let $x+1 = y \Rightarrow x = y-1$ or $x \to x-1$

Therefore,

$$f(x) = (x-1)^2 - 3(x-1) + 2$$
$$= x^2 + 1 - 2x - 3x + 3 + 2$$
$$= x^2 - 5x + 6$$

Hence, option *(b)* is correct.

35. *(c)* If $x^2, x, -8$ are in AP, then

$\Rightarrow 2x = x^2 - 8$

$\Rightarrow x^2 - 2x - 8 = 0$

$\Rightarrow (x-4)(x+2) = 0$

$\therefore \quad x \in \{-2, 4\}$

Hence, option *(c)* is correct.

36. *(a)* Given $a_{ij} = 2(i+j)$

$\therefore a_{11} = 2(1+1) = 4,\ a_{21} = 2(2+1) = 6$

$a_{12} = 2(1+2) = 6,\ a_{22} = 2(2+2) = 8$

$a_{13} = 2(1+3) = 8,\ a_{23} = 2(2+3) = 10$

$a_{31} = 2(3+1) = 8,\ a_{32} = 2(3+2) = 10$

$a_{33} = 2(3+3) = 12$

Therefore,

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$= \begin{vmatrix} 4 & 6 & 8 \\ 6 & 8 & 10 \\ 8 & 10 & 12 \end{vmatrix} = 2\cdot2\cdot2 \begin{vmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{vmatrix}$$

$$= 8[2(24-25) - 3(18-20) + 4(15-16)]$$
$$= 8[-2+6-4]$$
$$= 0$$

Hence, option *(a)* is correct.

37. *(d)* Given numbers are 2, 4, 6, 8.
$\therefore$ We can form determinant of order 2.
Number of determinants $= 4\times3\times2\times1 = 24$
Let's observe some determinants:

$\begin{vmatrix} 2 & 6 \\ 8 & 4 \end{vmatrix} = 8 - 48 = -40$, $\begin{vmatrix} 6 & 2 \\ 4 & 8 \end{vmatrix} = 48 - 8 = 40$

$\begin{vmatrix} 2 & 8 \\ 6 & 4 \end{vmatrix} = 8 - 48 = -40$, $\begin{vmatrix} 6 & 4 \\ 2 & 8 \end{vmatrix} = 48 - 8 = 40$

$\begin{vmatrix} 4 & 8 \\ 6 & 2 \end{vmatrix} = 8 - 48 = -40$, $\begin{vmatrix} 8 & 4 \\ 2 & 6 \end{vmatrix} = 48 - 8 = 40$

$\begin{vmatrix} 4 & 6 \\ 8 & 2 \end{vmatrix} = 8 - 48 = -40$, $\begin{vmatrix} 8 & 2 \\ 4 & 6 \end{vmatrix} = 48 - 8 = 40$

Hence, we can see that we are getting a pattern where each determinant value will be neutralised by other value.
So, sum of all the values of all determinant $= 0$
Hence, option *(d)* is correct.

38. *(d)* Given equation of the circle is:

$4x^2 + 4y^2 - 20x + 12y - 15 = 0$

$\Rightarrow x^2+y^2-5x+3y-\frac{15}{4}=0$

On comparing with the general equation of circle

$x^2+y^2+2gx+2fy+c=0$

$g=-\frac{5}{2}, f=\frac{3}{2}, c=-\frac{15}{4}$

Therefore,

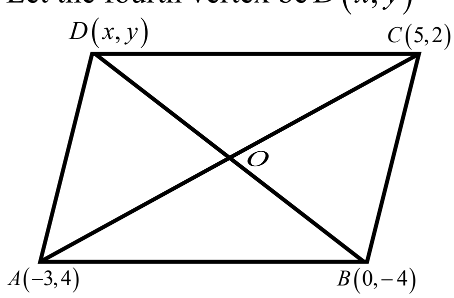
Radius $=\sqrt{g^2+f^2-c}$

$=\sqrt{\frac{25}{4}+\frac{9}{4}+\frac{15}{4}}=\frac{7}{2}$

$=3.5$ unit

Hence, option *(d)* is correct.

39. *(a)* Let the fourth vertex be $D(x,y)$



Since, the diagonals of a parallelogram bisect each other and O is mid-point of AC.

$\Rightarrow$ Coordinate of $O\left(\frac{-3+5}{2},\frac{4+2}{2}\right)=(1,3)$

Also, O is mid-point of BD.

$\Rightarrow$ Coordinate of $O\left(\frac{x+0}{2},\frac{y-4}{2}\right)=\left(\frac{x}{2},\frac{y-4}{2}\right)$

Therefore, compare the coordinate of O

$\frac{x}{2}=1 \Rightarrow x=2$

And,

$\frac{y-4}{2}=3 \Rightarrow y=10$

Thus, the fourth vertex is $(2,10)$.

Hence, option *(a)* is correct.

40. *(c)* We have given that

$y+px=1$ ....(i)

$y-qx=2$ ....(ii)

Let m_1 and m_2 are slopes of equation (i) and (ii) and they are perpendicular, so

$\Rightarrow m_1 \cdot m_2=-1$ and $m=\frac{-\text{coefficient of } x}{\text{coefficient of } y}$

$\Rightarrow \frac{-p}{1}\times\frac{-(-q)}{1}=-1$

$\Rightarrow -pq=-1$

$\Rightarrow pq-1=0$

Hence, option *(c)* is correct.

41. *(d)* Given that A,B,C are in AP.

$\Rightarrow 2B=A+C$

$\Rightarrow A-2B+C=0$

On comparing $A-2B+C=0$ with the given line $Ax+2By+C=0$

We get, $x=1, y=-1$

Hence, line $Ax+2By+C=0$ will pass through $(1,-1)$

Hence, option *(d)* is correct.

42. *(a)* Given points are $A(p,p-3), B(q+3,q)$ and $C(6,3)$

As points lies on a straight line, so

slope of AB = slope of BC

$\Rightarrow \frac{q-p+3}{q+3-p}=\frac{3-q}{6-q-3}$ $\left[\because \text{slope of line}=\frac{y_2-y_1}{x_2-x_1}\right]$

$\Rightarrow 1=1$

$\therefore$ Statement 1 is correct.

But it's not necessary that the collinear points lie in the first quadrant only.

$\therefore$ Statement 2 is correct.

Hence, option *(a)* is correct.

43. *(b)* Given that $l_1: x-2=0$ and $l_2:\sqrt{3}x-y-2=0$

$\therefore$ Slope of line $l_1, m_1=\frac{-\text{coefficient of } x}{\text{coefficient of } y}=\frac{-1}{0}=\infty$

The line l_1 is parallel to Y-axis or perpendicular to X-axis.

And, Slope of line $l_2, m_2=\frac{-\sqrt{3}}{-1}=\sqrt{3}$

The line l_2 makes an angle $60°$ from positive X-axis.

$\therefore$ Angle between l_1 and $l_2=90°-60°=30°$

Hence, option *(b)* is correct.

