HINTS & SOLUTION

1. *(b)* If $x = 2$, then, x, $x + 1$ and $x + 3$ are all prime numbers.

2. *(c)*
$$\sqrt{[0.04\times 0.4\times x]} = 0.4\times 0.04\times \sqrt{y}$$
$$\therefore \frac{\sqrt{x}}{\sqrt{y}} = \frac{0.4\times 0.04}{\sqrt{0.04\times 0.4}}$$
(squaring both sides)
$$\frac{x}{y} = \frac{0.4\times 0.4\times 0.04\times 0.04}{0.04\times 0.4} = 0.016$$

3. *(d)* A number is divisible by 25 when its last 2 digits are either zero or divisible by 25.

4. *(c)* I. The product of any three consecutive integers is divisible by 6.
II. Here, $3k = \{\ldots -6, -3, 0, 3, 6, \ldots\}$
$3k + 1 = \{\ldots -5, -2, 1, 4, 7, \ldots\}$
and $3k + 2 = \{\ldots -4, -1, 2, 5, 8, \ldots\}$
$\therefore \{3k, 3k + 1, 3k + 2\}$
$= \{\ldots -6, -5, -4, -3, -2, -1$
$0, 1, 2, 3, 4, 5, 6 \ldots\}$
Hence, it is true.

5. *(c)* a, $2a + 2$, $3a + 3$ are in GP.
$$\Rightarrow (2a+2)^2 = (3a+3)a$$
$$\Rightarrow 4a^2+4+8a = 3a^2+3a$$
$$\Rightarrow a^2+5a+4 = 0$$
$$\Rightarrow a = -1, -4$$
Now, $a = -1$ does not satisfy the given series.
$\therefore -4, -6, -9$ are in G.P.
$$\therefore t_4 = -4\left(\frac{3}{2}\right)^3 = -13.5$$

6. *(a)* Since, $2b = a + c$. …(i)
and $2\tan^{-1}b = \tan^{-1}a + \tan^{-1}c$
$$\Rightarrow \frac{2b}{1-b^2} = \frac{a+c}{1-ac}$$
$\Rightarrow b^2 = ac$ [from Eq. (i)]
$$\Rightarrow 4b^2 = 4ac$$
$\Rightarrow (a+c)^2 - 4ac = 0$ [from Eq. (i)]
$$\Rightarrow (a-c)^2 = 0$$
$$\Rightarrow a = c = b$$

7. *(a)* Here, LCM of 5, 6, 8 and 12 is 360, so the bells will toll after 360 s. So, in an hour they will toll together
$$= \frac{60\times 60}{360} = 10 \text{ times}$$

8. *(b)* Let x should be subtracted
$$(0.527\times 2.013) - x > 1$$
$$1.060851 - x > 1$$
$$x = 0.060851$$

9. *(b)* Here, $10^k = \dfrac{0.0003245}{3.245}$
$$= \frac{3.245\times 10^{-4}}{3.245}$$
$$10^k = 10^{-4}$$
So, $k = -4$

10. *(c)* In $\dfrac{1}{22}$ and $\dfrac{2}{15}$, 22 and 15 are not in the form of $2^m\times 5^n$ but in $\dfrac{1}{16}$.
16 in the form of $2^4\times 5^0$. So, $\dfrac{1}{16}$ can be written as a terminating decimal.
Hence, statements I and III are correct.

11. *(b)* If m and n are natural numbers, then $\sqrt[m]{n}$ is irrational unless n is mth power of an integer.

12. *(c)* Total rupees collected = ₹ 32.49

$= 32.49 \times 100$ paise $= 3249$ paise

$\therefore$ Number of members in the group

$= \sqrt{3249} = 57$

13. *(c)* After Ist hit, height of the ball

$= \frac{1}{2}(64)$

After IInd hit, height of the ball

$= \left(\frac{1}{2}\right)^2 (64)$

After 16th hit, height of the ball

$= \left(\frac{1}{2}\right)^{16} (64)$

$= \frac{1}{2^{16}}\left(2^6\right) = 2^{-10}$ m

14. *(d)* Distance travelled by thief car in one hour = 80 km

Distance travelled in one hour by police car = 100 km

So, police travels extra 20 km in 1 h.

So, to overtake thief, police car has to travel 5 km extra.

$\therefore$ Time $= \frac{5}{20} = \frac{1}{4}$ h $= \frac{60}{4}$ min

$= 15$ min

15. *(c)* Here, $x = 3$ m/s, $y = 4$ m/s and $t = 25$ min

$\therefore$ Required time $= \frac{x \times t}{y - x}$

$= \frac{3 \times 25}{4 - 3} = 75$ min

16. *(b)* Relative speed when trains are in opposite direction = $V_1 + V_2$

= 75 + 50 125 km/h

$= \frac{125 \times 5}{18}$ m/s

and total distance covered

= (100 + 150) = 250 m

$\therefore$ Time taken to cross each other

$= \frac{\text{Total covered distance}}{\text{Relative speed}}$

$= \frac{250 \times 18}{125 \times 5} = 7.2$ s

17. *(d)* Using the formula

$$\frac{M_1 D_1}{W_1} = \frac{M_2 D_2}{W_2}$$

18. *(a)* Since, X can do $\frac{3}{4}$ of work in 12 days.

