HINTS & SOLUTION

1. *(c)* $\because$ Dividend $= D \times Q + R$
Given, $D = 5Q$ and $D = 2R$
When $R = 15$, $D = 2 \times 15 = 30$
$\therefore \quad Q = \dfrac{D}{5} = \dfrac{30}{5} = 6$
$\therefore$ Dividend $= 30 \times 6 + 15 = 195$

2. *(a)* $\dfrac{a}{4} = \dfrac{b}{5} = \dfrac{c}{6} = k$ (say)
$\Rightarrow \quad a = 4k, \; b = 5k \; and \; c = 6k$
So, $\dfrac{a+b+c}{b} = \dfrac{4k+5k+6k}{5k}$
$\dfrac{15k}{5k} = 3$

3. *(b)* As, n is divided by 4 the remainder is 3, so
$n = 4q + 3$, where q is quotient.
$\Rightarrow 2n = 8q + 6$
$\Rightarrow 2n = (8k + 4) + 2 = 4\,(2k + 1) + 2$
So, if $2n$ is divided by 4 the quotient is $2k + 1$ and remainder is 2.

4. *(c)* I. If $x = 15$ and y = 14, then $x + y = 15 + 14 = 29$, which is a prime number. So, if x and y are composite, then $x + y$ is not always composite.
II. If $x = 15$ and $y = 14$, then $x - y = 15 - 14 = 1$ which is neither prime nor composite, hence again $x - y$ is not always composite.
III. Third condition is satisfied for all measure. Hence, only III is correct.

5. *(b)* Clearly, absolute value is defined by
$|x| = -x$

6. *(c)* Middle term = $T_{\frac{n+1}{2}}$
$\therefore \; a + \left(\dfrac{n+1}{2} - 1\right) d = m \quad$ (given)
$2a + (n-1)\,d = 2m \qquad \ldots$(i)
Now, $S_n = \dfrac{n}{2}\left[2a + (n-1)\,d\right] = nm$

7. *(c)* Given, $\dfrac{1}{b-a} + \dfrac{1}{b-c} = \dfrac{1}{a} + \dfrac{1}{c}$
$\Rightarrow \dfrac{1}{b-a} - \dfrac{1}{c} = \dfrac{1}{a} - \dfrac{1}{b-c}$
$\Rightarrow \dfrac{(c-b+a)}{c(b-a)} = \dfrac{(b-c-a)}{a(b-c)}$
$\Rightarrow \dfrac{1}{c(b-a)} = -\dfrac{1}{a(b-c)}$
$\Rightarrow ba - ca = -cb + ac$
$\Rightarrow ab + bc = 2ac$
$\therefore \qquad b = \dfrac{2ac}{a+c}$
Hence, a, b, c are in HP.

8. *(a)* $\because \; \dfrac{\frac{n}{2}\left[2 \times 3 + (n-1)\,2\right]}{\frac{10}{2}\left[2 \times 5 + (10-1) \times 3\right]} = 7$
$\Rightarrow \dfrac{n(n+2)}{5 \times 37} = 7$
$\Rightarrow \qquad n^2 + 2n - 1295 = 0$
$\Rightarrow \; n^2 + 37n - 35n - 1295 = 0$
$\Rightarrow \qquad (n+37)\,(n-35) = 0$
$\therefore \qquad n = 35$

9. *(b)* The numbers are $13x$ and $15x$.
So, x is the HCF. Now,
HCF $\times$ LCM = Product of numbers

$x \times 39780 = 13x \times 15x$

$\Rightarrow x \times 39780 = 13 \times 15 \times x^2$

$\Rightarrow \quad x = \dfrac{39780}{13 \times 15} = 204$

$\therefore$ Numbers are $13 \times 204 = 2652$ and $15 \times 204 = 3060$

10. *(c)* I. Let $a = 4$, and $b = 10$

$\therefore \quad a + b = 14$

HCF (4, 10) = 2

and HCF (14, 10) = 2

$\therefore$ HCF (a,b) = HCF $(a + b, b)$

II. Let $a = 6$ and $b = 15$

$\therefore b - a = 15 - 6 = 9$

HCF (6, 15) = 3

HCF (6, 9) = 3

$\therefore$ HCF (a,b) = HCF $(a, b - a)$

11. *(c)* Required time = LCM of 42,56 and 63 s LCM of 42, 56 and 63 is

2	42, 56, 63
3	21, 28, 63
7	7, 28, 21
	1, 4, 3

$\therefore$ Required time

$= 2 \times 3 \times 7 \times 4 \times 3 = 504$ s.

12. *(a)* LCM of 6, 9 and 12 = 36

13. *(a)* $x = 15.9273 - 11.0049 = 4.9224$

14. *(c)* $\dfrac{3}{4} = 0.75, \dfrac{5}{6} = 0.833$

$\dfrac{1}{2} = 0.5, \dfrac{2}{3} = 0.66, \dfrac{4}{5} = 0.8$ and $\dfrac{9}{10} = 0.9$

Clearly, 0.8 lies between 0.75 and 0.8333.

$\therefore \dfrac{4}{5}$ lies between $\dfrac{3}{4}$ and $\dfrac{5}{6}$.