44. *(d)* Since, we know that circle has eccentricity $e=0$
And parabola has eccentricity $e=1$
And ellipse has eccentricity $e<1$
And hyperbola has eccentricity $e>1$
Hence, option *(d)* is correct.

45. *(b)* Direction ratios of line $l_1=\langle 6,3,6\rangle$

$\Rightarrow a_1=6,\ a_2=3,\ a_3=6$

Direction ratios of line $l_2=\langle 3,3,0\rangle$

$\Rightarrow a_2=3,\ b_2=3,\ c_2=0$

Since,

$$\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$=\frac{6\times3+3\times3+6\times0}{\sqrt{6^2+3^2+6^2}\sqrt{3^2+3^2+0^2}}$$

$$=\frac{27}{9\times3\sqrt{2}}$$

$$=\frac{1}{\sqrt{2}}$$

$$\Rightarrow\theta=\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)=\cos^{-1}\left(\cos\frac{\pi}{4}\right)$$

$$\Rightarrow\theta=\frac{\pi}{4}$$

Hence, option *(b)* is correct.

46. *(c)* Let $A=(1,7,-5)$ and $B=(-3,4,-2)$

$\therefore$ Direction ratios of AB are given by

$AB=\langle(-3-1),(4-7),(-2+5)\rangle=\langle -4,-3,3\rangle$

$\Rightarrow a=-4,\ b=-3,\ c=3$

Direction cosines of Y-axis $=\langle 0,1,0\rangle$

$\Rightarrow l=0,\ m=1,\ n=0$

$\therefore$ Projection of AB on Y-axis

$=|al+bm+cn|$

$=|-4\times0+(-3)\times1+3\times0|$

$=3$

Hence, option *(c)* is correct.

47. *(c)* We have $c=a+b$ where $|a|=|b|\neq0$

Therefore, consider

$c\cdot(a-b)=(a+b)\cdot(a-b)$

$=|a|^2-|b|^2=|b|^2-|b|^2=0$

$\Rightarrow c$ is perpendicular to $(a-b)$.

Also,

$c\cdot(a\times b)=(a+b)(a\times b)$

$=a\cdot(a\times b)+b\cdot(a\times b)$

$=0+0=0$

$\Rightarrow c$ is perpendicular to $(a\times b)$.

Hence, option *(c)* is correct.

48. *(c)* Given that $|a+b|=|a-b|=4$

$\Rightarrow|a+b|^2=|a-b|^2$

$\Rightarrow|a|^2+|b|^2+2a\cdot b=|a|^2+|b|^2-2a\cdot b$

$\Rightarrow4a\cdot b=0$

$\Rightarrow a\cdot b=0$

So, a must be perpendicular to b
Hence, option *(c)* is correct.

49. *(c)* Given that a,b and c are coplanar.

$\Rightarrow[a\ b\ c]=0$

Therefore,

$(2a\times3b)\cdot4c+(5b\times3c)\cdot6a$

$=2\cdot3\cdot4[a\ b\ c]+5\cdot3\cdot6[b\ c\ a]$

$=24[a\ b\ c]+90[a\ b\ c]\qquad\{\because[a\ b\ c]=[b\ c\ a]\}$

$=24\times0+90\times0$

$=0$

Hence, option *(c)* is correct.

50. *(c)* Given that $\lim\limits_{x\to a}\frac{a^x-x^a}{x^a-a^a}=-1$

$\Rightarrow\lim\limits_{x\to a}\frac{a^x-x^a}{x^a-a^a}=-1\left(\frac{0}{0}\text{ form}\right)$

BY using L'Hospital rule,

$\Rightarrow\lim\limits_{x\to a}\frac{a^x\log_e a-ax^{a-1}}{ax^{a-1}-0}=-1$

$\Rightarrow \frac{a^a \log_e a - a \cdot a^{a-1}}{a \cdot a^{a-1}} = -1$

$\Rightarrow \frac{a^a (\log_e a - 1)}{a^a} = -1$

$\Rightarrow \log_e a = -1 + 1 = 0$

$\Rightarrow a = e^0 = 1$

$\therefore a = 1$

Hence, option *(c)* is correct.

51. *(b)* Given curve $y = me^{mx}$ where $m > 0$

$\because$ Curve intersects Y-axis at a point P, then $x = 0$

$\therefore \quad y = me^0 \Rightarrow y = m$

$\therefore$Point $P(0, m)$

Now, differentiation w.r.t. x of the given curve:

$\frac{dy}{dx} = me^{mx} \cdot m = m^2 e^{mx}$

Now, slope of the curve at the point $P(0, m)$

$= \frac{dy}{dx} = m^2 e^0 = m^2$

Hence, option *(b)* is correct.

52. *(c)* Let the tangent makes the angle with *X*-axis be θ, then

$\tan\theta = \left(\frac{dy}{dx}\right)_{(0,m)} = m^2 \Rightarrow \theta = \tan^{-1} m^2$

Now, the tangent will make the angle with Y-axis

$= \frac{\pi}{2} - \theta = \frac{\pi}{2} - \tan^{-1} m^2$

$= \cot^{-1} m^2 \qquad \left\{\because \tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}\right\}$

$= \sin^{-1}\left(\frac{1}{\sqrt{1+m^2}}\right) \qquad \left\{\because \cot^{-1} x = \sin\left(\frac{1}{\sqrt{1+m^2}}\right)\right\}$

Hence, option *(c)* is correct.

53. *(d)* Equation of the tangent to curve P is:

$y - y_1 = \left(\frac{dy}{dx}\right)_{(x_1, y_1)} = (x - x_1)$

$\Rightarrow y - m = m^2 (x - 0)$

$\Rightarrow y = m^2 x + m$

Hence, option *(d)* is correct.

54. *(b)* Let $x = 2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \ldots\infty}}}$

$\Rightarrow x = 2 + \frac{1}{x} \Rightarrow x^2 = 2x + 1$

$\Rightarrow x^2 - 2x - 1 = 0$

$\Rightarrow x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2}$

$\Rightarrow x = 1 \pm \sqrt{2}$

$\therefore x = \sqrt{2} + 1 \qquad \{\because x > 2\}$

Hence, option *(b)* is correct.

55. *(c)* If $P(n, r) = 2520$ and $C(n, r) = 21$

$\therefore \quad {}^nP_r = 2520$

$\Rightarrow \quad \frac{n!}{(n-r)!} = 2520 \qquad \ldots.(\text{i})$

And, ${}^nC_r = 21$

$\Rightarrow \quad \frac{n!}{r!(n-r)!} = 2! \qquad \ldots.(\text{ii})$

From equation (i) and (ii), we get:

$\frac{2520}{r!} = 21$

$\Rightarrow \quad r! = \frac{2520}{21} = 120$

$\Rightarrow \quad r! = 5!$

$\therefore \quad r = 5$

Putting the value of r in equation (i):

$\frac{n!}{(n-5)!} = 2520$

$\Rightarrow \quad n(n-1)(n-2)(n-3)(n-4) = 7 \times 6 \times 5 \times 4 \times 3$

$\therefore \quad n = 7$

Now,

$C(n+1, r+1) = {}^{n+1}C_{r+1}$

$= {}^{7+1}C_{5+1} = {}^8C_6$

$= \frac{8!}{6!2!} = \frac{8 \times 7}{2}$

$= 28$

Hence, option *(c)* is correct.