So, X can do 1 work in days in $\frac{12 \times 4}{3}$ days.

$\therefore X$ can do $\frac{1}{2}$ work in $\frac{12 \times 4 \times 1}{3 \times 2} = 8$ days

19. *(c)* Here, $x = 12$, $y = 16$ and $t = 9$

Two pipes A and B can fill a tank in x h and y h, respectively. If both the pipes *ax* opened simultaneously, then the time after which B should be closed, so that the tank is filled in t h $= \left[y(1 - t/x)\right]^{n}$

Required time after which B should be closed $= y\left(1 - \frac{t}{x}\right) = 16\left(1 - \frac{9}{12}\right)$

$= 16 \times \frac{3}{12} = 4$ min

20. *(a)* Percentage of oxygen in water

$= 100 - 14\frac{2}{7} = 100 - \frac{100}{7} = 85\frac{5}{7}\%$

Percentage of oxygen in 350 g of water

$= 85\frac{5}{7}\%$ of 350

$= \frac{600}{7} \times \frac{350}{100} = 300$ g

21. *(b)* Let income of man be ₹100.

Then, his expenditure = ₹ 75

and savings = ₹ 25

New income = ₹ (100 + 20) = ₹ 120

New expenditure = ₹ (75 + 7.5) = ₹ 82.50

New saving = ₹ (120 − 82.5) = ₹ 37.50

Saving difference = 37.50 − 25.0 = 12.5

∴ Percentage increase saving

$$= \frac{12.5}{25} \times 100 = 50\%$$

22. *(b)* Total borrowed money = ₹ 40000 and rate of interest = 8%

The interest for 2 yr

$= \frac{40000 \times 8 \times 2}{100} = ₹\ 6400$

Let he paid ₹ x at the end of second year.

∴ Interest will be calculated on

₹ $(40000 – x + 6400)$

Interest for 3 yr $= \frac{(46400 - x) \times 3 \times 8}{100}$

$= ₹\ \frac{6}{25}(46400 - x)$

$\therefore \frac{6}{25}(46400 - x) + 46400 - x$

$= 35960$

$\Rightarrow \quad x = \frac{21576 \times 25}{31} = ₹\ 17400$

23. *(c)* Let the sum be ₹ 100.

Then, amount = (Sum + SI)

$= \left(100 + \frac{100 \times 8 \times 5}{100}\right)$

$= (100 + 8 \times 5)$

So, when the amount is $(100 + 8 \times 5)$, then sum = 100

When the amount is ₹ 840, then sum

$= ₹\ \left(\frac{100 \times 840}{100 + 8 \times 5}\right)$

24. *(c)* Let the principal be ₹ x.

∴ Rate = ₹ R, Amount = ₹ $2x$ and Time = n yr

$\because A = P\left(1 + \frac{R}{100}\right)^n$

$\therefore x\left(1 + \frac{R}{100}\right)^n = 2x$

$\Rightarrow \left(1 + \frac{R}{100}\right)^n = 2 \quad \ldots(i)$

Let it becomes four fold in N yr.

Then,

$$x\left(1 + \frac{R}{100}\right)^N = 4x$$

$\Rightarrow \left(1 + \frac{R}{100}\right)^N = 4$

$\Rightarrow 2^2 = \left(1 + \frac{R}{100}\right)^N$

$\Rightarrow \left(1 + \frac{R}{100}\right)^{2n} = \left(1 + \frac{R}{100}\right)^N$

[from Eq. (i)]

$\therefore N = 2n$ yr

25. *(a)* Let the selling price be ₹ 100.

∴ Cost price $= \frac{SP \times 100}{100 + 10}$

$\frac{100 \times 100}{110} = ₹\ \frac{1000}{11}$

Now, if SP is ₹ 200.

$\therefore$ Gain

$$= ₹\left(200 - \frac{1000}{11}\right) = ₹\frac{1200}{11}$$

Gain percent

$$= \frac{1200/11}{1000/11} \times 100 = 120\%$$

26. *(d)* Let the numbers be A and B.