15. *(b)* $\dfrac{0.004 \times 0.0008}{0.002} = \dfrac{0.0000032}{0.02}$

$= 0.00016$

16. *(c)* Given expression

$= (11.98)^2 + (0.02)^2 + 11.98 \times x$

For the given expression to be a perfect square, we must have

$11.98 \times x = 2 \times 11.98 \times 0.02$

$\Rightarrow \quad x = 0.04$

[by using $(a + b)^2 = a^2 + b^2 + 2ab$]

17. *(a)* $N = 2^{0.15}$

$\Rightarrow N = (2)^{3/20}$

$\Rightarrow (N)^b = (2)^{3b/20}$

But, $N^b = 16$

$\therefore \quad 16 = (2)^{3/20}$

$\Rightarrow 2^4 = (2)^{3b/20}$

$\Rightarrow \quad 4 = \dfrac{3b}{20} \Rightarrow b = \dfrac{80}{3}$

18. *(d)* We know that, $(42)^2 = 1764$ and $(43)^2 = 1849$

Since, 1764 < 1780 < 1849

Hence, the smallest number that must be added to 1780 is (1849 − 1780), i.e. 69.

19. *(d)* $3\sqrt{5} + \sqrt{125} = 17.88$

$\Rightarrow \quad 3\sqrt{5} + 5\sqrt{5} = 17.88$

$\Rightarrow \quad 8\sqrt{5} = 17.88$

$\Rightarrow \quad \sqrt{5} = \dfrac{17.88}{8} = 2.235$

$\therefore \quad \sqrt{80} + 6\sqrt{5} = 4\sqrt{5} + 6\sqrt{5}$

$= 10\sqrt{5} = 10 \times 2.235$

$= 22.35$

20. *(b)* Total distance covered = 50 + 40 + 90 = 180 km

Total time taken

$$= \left(\frac{50}{25} + \frac{40}{20} + \frac{90}{15}\right) = 10 \text{ h}$$

$\therefore$ Average speed for the whole journey

$$= \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

$$= \frac{180}{10} = 18 \text{ km/h}$$

$$\therefore \ 18 \text{ km/h} = \frac{18 \times 5}{18} \text{ m/s} = 5 \text{ m/s}$$

21. *(c)* I. Here, $x = 20$ km/h, $y = 4$ km/h,

$t_1 = 30$ min, $t_2 = 10$ min

According to formula,

$\therefore$ Required distance

$$= \left(t_1 - t_2\right)(x + y)\frac{x}{y}$$

$$= \frac{(30 - 10)}{60}(20 + 4)\left(\frac{20}{4}\right)$$

$$= \frac{20}{60} \times 24 \times \frac{20}{4} = 5 \times 8 = 40 \text{ km}$$

So, I is incorrect.

II. Here, $x = 20$ km/h, $y = 10$ km/h,

$t_1 = 30$ min, $t_2 = 10$ min

According to formula,

$\therefore$ Required distance

$$= \left(\frac{30 - 10}{60}\right)(20 + 10)\left(\frac{20}{10}\right)$$

$$= \frac{20}{60} \times 30 \times \frac{20}{10} = 20 \text{ km}$$

So, II is correct.

22. *(d)* Actual speed of boy = $(p - q)$ km/h

Time taken to cover 1 km = $\frac{1}{p - q}$

$$\therefore \frac{1}{p - q} = r$$

$$\Rightarrow \frac{1}{r} = p - q$$

23. *(d)* Let the distance between P and $Q = d$ km. Total time taken by Pranit

$$= \frac{d}{10} + \frac{d}{15} = \frac{25d}{150}$$

Total time taken by Harish.

$$= \frac{2d}{12.5} = \frac{4d}{25}$$

According to question,

$$\frac{25d}{150} - \frac{4d}{25} = \frac{12}{60}$$

$$\Rightarrow \ d\left[\frac{25 - 24}{150}\right] = \frac{1}{5}$$

$$\Rightarrow \frac{d}{150} = \frac{1}{5} \Rightarrow d = 30 \text{ km}$$

24. *(a)* Let x km distance be covered in y h.

Then, speed of object in first case

$$= \frac{x}{y} \text{ km/h}$$

As, half of this distance is covered in double time.

Then, speed of object in second case

$$= \frac{x}{y} \div 2y = \frac{x}{2} \times \frac{1}{2y} = \frac{x}{4y} \text{ km/h}$$

$\therefore$ Ratio of first and second speeds

$$= \frac{x}{y} : \frac{x}{4y} = 1 : \frac{1}{4} = 4 : 1$$

25. *(d)* Distance travelled in 44 s = $2\pi r$

$$= 2 \times \frac{22}{7} \times 21 = 132 \text{ m}$$

$\therefore$ Speed $= \frac{132}{4} = 3$ m/s

$\left[\because \text{speed} = \frac{\text{distance}}{\text{time}}\right]$

Time taken to travel 3 km $= \frac{3000}{3}$

$= 1000$ s $= \frac{1000}{60}$ min

$= 16$ min 40 s

26. *(d)* I. 18 men can earn in 5 days = ₹ 1440

1 man can earn in 1 day = ₹ $\frac{1440}{18 \times 5}$

$\therefore$ 10 men can earn in 6 days

$= \frac{1440}{18 \times 5} \times 6 \times 10$

$= ₹960 \neq ₹1280$

II. 16 men can earn in 7 days = ₹ 1120

1 man can earn in 1 day = $\frac{1120}{16 \times 7}$

$\therefore$ 21 men can earn in 4 days

$= \frac{1120}{16 \times 7} \times 21 \times 4 = ₹840 \neq ₹800$

So, neither statement I nor II is correct.