56. *(d)* From the given expansion, we have:

$$\left(1+2x+x^2\right)^5+\left(1+4y+4y^2\right)^5$$
$$=\left[\left(1+x\right)^2\right]^5+\left[\left(1+2y\right)^2\right]^5$$
$$=\left(1+x\right)^{10}+\left(1+2y\right)^{10}$$

$\therefore$ Total number of terms in given expansion

$$=\left(10+1\right)+\left(10+1\right)=22$$

{$\because$ Total number of terms in expansion of $\left(1+x\right)^n=n+1$}

Hence, option *(d)* is correct.

57. *(c)* Let $A\cup B=\left\{x:\left(x-a\right)\left(x-b\right)>0 \text{ where } a>b\right\}$

It is possible if $x-a<0$ and $x-b<0$ or

$x<a$ and $x<b$

$\therefore$ $A=\left\{x:x<a\right\}$ and $B=\left\{x:x<b\right\}$

Hence, option *(c)* is correct.

58. *(a)* The middle term in the expansion of

$$\left(x^2+\frac{1}{x}\right)^{2n}=\left(\frac{2n}{2}+1\right)^{\text{th}} \text{ term} \quad \left\{\because 2n \text{ is even}\right\}$$
$$=\left(n+1\right)^{\text{th}} \text{ term}$$

As per question, value of the middle term $=184756x^{10}$

$$\Rightarrow {}^{2n}C_n\left(x^2\right)^{2n-n}\left(\frac{1}{x}\right)^n=184756x^{10}$$
$$\left\{\because T_{r+1}={}^nC_r x^{n-r}a^r \text{ in expansion of } \left(x+a\right)^n\right\}$$
$$\Rightarrow {}^{2n}C_n\left(x\right)^{4n-2n-n}=184756x^{10}$$
$$\Rightarrow {}^{2n}C_n\left(x\right)^n=184756x^{10}$$

Comparing the power of x both sides, we get

$$n=10$$

Hence, option *(a)* is correct.

59. *(c)* Given that:

$$C\left(47,4\right)+C\left(51,3\right)+C\left(50,3\right)+C\left(49,3\right)+C\left(48,3\right)+C\left(47,3\right)$$
$$={}^{47}C_4+{}^{51}C_3+{}^{50}C_3+{}^{49}C_3+{}^{48}C_3+{}^{47}C_3$$
$$={}^{51}C_3+{}^{50}C_3+{}^{49}C_3+{}^{48}C_3+{}^{47}C_3+{}^{47}C_4$$
$$={}^{51}C_3+{}^{50}C_3+{}^{49}C_3+{}^{48}C_3+{}^{48}C_4$$
$$\left\{\because {}^nC_r+{}^nC_{r-1}={}^{n+1}C_r\right\}$$
$$={}^{51}C_3+{}^{50}C_3+{}^{49}C_3+{}^{49}C_4$$
$$={}^{51}C_3+{}^{50}C_3+{}^{50}C_4$$
$$={}^{51}C_3+{}^{51}C_4$$
$$={}^{52}C_4=C\left(52,4\right)$$

Hence, option *(c)* is correct.

60. *(c)* We have $S_{2n}=3n+14n^2$

(S_n be the sum of first n terms of an AP)

$$\Rightarrow S_{2n}=\frac{3}{2}\left(2n\right)+\frac{7}{2}\left(2n\right)^2$$

Put $2n=n$, we get

$$\Rightarrow S_n=\frac{3n}{2}+\frac{7n^2}{2}$$

Therefore,

$$T_n=S_n-S_{n-1}$$
$$T_n=\frac{3}{2}n+\frac{7n^2}{2}-\frac{3}{2}\left(n-1\right)-\frac{7}{2}\left(n-1\right)^2$$
$$T_n=\frac{3}{2}n+\frac{7}{2}n^2-\frac{3}{2}n+\frac{3}{2}-\frac{7}{2}n^2-\frac{7}{2}+\frac{7}{2}\left(2n\right)$$
$$T_n=7n-2$$

Put $n=1,2,....$

$$T_1=7\left(1\right)-2=5$$
$$T_2=7\left(2\right)-2=12$$
$$\therefore d=T_2-T_1=12-5=7$$

Hence, option *(c)* is correct.

61. *(b)* Series of two-digit number that divisible by 4 is 12, 16, 20,......,96

This series is an AP, here $a=12$, $d=4$ and $l=96$

Therefore,

$\therefore \quad l = a + (n-1)d$

$\Rightarrow 96 = 12 + (n-1)4$

$\Rightarrow 84 = (n-1)4$

$\Rightarrow n - 1 = 21$

$\Rightarrow n = 22$

Hence, option *(b)* is correct.

62. *(b)* We have

$$\tan\left\{2\tan^{-1}\left(\frac{1}{3}\right)\right\} = \tan\left\{2\tan^{-1}\frac{2\times\frac{1}{3}}{1-\left(\frac{1}{3}\right)^2}\right\}$$

$$\left\{\because 2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}\right\}$$

$$= \tan\tan^{-1}\left(\frac{\frac{2}{3}}{\frac{8}{9}}\right) = \frac{2\times 9}{3\times 8} = \frac{3}{4}$$

Hence, option *(b)* is correct.

63. *(d)* Let AP be a ladder and QR be a vertical flagstaff. P is a point 9m below the top on flagstaff. A is the foot of the ladder and h is the height of point P from the ground.

$\therefore AP = 9\text{m},\ PR = 9\text{m},\ PQ = h\text{m}$

In ΔAQP, $\sin\theta = \frac{PQ}{AP}$

$\Rightarrow \sin 60° = \frac{h}{9} \Rightarrow \frac{\sqrt{3}}{2} = \frac{h}{9}$

$\Rightarrow 9\sqrt{3} = 2h \quad \Rightarrow h = \frac{9\sqrt{3}}{2} = 7.7\text{m}$

$\therefore$ Height of flagstaff

$= h + 9 = 7.7 + 9 = 16.7\text{m}$

Hence, option *(d)* is correct.

64. *(d)* We have a binary number $(cdccddcccddd)_2$ where $c > d$. We know that only two bit (digits) 0 and 1 be any binary number.

$\therefore$ Given binary number is

$$(101100111000)_2 = (1\times 2^{11} + 0\times 2^{10} + 1\times 2^9 + 1\times 2^8 + 0\times 2^7 + 0\times 2^6 + 1\times 2^5 + 1\times 2^4 + 1\times 2^3 + 0\times 2^2 + 0\times 2^1 + 0\times 2^1)_{10}$$

$$= (2048 + 512 + 256 + 32 + 16 + 8)_{10}$$

$$= (2872)_{10}$$

Hence, option *(d)* is correct.