27. *(d)* 1 day work of $A = \frac{1}{x}$

1 day work of $B = \frac{1}{3x}$

$\therefore$ 1 day work of both A and B

$= \frac{1}{x} + \frac{1}{3x} = \frac{4}{3x}$

given, one day work of both A and B

$= \frac{1}{12}$

$\Rightarrow \frac{4}{3x} = \frac{1}{12} \Rightarrow 3x = 48$

$\Rightarrow x = 16$

Hence, the value of x is 16.

28. *(c)* $\therefore$ One day work of Rajesh = $\frac{1}{6}$

$\therefore$ One day work of Shailesh = $\frac{1}{12}$

Hence, one day work of, Rajesh and Shailesh

$= \frac{1}{6} + \frac{1}{12} = \frac{2+1}{12} = \frac{3}{12} = \frac{1}{4}$

$\therefore$ Two day's work = $\frac{1}{2}$

Thus, if they work together for 2 day's, then half of the work will be complete.

29. *(b)* Given, x% of $y = 13x$

$\Rightarrow \frac{x}{100} y = 13x$

$\therefore y = 13 \times 100 = 1300$

30. *(a)* Let fraction be $\frac{x}{y}$

New fraction = $\frac{120\% \text{ of } x}{90\% \text{ of } y} = \frac{4x}{3y}$

According to question,

$\frac{4x}{3y} = \frac{16}{27} \Rightarrow \frac{x}{y} = \frac{16}{27} \times \frac{3}{4} = \frac{4}{9}$

31. *(a)* Let the total number of questions in examination be x.

By given condition, 40% of $x = 10$

$\Rightarrow \frac{x \times 40}{100} = 10 \Rightarrow x = \frac{1000}{40} = 25$

32. *(a)* Water in the mixture = 10% of 140 L

$= \frac{10}{100} \times 140 = 14$ L

Let x L of water added in the mixture, then

$\left(\frac{14+x}{140+x}\right) \times 100 = 12.5$

$\Rightarrow 1400 + 100x = 1750 + 12.5x$

$\Rightarrow 87.5x = 350$

$$\Rightarrow\ x = \frac{350}{87.5} = 4\text{L}$$

33. *(a)* Let monthly income be ₹ x

$$\Rightarrow\ 87\frac{1}{2}\%\ \text{of}\ x = ₹\ 3500$$

$$\Rightarrow\ \frac{175}{2\times 100}x = ₹\ 3500$$

$$\therefore\ x = \frac{3500\times 2\times 100}{175} = ₹\ 4000$$

34. *(a)* The price of item first increased by 20% and then decreased by 20%.

$\therefore$ Net effect

$$= \left(20 - 20 + \frac{20\times(-20)}{100}\right)$$

$$= \frac{-400}{100} = -4\%$$

35. *(c)* Glycerine in the given sample = 80% of 5 L $= \frac{80}{100}\times 5 = 4\text{L}$

Let x L of glycerine be added, then

$$\frac{4+x}{(5+x)}\times 100 = 95$$

$$\Rightarrow\ 80 + 20x = 95 + 19x$$

$$\therefore\ x = 15\text{ L}$$

36. *(c)* Let the sum be ₹ x.

Then, $\frac{x\times 13\times 1}{100} - \frac{x\times 12\times 1}{100} = 110$

$$\Rightarrow\ \frac{x}{100} = 110$$

$$\Rightarrow\ x = 110\times 100 = ₹\,11000$$

37. *(b)* Let the amount of A = ₹ a, time = 2 yr and rate = 5%

$\therefore$ Simple Interest of

$$A = \frac{a\times 2\times 5}{100} = \frac{10a}{100}$$

Let the amount of B = ₹ b, rate = 5% and time = 3 yr.

$\therefore$ Simple interest of

$$B = \frac{b\times 3\times 5}{100} = \frac{15b}{100}$$

Let the amount of C = ₹ c, time = 4 yr and rate = 5%

$\therefore$ Simple interest of

$$C = \frac{c\times 4\times 5}{100} = \frac{20c}{100}$$

But $\frac{a\times 10}{100} = \frac{b\times 15}{100} = \frac{c\times 20}{100}$

$$\Rightarrow\ 10a = 15b = 20c = k$$

So, $a = \frac{k}{10},\ b = \frac{k}{15},\ c = \frac{k}{20}$

$$\therefore\ a:b:c = \frac{1}{10}:\frac{1}{15}:\frac{1}{20}$$

38. *(a)* Given, P = ₹ 400, R = 5% and T = 3 yr

$$\text{Simple interest} = \frac{P\times R\times T}{100}$$

$$SI = \frac{400\times 3\times 5}{100} = ₹\,60$$

$\therefore$ Amount = $P + SI$ = 400 + 60 = ₹ 460

39. *(c)* Here, rate of interest

$$= 3\frac{1}{8}\% = \frac{25}{8}\%$$

Let principal be ₹ x.

and simple interest = ₹ $\frac{3}{8}x$

$$\therefore\ \frac{3}{8}x = \frac{x\times\frac{25}{8}\times T}{100}$$

$$\Rightarrow \frac{300}{25} = T \Rightarrow\ T = 12\text{ yr}$$

40. *(d)* Let the person invest amount x and y into two different rates of interest.

$$\therefore \frac{x\times 12\times 1}{100}+\frac{y\times 10\times 1}{100}=130$$

$$\Rightarrow 12x+10y=13000 \quad \ldots(i)$$

and $\frac{y\times 12\times 1}{100}+\frac{x\times 10\times 1}{100}=134$

$$\Rightarrow 12y+10x=13400 \quad \ldots(ii)$$

On solving Eqs. (i) and (ii), we get

x = ₹ 500 and y = ₹ 700

41. *(a)* Given, P = ₹ 24000, R = 5% per annum and n = 3 yr

$$\therefore A=P\left(1-\frac{R}{100}\right)^n$$

$$=24000\left(1-\frac{5}{100}\right)^3$$

$$=24000\left(\frac{95}{100}\right)^3$$

$$=₹\ 20577$$

42. *(d)* I. Given, R = 4%, n = 2 yr and A = ₹ 169, P = ?

$$A=P\left(1+\frac{R}{100}\right)^n$$

$$169=P\left(1+\frac{4}{100}\right)^2$$

$$\Rightarrow 169=P\left(\frac{26}{25}\right)^2$$

$$P=\frac{169\times 25\times 25}{26\times 26}=₹\ 156.25$$

II. Given, SI = ₹ 120, n = 2 yr and CI = ₹ 129

$$SI=\frac{P\times R\times T}{100}$$

$$120=\frac{P\times R\times 2}{100}\Rightarrow PR=₹\ 6000$$

$$\therefore P=\frac{6000}{R} \quad \ldots(i)$$

$$CI=P\left[\left(1+\frac{R}{100}\right)^n-1\right]$$

$$129=P\left[\left(1+\frac{R}{100}\right)^2-1\right]$$

$$1290000=P\left[(100+R)^2-100^2\right]$$

$$=P\left[R^2+R\times 200\right]$$

$$=\frac{6000}{R}\left[R^2+R\times 200\right]$$

[from eqn (i)]

1290000 = 6000R + 1200000

$$R=\frac{90000}{6000}=15\%$$

Hence, both statement are correct.

43. *(b)* Give, CI = ₹ 832, SI = ₹ 800, n = 2 yr

$$CI=P\left\{\left(1+\frac{R}{100}\right)^2-1\right\}$$

$$\therefore 832=P\left\{\left(1+\frac{R}{100}\right)^2-1\right\} \ldots(i)$$

Also, $SI=\frac{P\times R\times T}{100}$

$$\Rightarrow 800=\frac{P\times R\times T}{100}$$

$$\Rightarrow P=\frac{40000}{R}$$

From Eq. (i),

$$832=\frac{40000}{R}\left(\frac{R^2}{10000}+\frac{2R}{100}\right)$$

$$\Rightarrow 832=\frac{40000}{100}\left(\frac{R}{100}+2\right)$$

$$\Rightarrow 832=4R+800$$

$$\therefore R=\frac{32}{4}=8\%$$

44. *(a)* Given, selling price of article = ₹ 247.50 and gain = $\frac{25}{2}$ %

$\therefore$ Cost price

$$= ₹ \left\{ \frac{100}{\left(100 + \frac{25}{2}\right)} \times 247.50 \right\}$$

$$= ₹ \frac{100 \times 2 \times 247.50}{225} = ₹ 220$$

45. *(a)* To calculate overall percentage is:

$$\text{Overall \%} = \frac{Gain\% \times Loss\%}{100}$$

Gain% = 20%

Loss% = 20%

Substituting,

$$\text{Overall Loss\%} = \frac{20 \times 20}{100} = 4\%$$

Thus the man incurs a loss of 4%.

46. *(c)* Let marked price be ₹ x.

Selling price after 5% discount

$$= x - \frac{5}{100}x = \frac{19}{20}x$$

$$\text{Profit} = SP - CP = \frac{19}{20}x - 380$$

$$\text{Profit \%} = \frac{\frac{19}{20}x - 380}{380} \times 100$$

$$25 = \frac{\frac{19}{20}x - 380}{380} \times 100$$

$$x = 475 \times \frac{20}{19} = ₹ 500$$

47. *(b)* Given, cost of article = ₹ 200

Selling price of article = 95% of (90% of 200)

$$= \frac{95}{100} \times \frac{90}{100} \times 200 = ₹ 171$$

48. *(c)* Given, cost price of 1L = ₹ 8.50

$\therefore$ Total CP of milk = 28 × 8.50 = ₹ 238

$\therefore$ Profit = 12.5% of 238

$$= \frac{12.5}{100} \times 238 = ₹ 29.75$$

Let he added x L of water.