65. *(b)* Given that $\operatorname{cos}ec\theta = \frac{29}{21}$, where $0 < \theta < 90°$

$\because \operatorname{cos}ec\theta = \frac{H}{P} = \frac{29}{21} = \text{k} \ (\text{let})$

$\therefore H = 29k,\ P = 21k$

$\therefore B = \sqrt{(H)^2 - (P)^2} = \sqrt{(29k)^2 - (21k)^2}$

$= \sqrt{841k^2 - 441k^2}$

$= \sqrt{400k^2} = 20k$

$\therefore \sec\theta = \frac{H}{B} = \frac{29k}{20k} = \frac{29}{20}$ and $\tan\theta = \frac{P}{B} = \frac{21k}{20k} = \frac{21}{20}$

Now,

$$4\sec\theta + 4\tan\theta = 4\left(\frac{29}{20}\right) + 4\left(\frac{21}{20}\right)$$

$$= 4\left(\frac{50}{20}\right)$$

$$= 10$$

Hence, option *(b)* is correct.

66. *(d)* Since, $g(x) = \sin\left(\frac{x}{4}\right)$

Then,

$$g(x + 8\pi) = \sin\left(\frac{x + 8\pi}{4}\right) = \sin\left(2\pi + \frac{x}{4}\right)$$

$$= \sin\left(\frac{x}{4}\right)$$

$$= g(x)$$

$\therefore$ Period of the function $g(x) = 8\pi$

Hence, option *(d)* is correct.

67. *(c)* Since, $h(x)=\cos\left(\frac{4x}{5}\right)$

Then,

$$h\left(x+\frac{5\pi}{2}\right)=\cos\frac{4}{5}\left(x+\frac{5\pi}{2}\right)=\cos\left(2\pi+\frac{4x}{5}\right)$$

$$=\cos\left(\frac{4x}{5}\right)=h(x)$$

$\therefore$ Period of the function $h(x)=\frac{5\pi}{2}$

Hence, option *(c)* is correct.

68. *(c)* Since, $f(x)=g(x)+h(x)$

Then,

$$f(x+40\pi)=\sin\left(\frac{x+40\pi}{4}\right)+\cos\frac{4}{5}(x+40\pi)$$

$$=\sin\left(10\pi+\frac{x}{4}\right)+\cos\left(32\pi+\frac{4x}{5}\right)$$

$$=\sin\left(5\times 2\pi+\frac{x}{4}\right)+\cos\left(16\times 2\pi+\frac{4x}{5}\right)$$

$$=\sin\left(\frac{x}{4}\right)+\cos\left(\frac{4x}{5}\right)=f(x)$$

$\therefore$ Period of the function $f(x)=40\pi$

Hence, option *(c)* is correct.

69. *(b)* We have $y=\cos^{-1}(\sin x)$

Differentiate w.r.t. x, we get

$$\frac{dy}{dx}=\frac{-1}{\sqrt{1-\sin^2 x}}\cos x$$

$$\frac{dy}{dx}=\frac{-\cos x}{\cos x}=-1 \qquad [\because \sin^2 x+\cos^2 x=1]$$

Slope of the curve $=\tan\theta$

$\therefore \quad \tan\theta=-1$

$\Rightarrow \quad \tan\theta=-\tan\left(\frac{\pi}{4}\right)$

$\Rightarrow \quad \tan\theta=\tan\left(\pi-\frac{\pi}{4}\right) \qquad [\because \theta\in(0,\pi)]$

$\Rightarrow \quad \theta=\frac{3\pi}{4}$

Hence, option *(b)* is correct.

70. *(d)* We have $f(x)=\frac{\sqrt{x-1}}{x-4}$

$\therefore \quad x-1\geq 0$ and $x-4\neq 0$

$\Rightarrow \quad x\geq 1$ and $x\neq 4$

So, $\quad x\in[1,4)\cup(4,\infty)$

$\therefore$ Domain $=x\in[1,4)\cup(4,\infty)$

Hence, option *(d)* is correct.

71. *(a)* We have $f(x)=\begin{cases}\frac{\sin 2x}{5x}, & \text{if } x\neq 0\\ \frac{2}{15}, & \text{if } x=0\end{cases}$

at $x=0$

$$\lim_{x\to 0} f(x)=\lim_{x\to 0}\frac{\sin 2x\times 2}{5x\times 2}=\frac{2}{5}$$

$$f(0)=\frac{2}{5}$$

$\therefore \lim_{x\to 0} f(x)\neq f(0)$

So, at $x=0$ function is discontinuous.

Hence, option *(a)* is correct.

72. *(a)* We have $f(x)=|x-3|$

We know that, modulus function is continuous on R

Hence, option *(a)* is correct.

73. *(b)* We have $f(x)=\frac{2x-\sin^{-1}x}{2x+\tan^{-1}x}$

Function is continuous at each point

$\therefore \quad f(0)=\lim_{x\to 0} f(x)$

Now,

$$\lim_{x\to 0} f(x)=\lim_{x\to 0}\frac{2x-\sin^{-1}x}{2x+\tan^{-1}x}$$

$$=\lim_{x\to 0}\frac{2-\frac{1}{\sqrt{1-x^2}}}{2+\frac{1}{1+x^2}} \qquad [\text{using L'Hospital rule}]$$

$$=\lim_{x\to 0}\frac{2-1}{2+1}=\frac{1}{3}$$

$\therefore \quad f(0)=\frac{1}{3}$

Hence, option *(b)* is correct.

74. *(a)* We have $f(x)=\sqrt{25-x^2}$

Now,

$$\lim_{x\to 1}\frac{f(x)-f(1)}{x-1}=\lim_{x\to 1}\frac{\sqrt{25-x^2}-\sqrt{24}}{x-1}$$

$$=\lim_{x\to 1}\frac{\frac{-2x}{2\sqrt{25-x^2}}-0}{1-0}\quad \text{[using L'Hospital rule]}$$

$$=-\frac{1}{\sqrt{24}}$$

Hence, option *(a)* is correct.

75. *(a)* We have $y=\tan^{-1}\left(\frac{5-2\tan\sqrt{x}}{2+5\tan\sqrt{x}}\right)$

$$\Rightarrow\quad y=\tan^{-1}\left(\frac{\frac{5}{2}-\tan\sqrt{x}}{1+\frac{5}{2}\tan\sqrt{x}}\right)$$

Let $\tan A=\frac{5}{2}$, therefore,

$$\Rightarrow\quad y=\tan^{-1}\left(\frac{\tan A-\tan\sqrt{x}}{1+\tan A\tan\sqrt{x}}\right)$$

$$\Rightarrow\quad y=\tan^{-1}\left(\tan\left(A-\sqrt{x}\right)\right)$$

$$\Rightarrow\quad y=A-\sqrt{x}$$

$$\Rightarrow\quad y=\tan^{-1}\frac{5}{2}-\sqrt{x}$$

Differentiate w.r.t. x, we get

$$\frac{dy}{dx}=-\frac{1}{2\sqrt{x}}$$

Hence, option *(a)* is correct.