$\therefore$ Profit = $x \times 8.5 \Rightarrow 29.75 = x \times 8.5$

$\therefore$ $x = 3.5$L

49. *(c)* Let the cost price be ₹ x.

$$\text{Marked price} = \frac{x \times 100}{100} = ₹ \frac{11x}{10}$$

$$\therefore \quad SP = \frac{11x}{10} \times \frac{90}{100} = \frac{99x}{100}$$

$\therefore$ Required gain/loss per cent

$$= \frac{\frac{99x}{100} - x}{x} \times 100 = -1\%$$

50. *(a)* Given, $x : y = 1 : 3$, $y : z = 5 : k$, $z : t = 2 : 5$ and $t : x = 3 : 4$

$$\frac{x}{y} \times \frac{y}{z} \times \frac{z}{t} \times \frac{t}{x} = 1$$

$$\Rightarrow \frac{1}{3} \times \frac{5}{k} \times \frac{2}{5} \times \frac{3}{4} = 1$$

$$\Rightarrow k = \frac{1}{2}$$

51. *(b)* $Speed \propto \frac{1}{Time}$

$\therefore$ Required ratio

$$= \frac{1}{4} : \frac{1}{3} : \frac{1}{2} = 3 : 4 : 6$$

52. *(a)* Let the number of coins of ₹ 1, 50 paise and 10 paise be 3x, 8x and 10x, respectively According to the question,

$$\frac{3x}{1} + \frac{8x}{2} + \frac{10x}{10} = 112$$

$$\Rightarrow 3x + 4x + x = 112$$

$$\Rightarrow \quad x = \frac{112}{8} = 14$$

$\therefore$ Number of 50 paise coins
$= 14 \times 8 = 112$

53. *(b)* As $x \propto \frac{1}{y^2} \Rightarrow x = \frac{k}{y^2}$ …(i)
If $x = 1$
and $y = 6$
$1 = \frac{k}{6^2} \Rightarrow k = 36$
On putting the value of k in Eq. (i), we get
$x = \frac{36}{y^2}$ …(ii)
I. On putting $y = 3$ in Eq. (ii), $x = \frac{36}{9}$
$x = 4$
II. On putting $y = 6$ in Eq. (ii), we get
$x = \frac{36}{36} = 1$
$x = 1$
Both statements I and II are correct.

54. *(c)* Fresh grapes contain 10% pulp.
$\therefore$ 20 kg fresh grapes contain 2 kg pulp.
Dry grapes contain 80% pulp.
2 kg pulp would contain
$\frac{2}{0.8} = \frac{20}{8} = 2.5$ kg dry grapes

55. *(d)* $\left[\log_{10}\left(5 \log_{10} 100\right)\right]^2$
$= \left[\log_{10}\left(5 \log_{10} 10^2\right)\right]^2$
$= \left[\log_{10}\left(10 \log_{10} 10\right)\right]^2$
$= \left[\log_{10} 10\right]^2 \quad \left[\because \log_{10} 10 = 1\right]$
$= 1^2 = 1$

56. *(d)* The characteristic in $\log 6.7482 \times 10^{-5}$ is -5.

57. *(b)* $10^{\log_{10} m + 2\log_{10} n + 3\log_{10} p}$
$= 10^{\log_{10} m + \log_{10} n^2 + \log_{10} p^3}$
$\Rightarrow 10^{\log_{10} mn^2 p^3} = mn^2 p^3$
$\left[\because a^{\log_a p} = p\right]$

58. *(a)* $8 - 4x - 2x^3 + x^4 = 4(2 - x) - x^3(2 - x)$
$= (2 - x)(4 - x^3)$

59. *(d)* $\because (a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
$\therefore (6)^2 = 26 + 2(ab + bc + ca)$
$\Rightarrow 2(ab + bc + ca) = 10$
$\Rightarrow ab + bc + ca = 5$

60. *(a)* $x^4 + 4y^4$
$= x^4 + 4y^4 + 4x^2y^2 - 4x^2y^2$
$= \left(x^2 + 2y^2\right)^2 - (2xy)^2$
$= \left(x^2 + 2y^2 - 2xy\right)$
$\left(x^2 + 2y^2 + 2xy\right)$
From above it is clear that $x^4 + 4y^4$ is divisible by $x^2 + 2y^2 + 2xy$

61. *(b)* $x^{3/2} - xy^{1/2} + x^{1/2}y - y^{3/2}$
$= x\left(x^{1/2} - y^{1/2}\right) + y\left(x^{1/2} - y^{1/2}\right)$
$= \left(x^{1/2} - y^{1/2}\right)(x + y)$
$\Rightarrow$ Quotient
$= \frac{\left(x^{1/2} - y^{1/2}\right)(x + y)}{\left(x^{1/2} - y^{1/2}\right)} = x + y$

62. *(c)* Given, $f(x)$ and $g(x)$ vanish at $x = 1/2$
So, $(2x - 1)$ is a factor of $f(x)$ and $g(x)$ both.
Hence, HCF of $f(x)$ and $g(x) = 2x - 1$

63. *(c)* We know that, $(x + y)$ and $(x - y)$ are the factors of $(x^{10} - y^{10})$.

64. *(b)* $4y^4x - 9y^2x^3 = y^2x\left(4y^2 - 9x^2\right)$

$= y^2x(2y-3x)(2y+3x)$

$4y^2x^2+6yx^3 = 2yx^2(2y+3x)$

$\therefore$ Required HCF $= xy(2y+3x)$

65. *(b)* Since, HCF of x^2+x-12 and $2x^2-kx-9$ is $(x-k)$, then $(x-k)$ will be the factor of $2x^2-kx-9$.

$\therefore 2x^2-kx-9=0$

$\Rightarrow k^2-9=0$

$\Rightarrow k=\pm 3$

and factor of $2x^2-kx-9$ are $(x+4)(x-3)$.

Hence, value of k is 3.

66. *(a)* Here, $\dfrac{x^3+3x^2-1}{x^2+\sqrt{x-1}}$ is not rational expression, since the denominator is not a polynomial.

67. *(d)* Given, $x+y+z=0 \Rightarrow x+y=-z$.

On squaring both sides, we get

$x^2+y^2+2xy=z^2$

Similarly, $y^2+z^2-x^2=-2yz$ and $z^2+x^2-y^2=-2zx$

$$\therefore \frac{1}{x^2+y^2-z^2}+\frac{1}{y^2+z^2-x^2}+\frac{1}{z^2+x^2-y^2}$$

$$=\frac{1}{-2xy}+\frac{1}{-2yz}+\frac{1}{-2zx}$$

$$=\frac{1}{2}\left(\frac{z+x+y}{xyz}\right)=0$$

68. *(c)* Given equations are,

$\alpha x+3y=\alpha-3$ and $12x+\alpha y=\alpha$

Here, $a_1=\alpha,\ b_1=3,\ c_1=\alpha-3$

$a_2=12,\ b_2=\alpha,\ c_2=\alpha$

Since, system has unique solution,

So, $\dfrac{a_1}{a_2}\neq\dfrac{b_1}{b_2} \Rightarrow \dfrac{\alpha}{12}\neq\dfrac{3}{\alpha}$

$\Rightarrow \alpha^2\neq 36 \Rightarrow \alpha\neq\pm 6$

69. *(a)* Given, $ax^2-2\sqrt{5}x+4=0$ has equal roots.

$\therefore$ Discriminant

$$=\left(-2\sqrt{5}\right)^2-4(a)4=0 \quad [\because D=B^2-4AC]$$

$\Rightarrow 20-16a=0 \Rightarrow a=5/4$

70. *(b)* Let roots of equation be α and $\dfrac{1}{\alpha}$.

$\therefore$ Product of roots

$$=\alpha\times\frac{1}{\alpha}=\frac{\text{Constant term}}{\text{Coefficient of } x^2}=\frac{r}{p}$$

$\Rightarrow 1=\dfrac{r}{p} \Rightarrow r=p$

71. *(d)* Given, $\alpha+\beta=24$ and $\alpha-\beta=8$

On solving, we get $\alpha=16$ and $\beta=8$

Sum of roots $=\alpha+\beta=24$

and product of roots $= 16\times 8 = 128$

So, required equation is

$$x^2-24x+128=0$$

72. *(b)* Given, $2x^2-3x-4=0$

For getting a reciprocal roots, we replace x by $\dfrac{1}{x}$, we get

$$2\left(\frac{1}{x}\right)^2-3\left(\frac{1}{x}\right)-4=0$$

$$\Rightarrow \frac{2}{x^2}-\frac{3}{x}-4=0$$

$\Rightarrow \quad -4x^2-3x+2=0$

$\Rightarrow \quad 4x^2+3x-2=0$

73. *(b)* Here, $\alpha+\beta=b/a$ and $\alpha\beta=b/a$

So, $\sqrt{\frac{\alpha}{\beta}}+\sqrt{\frac{\beta}{\alpha}}=\frac{\alpha+\beta}{\sqrt{\alpha\beta}}=\frac{b/a}{\sqrt{b/a}}$

$=\sqrt{\frac{b}{a}}$

74. *(c)* Given, $\log_{10}\left(x^2-6x+45\right)=2$

$\Rightarrow \left(x^2-6x+45\right)=10^2=100$

$\Rightarrow \quad x^2-6x-55=0$

$\Rightarrow x^2-11x+5x-55=0$

$\Rightarrow x(x-11)+5(x-11)=0$

$\Rightarrow \quad (x+5)(x-11)=0$

$\therefore \quad x=11, \ -5$

75. *(b)* $\frac{1}{2}\left(\frac{3}{5}x+4\right)\geq\frac{1}{3}(x-6)$

$\Rightarrow \quad \frac{3}{10}x+2\geq\frac{1}{3}x-2$

$\Rightarrow \quad 9x+60\geq 10x-60$

$\Rightarrow \quad -x\geq -120$

[multiplying both sides by -1]

$\Rightarrow \quad x\leq 120$

Thus, all real numbers x which are less than or equal to 120 satisfies the inequality.