76. *(a)* We have $f(x)=x\sin x+\cos x+\frac{1}{2}\cos^2 x$

$$\therefore\ f'(x)=x(\cos x)+\sin x-\sin x+\frac{1}{2}\cdot 2\cos x(-\sin x)$$

$$\Rightarrow f'(x)=x\cos x-\sin x\cos x$$

By checking options, we put $x=\frac{\pi}{4}$

$$\therefore\ f'(x)=\frac{\pi}{4}\cos\frac{\pi}{4}-\sin\frac{\pi}{4}\cos\frac{\pi}{4}$$

$$=\frac{\pi}{4}\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}$$

$$=\frac{\pi}{4\sqrt{2}}-\frac{1}{2}>0$$

So, $f(x)$ is increasing in the interval $\left(0,\frac{\pi}{2}\right)$

Hence, option *(a)* is correct.

77. *(c)* We have $\lim_{\theta\to 0}\frac{\sqrt{1-\cos\theta}}{\theta}$

$$\therefore\ \lim_{\theta\to 0}\frac{\sqrt{1-\cos\theta}}{\theta}=\lim_{\theta\to 0}\frac{\sqrt{1-\left(1-2\sin^2\frac{\theta}{2}\right)}}{\theta}$$

$$=\lim_{\theta\to 0}\frac{\sqrt{2\sin^2\frac{\theta}{2}}}{\theta}$$

$$=\lim_{\theta\to 0}\frac{\sqrt{2}\sin\frac{\theta}{2}}{\frac{\theta}{2}\times 2}=\frac{1}{\sqrt{2}}$$

Hence, option *(c)* is correct.

78. *(c)* We have $f(x)=x^2-4x+5$

Let $y=x^2-4x+5\Rightarrow\frac{dy}{dx}=2x-4$

Now, $\frac{dy}{dx}=0\Rightarrow 2x-4=0\Rightarrow x=2$

Therefore, at $x=2\Rightarrow y=1$

At $x=1,\ y=(1)^2-4(1)+5=2$

At $x=4,\ y=(4)^2-4(4)+5=5$

So, $y\in[1,5)$

$\therefore$ Range $=[1,5)$

Hence, option *(c)* is correct.

79. *(b)* Consider the given integral

$$\int_a^b [x]dx + \int_a^b [-x]dx = \int_a^b ([x]+[-x])dx$$

$$\{\because [x]+[-x]=-1, \text{ if } x \notin z\}$$

$$= \int_a^b (-1)dx$$

$$= -(x)_a^b$$

$$= -(b-a) = a-b$$

Hence, option *(b)* is correct.

80. *(d)* Consider the given integral

$$\int_2^8 |x-5|dx = \int_2^5 |x-5|dx + \int_5^8 |x-5|dx$$

$$\left\{\because \int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx\right\}$$

$$= -\int_2^5 (x-5)dx + \int_5^8 (x-5)dx$$

$$= -\left(\frac{x^2}{2}-5x\right)_2^5 + \left(\frac{x^2}{2}-5x\right)_5^8$$

$$= -\left[\left(\frac{25}{2}-25\right)-(2-10)\right] + \left[(32-40)-\left(\frac{25}{2}-25\right)\right]$$

$$= \frac{25}{2} - 8 - 8 + \frac{25}{2} = 9$$

Hence, option *(d)* is correct.

81. *(d)* We have $\int \sin^3 x \cos x dx$

Let $\sin x = t \Rightarrow \cos x dx = dt$

Therefore, from the given integral, we have

$$\int \sin^3 x \cos x dx = \int t^3 dt = \frac{t^4}{4} + C$$

Since, $t = \sin x$, therefore

$$\int \sin^3 x \cos x dx = \frac{\sin^4 x}{4} + C = \frac{(\sin^2 x)^2}{4} + C$$

$$= \frac{(1-\cos^2 x)^2}{4} + C \quad [\because \sin^2 x + \cos^2 x = 1]$$

Hence, option *(d)* is correct.

82. *(b)* We have the following integral

$$\int e^{\ln(\tan x)} dx = \int \tan x dx \quad [\because e^{\ln(x)} = x]$$

$$= \ln|\sec x| + C$$

Hence, option *(b)* is correct.

83. *(d)* We have the following integral

$$\int_{-1}^1 \left[\frac{d}{dx}\left(\tan^{-1}\frac{1}{x}\right)\right]dx = \left[\tan^{-1}\frac{1}{x}\right]_{-1}^1$$

$$= \tan^{-1}(1) - \tan^{-1}(-1)$$

$$= \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$$

Hence, option *(d)* is correct.

84. *(c)* Let A be event of dice shows 5 and B be the event that sum is 10 or more.

Here $n(S) = 36$

$$n(A) = \{(1,5),(2,5),(3,5),(4,5),(5,5),(6,5),(5,1),(5,2)$$
$$(5,3),(5,4),(5,6)\}$$

$$n(B) = \{(5,5),(6,4),(4,6),(6,5),(5,6),(6,6)\}$$

$$n(A \cap B) = \{(5,5),(6,5),(5,6)\}$$

$$\therefore P\left(\frac{B}{A}\right) = \frac{\frac{3}{36}}{\frac{11}{36}} = \frac{3}{11} \quad \left[\because P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)}\right]$$

Hence, option *(c)* is correct.

85. *(d)* Consider the given differential equation

$$\ln\left(\frac{dy}{dx}\right) = ax + by$$

$$\Rightarrow \frac{dy}{dx} = e^{ax+by}$$

$$\Rightarrow \frac{dy}{dx} = e^{ax} \cdot e^{by}$$

$$\Rightarrow \frac{dy}{e^{by}} = e^{ax}dx$$

Integrating both sides,

$$\int e^{-by}dy = \int e^{ax}dx$$

$$\Rightarrow \frac{e^{-by}}{-b} = \frac{e^{ax}}{a} + C$$

$$\Rightarrow \frac{e^{ax}}{a} + \frac{e^{-by}}{b} + C = 0$$

Hence, option *(d)* is correct.

86. *(d)* In group of men, let number of men $= a$

$$\overline{X_1} = 26 \text{ yr and } n_1 = a(\text{let})$$

And in group of women, number of women $= b$

And combined mean $\overline{X} = 25$

Now, $\overline{X} = \dfrac{n_1\overline{X_1} + n_2\overline{X_2}}{n_1 + n_2}$

$\Rightarrow \quad 25 = \dfrac{26a + 21b}{a+b}$

$\Rightarrow \quad 25a + 25b = 26a + 21b$

$\Rightarrow \quad 4b = a$

$\Rightarrow \quad \dfrac{a}{b} = \dfrac{4}{1}$

Hence, option *(d)* is correct.

87. *(a)* We know that $\vec{n} = \dfrac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$

$$\therefore \quad \vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -4 & -1 \end{vmatrix} = 5\hat{i} + 5\hat{j} - 5\hat{k}$$

$$\Rightarrow |\vec{a}\times\vec{b}| = \sqrt{(5)^2 + (5)^2 + (-5)^2} = 5\sqrt{3}$$

Therefore,

$$\vec{n} = \frac{5\hat{i} + 5\hat{j} - 5\hat{k}}{5\sqrt{3}} = \frac{\hat{i} + \hat{j} - \hat{k}}{\sqrt{3}} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$$

Hence, option *(a)* is correct.

88. *(d)* Given lines,

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda \ (\text{let})$$

$\therefore \dfrac{x-1}{2} = \lambda \Rightarrow x = 2\lambda + 1$

And,

$\dfrac{y-2}{3} = \lambda \Rightarrow y = 3\lambda + 2$

And,

$\dfrac{z-3}{4} = \lambda \Rightarrow z = 4\lambda + 3$

By checking options if $3x + 2y - 3z$, then

$$3(2\lambda+1) + 2(3\lambda+2) - 3(4\lambda+3)$$
$$= 6\lambda + 3 + 6\lambda + 4 - 12\lambda - 9 = -2$$

and,

$$3(2\lambda+1) - 6(3\lambda+2) + 3(4\lambda+3)$$
$$= 6\lambda + 3 - 18\lambda - 12 + 12\lambda + 9 = 0$$

Hence, option *(d)* is correct.

89. *(d)* Statement I If the line segment joining the point $P(m,n)$ and $Q(r,s)$ subtends angle α at origin, then

$$\cos\alpha = \frac{mr + ns}{\sqrt{m^2 + n^2}\sqrt{r^2 + s^2}}$$

So, Statement I is not correct.

Statement II : In any triangle ABC

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statement II is correct.

Hence, option *(d)* is correct.

90. *(d)* We know that,

$$MD = \frac{4}{5}SD$$

$\Rightarrow \quad 5MD = 4SD$

$\Rightarrow \quad 5x = 4y \quad [\because MD = x \text{ and } SD = y]$

$\therefore \quad x < y$

Hence, option *(d)* is correct.

91. *(c)* We have, $z = \dfrac{1+2i}{1-(1-i)^2}$

$\Rightarrow \quad z = \dfrac{1+2i}{1-(1-1-2i)} \quad [\because i^2 = -1]$

$\Rightarrow \quad z = \dfrac{1+2i}{1+2i} = 1$

$\therefore \quad |z| = 1$

Hence, option *(c)* is correct.

92. *(a)* Here, $\arg(z) = \tan^{-1}\left(\dfrac{\text{Im}(z)}{\text{Re}(z)}\right)$

$$\Rightarrow \quad \arg(z) = \tan^{-1}\left(\frac{0}{1}\right)$$
$$= \tan^{-1}(0)$$
$$= 0$$

Hence, option *(a)* is correct.

93. *(b)* We have, $f(x)=\log\left(\sqrt{x^2+1}-x\right)$

$\therefore \quad f(-x)=\log\left(\sqrt{x^2+1}+x\right)$

$$=\log\left(\frac{\left(\sqrt{x^2+1}+x\right)\left(\sqrt{x^2+1}-x\right)}{\left(\sqrt{x^2+1}-x\right)}\right)$$

$$=\log\left(\frac{x^2+1-x^2}{\sqrt{x^2+1}-x}\right)$$

$$=\log\left(\frac{1}{\sqrt{x^2+1}-x}\right)$$

$$=-\log\left(\sqrt{x^2+1}-x\right)$$

$$=-f(x)$$

$\therefore f(x)$ is an odd function.

Hence, option *(b)* is correct.

94. *(d)* We have, $f(x)=\log_x 10$

$\Rightarrow \quad f(x)=\dfrac{\log 10}{\log x}=\dfrac{1}{\log x}$

$\therefore f(x)$ is defined when $x>0$ and $x\neq 1$.

Hence, option *(d)* is correct.

95. *(a)* Given, $\dfrac{dy}{dx}=\cos(y-x)+1$ ....(i)

Let $y-x=t \Rightarrow \dfrac{dy}{dx}-1=\dfrac{dt}{dx} \Rightarrow \dfrac{dy}{dx}=1+\dfrac{dt}{dx}$

From Eq. (i), we get:

$$1+\frac{dt}{dx}=\cos t+1$$

$$\Rightarrow \quad \frac{dt}{dx}=\cos t$$

$$\Rightarrow \quad \sec t\,dt=dx$$

On integrating both sides, we get:

$$\int \sec t\,dt=\int dx$$

$$\Rightarrow \log(\sec t+\tan t)=x+a$$

$$\Rightarrow \sec t+\tan t=e^x\cdot e^a$$

$$\Rightarrow \frac{e^x}{\sec t+\tan t}=e^{-a}$$

$$\Rightarrow \frac{e^x(\sec t-\tan t)}{(\sec t+\tan t)(\sec t-\tan t)}=e^{-a}$$

$$\Rightarrow \frac{e^x(\sec t-\tan t)}{\sec^2 t-\tan^2 t}=e^{-a}$$

$$\Rightarrow e^x(\sec t-\tan t)=e^{-a}$$

$$\Rightarrow e^x\left[\sec(y-x)-\tan(y-x)\right]=e^{-a}$$

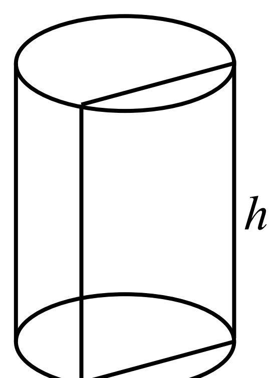
$$\therefore \quad e^x\left[\sec(y-x)-\tan(y-x)\right]=c$$

$$\left[\text{where } c=e^{-a}\right]$$

Hence, option *(a)* is correct.

96. *(a)* Let r be the radius and h be the height of the half cylinder.

Then, surface area, $S=\pi rh+\pi r^2+2rh$



$$\therefore \quad \frac{dS}{dr}=\pi h+2\pi r+2h$$

On putting $\dfrac{dS}{dr}=0$

$$\Rightarrow \quad 2r=-\frac{(\pi h+2h)}{\pi}$$

$$\Rightarrow \quad 2r=-\frac{h(\pi+2)}{\pi}$$

$$\Rightarrow \quad \frac{2r}{h}=-\frac{(\pi+2)}{\pi}$$

$$\Rightarrow \quad \frac{h}{2r}=-\frac{\pi}{(\pi+2)}$$

Neglecting the $-$sign as r and h cannot be negative.

$$\therefore \quad \frac{h}{2r}=\frac{\pi}{\pi+2}$$

Hence, option *(a)* is correct.

97. *(c)* Total number of possible outcomes $=2\times 6=12$

And favourable outcomes $=(H,2),(H,4),(H,6)$

Therefore, total number of possible outcomes $=3$

$$\therefore \text{Required probability}=\frac{3}{12}=\frac{1}{4}$$

Hence, option *(c)* is correct.

98. *(c)* Here, we have given mean $(\bar{x})=50$

And, the new mean $=\dfrac{50-5}{4}=\dfrac{45}{4}=11.25$

And standard deviation $(\sigma)=10$

Therefore, the new standard deviation is

$$=\frac{10}{4}=2.5$$

Since, addition and subtraction does not affect standard deviation.