76. *(a)* $A\cup B=\{5,6,7\}\cup\{7,8,9\}$

$=\{5,6,7,8,9\}$

77. *(a)* Clearly, $A\cap(A\cup B)=A$

78. *(c)* Equivalent sets have same cardinal numbers. Here, cardinal numbers of I, III, IV sets are same.

79. *(d)* $\{x : x$ is an integer and less than $1000\}=[\ldots,998,999]$

i.e. $x\in(-\infty, 1000)$ is an infinite set.

80. *(c)* $n(A\cup B)=n(A)+n(B)-n(A\cap B)$

$\therefore \ n(A\cap B)=n(A)+n(B)-n(A\cup B)$

$=17+23-38=2$

81. *(b)* $\cos 15^\circ-\sin 15^\circ=\cos 15^\circ$

$-\sin(90^\circ-75^\circ)$

$=\cos 15^\circ-\cos 75^\circ$

$=2\sin\frac{15^\circ+75^\circ}{2}.\sin\frac{75^\circ-15^\circ}{2}$

$\left(\because \cos C-\cos D=2\sin\frac{C+D}{2}\sin\frac{D-C}{2}\right)$

$=2\sin 45^\circ.\sin 30^\circ$

$=2\frac{1}{\sqrt{2}}.\frac{1}{2}=\frac{1}{\sqrt{2}}$

82. *(c)* $\because$ Given, $\tan\theta+\frac{1}{\tan\theta}=2$

On squaring both side, we get

$\left(\tan\theta+\frac{1}{\tan\theta}\right)^2=(2)^2$

$\Rightarrow \tan^2\theta+\frac{1}{\tan^2\theta}+2=4$

$\Rightarrow \tan^2\theta+\frac{1}{\tan^2\theta}=2$

83. *(c)* Given, $x=y\cos\frac{2\pi}{3}=z\cos\frac{4\pi}{3}$

$\Rightarrow x=\frac{-y}{2}=\frac{-z}{2}=k$ (let)

$x=k, \ y=-2k, \ z=-2k$

$\Rightarrow xy+yz+zx=k(-2k)+(-2k)(-2k)$

$+(-2k)k$

$=-2k^2+4k^2-2k^2=0$

84. *(a)* $\because A+B+C+D=360°$

$\therefore A+B=360°-(C+D)$

$\therefore \sin(A+B)=\sin[360°-(C+D)]$

$=-\sin(C+D)$

$\therefore \sin(A+B)+\sin(C+D)=0$

Also,

$\cos(A+B)=\cos[360°-(C+D)]$

$\cos(A+B)=\cos(C+D)$

Hence, only statement I is correct.

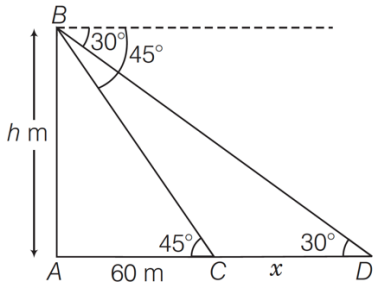
85. *(c)* $\cos 4x=1-2\sin^2 2x$

$=1-2(2\sin x\cos x)^2$

$=1-2(4\sin^2 x\cos^2 x)$

$=1-8\sin^2 x\cos^2 x$

86. *(a)* Let height of tower AB be h m and distance between C and D be x m.



In right angled ΔACB, $\tan 45°=\frac{AB}{AC}$

$\Rightarrow 1=\frac{h}{60} \Rightarrow h=60$ m …(i)

Now, in right angled ΔADB,

$\tan 30°=\frac{AB}{AD}=\frac{AB}{AC+CD}$

$=\frac{60}{60+x}$ [from Eq. (i)]

$\Rightarrow \frac{1}{\sqrt{3}}=\frac{60}{60+x}$

$\Rightarrow 60+x=60\sqrt{3}$

$\Rightarrow x=60(\sqrt{3}-1)=60(1.73-1)$

$=60\times 0.73=43.8$ m

Now, given time = 5 s

We know that, Speed $=\frac{\text{Distance}}{\text{Time}}$

$\therefore$ Speed of boat

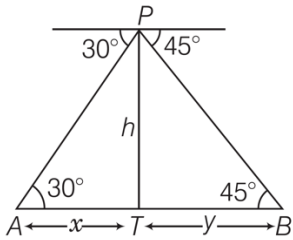
$=\frac{43.8}{5}\times\frac{18}{5}=\frac{788.4}{25}$

$=31.5$ km/h

87. *(a)* In right angled

ΔPBT, $\tan 45°=\frac{h}{y}$

$\Rightarrow 1=\frac{h}{y}$



$\therefore y=h$ …(i)

and in right angled ΔPTA,

$\tan 30°=\frac{h}{x} \Rightarrow x=\sqrt{3}h$ …(ii)

$\therefore$ Required distance, $AB=x+y$

$x+y=\sqrt{3}h+h=h(\sqrt{3}+1)$ m

88. *(b)* Given, angle = $\frac{3}{5}$ of right angle

$=\frac{3}{5}\times 90°=3\times 18°=54°$

Supplement of $54°=(180°-54°)$

= An angle of measure $126°$

89. *(a)* Only statements I and II are true.