Hence, option *(c)* is correct.

99. *(c)* Here, A and B be (3×3) matrices with $\det A=4$ and $\det B=3/$

We know that

$$\det(KAB)=K^n\det(A)\times\det(B)$$

where n is the order of A and B, K is a real number.

$$\therefore \det(2AB)=(2)^3\det A\times\det(B) \quad [\because n=3 \text{ and } K=2]$$

$$=8\times 4\times 3$$

$$=96$$

Hence, option *(a)* is correct.

100. *(c)* Here, A and B be (3×3) matrices with $\det A=4$ and $\det B=3$

We know that

$$\det(KAB^{-1})=K^n\det(A)\times\frac{1}{\det(B)}$$

where n is the order of A and B, K is a real number.

$$\therefore \det(3AB^{-1})=(3)^3\det A\times\frac{1}{\det(B)} \quad [\because n=3 \text{ and } K=3]$$

$$=27\times 4\times\frac{1}{3}=36$$

Hence, option *(c)* is correct.

101. *(a)* Given that $\cos(\theta-\alpha)=a$ and $\cos(\theta-\beta)=b$

Therefore, consider

$$\cos(\alpha-\beta)=\cos\{(\theta-\beta)-(\theta-\alpha)\}$$

$$=\cos(\theta-\beta)\cos(\theta-\alpha)+\sin(\theta-\beta)\sin(\theta-\alpha)$$

$$=ab+\sqrt{1-a^2}\sqrt{1-b^2} \qquad \left\{\because \sin x=\sqrt{1-\cos^2 x}\right\}$$

Hence, option *(a)* is correct.

102. *(a)* Consider that,

$$\sin^2(\alpha-\beta)+2ab\cos(\alpha-\beta)$$

$$=1-\cos^2(\alpha-\beta)+2ab\cos(\alpha-\beta)$$

$$=1-\left(ab+\sqrt{1-a^2}\sqrt{1-b^2}\right)^2+2ab\left(ab+\sqrt{1-a^2}\sqrt{1-b^2}\right)$$

$$=1-\left[a^2b^2+(1-a^2)(1-b^2)+2ab\sqrt{1-a^2}\sqrt{1-b^2}\right]+2a^2b^2+2ab\sqrt{1-a^2}\sqrt{1-b^2}$$

$$=1-\left[a^2b^2+1-a^2-b^2+a^2b^2+2ab\sqrt{1-a^2}\sqrt{1-b^2}\right]+2a^2b^2+2ab\sqrt{1-a^2}\sqrt{1-b^2}$$

$$=1-a^2b^2-1+a^2+b^2-a^2b^2-2ab\sqrt{1-a^2}\sqrt{1-b^2}+2a^2b^2+2ab\sqrt{1-a^2}\sqrt{1-b^2}$$

$$=a^2+b^2$$

Hence, option *(a)* is correct.

103. *(b)* Let $z=a^2x+b^2y$ ....(i)

Since, $xy=c^2 \Rightarrow y=\dfrac{c^2}{x}$

On putting $y=\dfrac{c^2}{x}$ in equation (i), we get:

$$\Rightarrow \quad z=a^2x+b^2\left(\frac{c^2}{x}\right) \qquad \text{....(ii)}$$

On differentiating equation (ii) both sides, we get:

$$\frac{dz}{dx} = a^2 - \frac{b^2c^2}{x^2} \quad ....(iii)$$

$$\Rightarrow \frac{d^2z}{dx^2} = \frac{2b^2c^2}{x^3} \quad ....(iv)$$

For maxima and minima, we put $\frac{dz}{dx} = 0$

$$\therefore \quad a^2 - \frac{b^2c^2}{x^2} = 0$$

$$\Rightarrow \quad \frac{b^2c^2}{x^2} = a^2$$

$$\Rightarrow \quad x = \pm\frac{bc}{a}$$

At $x = \frac{bc}{a}$, $\frac{d^2z}{dx^2} = \frac{2a^3}{bc} > 0$

which gives minimum value.

At $x = -\frac{bc}{a}$, $\frac{d^2z}{dx^2} = -\frac{2a^3}{bc} < 0$

which gives maximum value.

$\therefore$ Minimum value of z at $x = \frac{bc}{a}$ is $abc + abc = 2abc$

Hence, option *(b)* is correct.

104. *(c)* Variance of first n natural number is

$$= \frac{n^2 - 1}{12} = \frac{20^2 - 1}{12} = \frac{399}{12} = 33.25$$

If all the natural number between 1 and 20 multiplied by 3, then

$$\text{Required variance} = 9 \times 33.25$$

$$= 299.25$$

Hence, option *(c)* is correct.

105. *(b)* We have, $p(\text{success}) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$

$$\therefore \quad q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$$

Given $n = 8$

$\therefore$ Mean $= np = 8 \times \frac{1}{3} = \frac{8}{3}$

And,

$$\text{Standard deviation} = \sqrt{npq}$$

$$= \sqrt{8 \times \frac{1}{3} \times \frac{2}{3}}$$

$$= \sqrt{\frac{16}{9}} = \frac{4}{3}$$

Hence, option *(b)* is correct.

106. *(a)* We have, $f(x) = 2x - x^2$

Therefore,

$$f(x+2) + f(x-2) = \left[2(x+2) - (x+2)^2\right] + \left[2(x-2) - (x-2)^2\right]$$

$$= \left[2x + 4 - (x^2 + 4 + 4x)\right] + \left[2x - 4 - (x^2 + 4 - 4x)\right]$$

$$= \left[x^2 - 2x\right] + \left[-x^2 + 6x - 8\right]$$

$$= -2x^2 + 4x - 8$$

At $x = 0$, we get:

$$f(x+2) + f(x-2) = -2(0)^2 + 4(0) - 8$$

$$= -0 + 0 - 8$$

$$= -8$$

Hence, option *(a)* is correct.

107. *(b)* We know the domain of $\cos^{-1} x$ is $[-1, 1]$.

$\therefore$ Domain of $\cos^{-1}(x-2)$ is:

$$-1 \le x - 2 \le 1$$

$$\Rightarrow \quad -1 + 2 \le x \le 1 + 2$$

$$\Rightarrow \quad 1 \le x \le 3$$

Therefore, domain of the function is $[1, 3]$

Hence, option *(b)* is correct.

108. *(a)* We have, $\frac{dr}{dt} = 0.7\,\text{cm/sec}$

Now, circumference of circle, $C = 2\pi r$

On differentiating w.r.t. t, we get:

$\therefore$ Domain of $\cos^{-1}(x-2)$ is:

$$\frac{dC}{dt}=2\pi\frac{dr}{dt}$$

$$=2\times\frac{22}{7}\times0.7\text{ cm/sec}$$

=4.4 cm/sec

Hence, option *(a)* is correct.

109. *(b)* Plane parallel to *xy*-plane is $z=k$ ....(i)

Since, it is intercept 5 units on *Z*-axis.