90. *(b)* Let the angles of a triangle be $2x$, $3x$, $4x$, then $2x + 3x + 4x = 180°$
[by angle sum property of a triangle]

$9x = 180° \Rightarrow x = 20°$

So, angles are $2x = 40°$,

$3x = 60°$, $4x = 80°$.

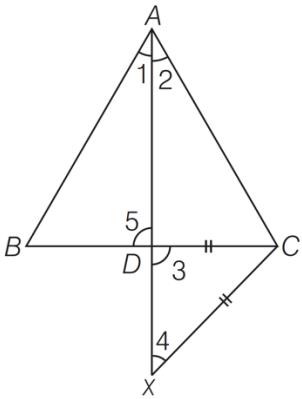
91. *(a)* In ΔDCX, $CD = CX$ [given]

$\angle 3 = \angle 4$
[opposite angle of same sides]

But $\angle 3 = \angle 5$, So, $\angle 4 = \angle 5$

In ΔABD and ΔACX,

$\angle 1 = \angle 2$ [given]

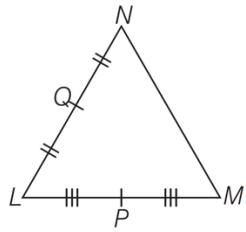


$\angle 4 = \angle 5$

$\therefore \quad \Delta ABD \sim \Delta ACX$
[by AA similarity]

92. *(c)* I. It is true that the three medians of a triangle divide it into six triangles of equal area.
II. It is also true that, the perimeter of a triangle is greater than the sum of the lengths of its three medians.
Hence, I and II are correct.

93. *(b)* Given,



I. $PQ^2 = MP^2 + NQ^2$

$\Rightarrow \quad PQ^2 = LP^2 + LQ^2$

[$\because LP = MP$ and $NQ = LQ$]

$\Rightarrow \quad \angle QLP = 90°$

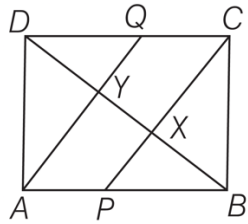
It means, ΔNLM is a right-angled triangle.

II. It also true that if in a ΔABC. $AB^2 > BC^2 + CA^2$, then $\angle ACB$ is obtuse.

Hence, both statements are individually true but statement II is not the correct explanation of statement I.

94. *(c)* As $ABCD$ is a ||gm

$\therefore$ $AB \| DC$ and $AB = DC$



and $\frac{1}{2}AB = \frac{1}{2}DC$

$\Rightarrow AP = QC$

$\therefore$ $APCQ$ is a ||gm

$\Rightarrow \quad AQ \| PC$

In ΔBAY, $XP \| AY$

and P is the mid-point of AB

$\therefore \quad BX = YX$

Similarly, in ΔDXC, $DY = YX$

$\therefore\ BX = XY = DY$

95. *(a)* In a rhombus $ABCD$, if AC and BD are two diagonals then

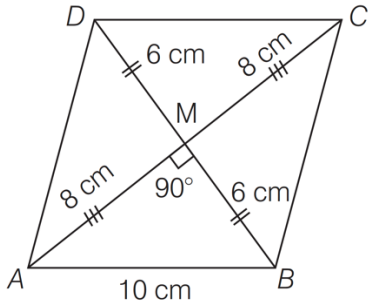
$$AB^2 + BC^2 + CD^2 + AD^2$$

$$= AC^2 + BD^2$$

$$\Rightarrow (10)^2 + (10)^2 + (10)^2 + (10)^2$$

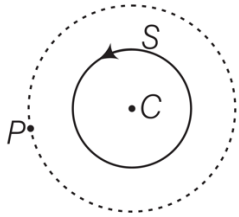
$$= (16)^2 + (12)^2$$

$$\Rightarrow 400 = 400$$



Hence, both I and II are true but III is false.

96. *(b)* If S is a circle with centre C and P be a movable point outside S, then the locus of P such that the tangent from P to S are of constant length is the circle through P with centre at C.



97. *(b)* Circumference of circle

$$= 2\pi r = 2 \times \frac{22}{7} \times 42 = 264 \text{ cm}$$

$\therefore$ Length of wire = 264 cm

Wire is bent into a square.

$\therefore$ Perimeter of square = 264 cm

$\Rightarrow 4 \times$ Sides of square = 264

$\therefore$ Side of square $= \frac{264}{4} = 66$ cm

98. *(b)* Let the radius of big drop and small drop be R and r respectively.

By given condition,

$27 \times$ Volume of smaller drops = Volume of bigger drop

$$\therefore\ 27 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$$

$$\Rightarrow 27 \times (0.2)^3 = R^3 \left[\because r = 0.2 \text{ cm}\right]$$

$$\Rightarrow (3 \times 0.2)^3 = R^3 \Rightarrow 0.6 \text{ cm}$$

99. *(c)* Lower class limits are obtained by subtracting 0.5 from the lower limit, so clearly 9.5, 19.5, 29.5 and 39.5 are the actual lower class limits.

100. *(a)* Required class boundary = Lower class boundary of lowest class + Width of class $\times$ Number of class

$= 5.1 + 2.5 \times 10 = 30.1$