$\therefore$ Point $(0,0,5)$ satisfy equation (i), we get

$$k=5$$

Put $k=5$ in equation (i), we get

$$z=5$$

Hence, option *(b)* is correct.

110. *(c)* Let $f(x)=\sin x\cdot\cos x$

$$\Rightarrow\quad f(x)=\frac{1}{2}\times2\sin x\cdot\cos x$$

$$\Rightarrow\quad f(x)=\frac{1}{2}\times\sin 2x$$

We know that the maximum value of $\sin 2x$ is 1.

$$\therefore\quad f(x)_{\max}=\frac{1}{2}\times1=\frac{1}{2}$$

Hence, option *(c)* is correct.

111. *(c)* We have given that $n(Z)=90$

$$\Rightarrow\quad 12+18+17+c=90$$

$$\Rightarrow\quad c=90-47=43$$

Also, given,

$$\frac{n(Y)}{n(Z)}=\frac{4}{5}$$

$$\Rightarrow\quad \frac{16+18+17+b}{90}=\frac{4}{5}$$

$$\Rightarrow\quad 51+b=72$$

$$\Rightarrow\quad b=72-51=21$$

Hence, option *(b)* is correct.

112. *(d)* Now,

$n(X)+n(Y)+n(Z)-n(X\cap Y)-n(Y\cap Z)-n(X\cap Z)$
$+n(X\cap Y\cap Z)$

$$=a+12+18+16+b+17+c$$

$$=a+b+c+63$$

$$=a+b+43+63\qquad[\because c=43]$$

$$=a+b+106$$

Hence, option *(d)* is correct.

113. *(a)* Complement of *X*

$$=p+b+c+17$$

$$=p+b+43+17\qquad[\because c=43]$$

$$=p+b+60$$

Hence, option *(a)* is correct.

114. *(c)* Given, matrix *A* is of order 3×5 and matrix *B* is of order 5×3.

$\therefore$ Order of matrix $AB=[A]_{3\times5}[B]_{5\times3}=[AB]_{3\times3}$

And order of matrix $BA=[B]_{5\times3}[A]_{3\times5}=[BA]_{5\times5}$

Hence, option *(c)* is correct.

115. *(c)* We have

$$I=\int\frac{3x^2+8-4k}{x}dx=\int\left(3x+\frac{8}{x}-\frac{4}{x}k\right)dx$$

Therefore,

$$I=\frac{3x^2}{2}+8\log|x|-4k\log|x|+C$$

$$=\frac{3x^2}{2}+(8-4k)\log|x|+C$$

Integration I become a rational, if

$8-4k=0\Rightarrow k=2$

Hence, option *(c)* is correct.

116. *(b)* Mean of the given observations is,

$$\bar{x}=\frac{-\sqrt{6}-\sqrt{5}-\sqrt{4}-1+1+\sqrt{4}+\sqrt{5}+\sqrt{6}}{8}$$

$$\bar{x}=0$$

Now, standard deviation σ is given by:

$$\sigma = \sqrt{\frac{\sum\left(x_1 - \bar{x}\right)^2}{n}}$$

$$= \sqrt{\frac{\left(-\sqrt{6}-0\right)^2 + \left(-\sqrt{5}-0\right)^2 + \left(-\sqrt{4}-0\right)^2 + \left(-1-0\right)^2 + \left(1-0\right)^2 + \left(\sqrt{4}-0\right)^2 + \left(\sqrt{5}-0\right)^2 + \left(\sqrt{6}-0\right)^2}{8}}$$

$$= \sqrt{\frac{6+5+4+1+1+4+5+6}{8}}$$

$$= \sqrt{\frac{32}{8}}$$

$$= 2$$

Hence, option *(b)* is correct.

117. *(d)* We have, coefficient of variation $= \frac{\sigma}{\bar{x}} \times 100$

Now, standard deviation,

$$\sigma = \frac{1}{n}\sqrt{n\sum x_i^2 - \left(\sum x_i\right)^2}$$

$$= \frac{1}{10}\sqrt{10 \times 200 - (20)^2}$$

$$= \frac{1}{10}\sqrt{2000 - 400}$$

$$= \frac{1}{10}\sqrt{1600}$$

$$= \frac{40}{10} = 4$$

And, mean, $\bar{x} = \frac{\sum x_i}{n} = \frac{20}{10} = 2$

$\therefore$ Coefficient of variation $= \frac{4}{2} \times 100 = 200$

Hence, option *(d)* is correct.

118. *(a)* Total number of gold coins $= 15$

And number of counterfeits coins $= 6$

$\therefore$ Probability of getting all four coins are counterfeits is

$$= \frac{{}^6C_4}{{}^{15}C_4}$$

$$= \frac{\frac{6!}{4!(6-4)!}}{\frac{15!}{4!(15-4)!}} \quad \left[\because {}^nC_r = \frac{n!}{r!(n-r)!}\right]$$

$$= \frac{\frac{6 \times 5}{2 \times 1}}{\frac{15 \times 14 \times 13 \times 12}{4 \times 3 \times 2 \times 1}}$$

$$= \frac{6 \times 5 \times 4 \times 3}{15 \times 14 \times 13 \times 12}$$

$$= \frac{1}{7 \times 13} = \frac{1}{91}$$

Hence, option *(a)* is correct.

119. *(d)* Here, $P(A) = 0.6, P(B) = 0.5$ and $P(A \cap B) = 0.4$

Therefore,

1. $P\left(\bar{A} \cup B\right) = 1 - P(A) + P(A \cap B)$

$= 1 - 0.6 + 0.4 = 0.8$ (which is incorrect)

2. $P\left(\bar{B}/\bar{A}\right) = \frac{P\left(\bar{B} \cap \bar{A}\right)}{P\left(\bar{A}\right)}$

$$= \frac{P\left(\bar{B} \cap \bar{A}\right)}{P\left(\bar{A}\right)}$$

$$= \frac{P\left(\overline{A \cup B}\right)}{P\left(\bar{A}\right)}$$

$$= \frac{1 - P(A \cup B)}{1 - P(A)}$$

$$= \frac{1 - \left[P(A) + P(B) - P(A \cap B)\right]}{1 - P(A)}$$

$$= \frac{1 - [0.6 + 0.5 - 0.4]}{1 - 0.6}$$

$$= \frac{1 - [0.7]}{1 - 0.6}$$

$$= \frac{0.3}{0.4} = 0.75 \text{(which is incorrect)}$$

Therefore, both the statements are incorrect.

Hence, option *(d)* is correct.

120. *(c)* Total number of lottery tickets $= 10$

The prime number from 1 to 10 are $\{2, 3, 5, 7\}$

$\therefore$ Probability of drawing two prime number tickets is

$$= \frac{{}^{4}C_2}{{}^{10}C_2}$$

$$= \frac{\frac{4!}{2!(4-2)!}}{\frac{10!}{2!(10-2)!}}$$

$$= \frac{\frac{4\times 3}{2\times 1}}{\frac{10\times 9}{2\times 1}} = \frac{4\times 3}{10\times 9} = \frac{2}{15}$$

Hence, option *(c)* is correct.