$\therefore 0.5A = 0.07B$ [by given condition]

$$\Rightarrow \frac{A}{B} = \frac{0.07}{0.5} = \frac{7}{50} \; i.e.\; 7 : 50$$

27. *(b)* Given, $x : 6 :: 5 : 3$

$$\therefore \frac{x}{6} = \frac{5}{3} \Rightarrow 3x = 30$$

$$\Rightarrow x = \frac{30}{3} = 10$$

28. *(b)* As, $A : (B + C) = 2 : 9$ [given]

Sum of ratios $= 2 + 9 = 11$

So, A's part $= 770 \times \frac{2}{11} = ₹\ 140$

29. *(d)* By given condition,

$$y = lx + \frac{m}{x} \quad \ldots\text{(i)}$$

where, l and m are proportionality constants. when $y = 6$ and $x = 4$,

then $6 = 4l + \frac{m}{4}$

$\Rightarrow 16l + m = 24 \quad \ldots$(ii)

when $y = \frac{10}{3}$ and $x = 3$,

then $\frac{10}{3} = 3l + \frac{m}{3}$

$\Rightarrow 9l + m = 10 \quad \ldots$(iii)

On solving Eqs. (ii) and (iii)

$l = 2$ and $m = -8$

From Eq. (i) $y = 2x - \frac{8}{x}$

30. *(a)* $\log_{10} 5 = \log_{10} \frac{10}{2}$

$= \log_{10} 10 - \log_{10} 2 = 1 - 0.3010$

$= 0.6990$

31. *(d)* Let $\log_3 \left(27 \times \sqrt[4]{9} \times \sqrt[3]{9}\right) = x$

$\Rightarrow 3^x = 27 \times \sqrt[4]{9} \times \sqrt[3]{9}$

$\Rightarrow 3^x = 3^3 \times 3^{2/4} \times 3^{2/3}$

$3^x = 3^{25/6}$

On comparing both sides, we get

$$\Rightarrow x = \frac{25}{6} = 4\frac{1}{6}$$

32. *(b)* This is the solution with explanation for

$\left(\log_3 x\right)\left(\log x^{2x}\right)\left(\log_{2x} y\right) = \log_x x^2$

$$\Rightarrow \frac{\log x}{\log 3} \times \frac{\log 2x}{\log x} \times \frac{\log y}{\log 2x} = \frac{\log x^2}{\log x}$$

$$\left[\because \log_b a = \frac{\log a}{\log b}\right]$$

$$\Rightarrow \frac{\log y}{\log 3} = \frac{2 \log x}{\log x} \left[\because \log a^b = b \log a\right]$$

$\Rightarrow \log y = 2 \log 3 \Rightarrow \log y = \log 3^2$

$\left[\because \log m = \log n \Rightarrow m = n\right]$

$\Rightarrow \log y = \log 9$

$\therefore \quad y = 9$

33. *(b)* $x^2 - 2\sqrt{3}x + 3$

$= x^2 - \sqrt{3}x - \sqrt{3}x + 3$

$= x\left(x - \sqrt{3}\right) - \sqrt{3}\left(x - \sqrt{3}\right)$

$= \left(x - \sqrt{3}\right)\left(x - \sqrt{3}\right) = \left(x - \sqrt{3}\right)^2$

34. *(b)*

$$\because \left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right)$$

$$\left(\sqrt{5}\right)^3 = x^3 + \frac{1}{x^3} + 3\left(\sqrt{5}\right)$$

$$\left[\because x+\frac{1}{x}=\sqrt{5}\right]$$

$$\Rightarrow x^3+\frac{1}{x^3} = 5\sqrt{5} - 3\sqrt{5}$$

$$= 2\sqrt{5}$$

35. *(c)* $f(x) = x^4 - 2x^3 + 3x^2 - ax + b$

Put $x = 1$ as $f(1) = 5$, $f(-1) = 19$

$f(1) = 1^4 - 2(1)^3 + 3(1)^2 - a + b$

$\Rightarrow 1 - 2 + 3 - a + b = 5$

$\Rightarrow \quad -a + b = 3 \quad \ldots$(i)

and

$f(-1) = (-1)^4 - 2(-1)^3 + 3\left(-1^2\right) - a(-1) + b$

$\Rightarrow 1 + 2 + 3 + a + b = 19$

$\Rightarrow a + b = 13 \quad \ldots$(ii)

On solving Eqs. (i) and (ii), we get

$a = 5, b = 8$

36. *(a)* Let $p(x) = x^4 - x^2 - 6$

$= x^4 - 3x^2 + 2x^2 - 6$

$= x^2\left(x^2 - 3\right) + 2\left(x^2 - 3\right)$

$= \left(x^2 + 2\right)\left(x^2 - 3\right)$

$q(x) = x^4 - 4x^2 + 3$

$= x^4 - 3x^2 - x^2 + 3$

$= x^2\left(x^2 - 3\right) - 1\left(x^2 - 3\right)$

$= \left(x^2 - 3\right)\left(x^2 - 1\right)$

HCF of $p(x)$, $q(x) = x^2 - 3$

37. *(b)* Since, $(z - 1)$ is the HCF, so it will divide each one of the given polynomials. So, $z = 1$ will make each one zero.

$\therefore p(1)^2 - q(1 + 1) = 0$

$\Rightarrow p = 2q$

38. *(b)* Since, $(x + k)$ is the HCF, it will divide both the polynomials without leaving any remainder, thus $x = -k$ will make both of them zero.

$\therefore k^2 - pk + q = k^2 - ak + b$

or $-ak + b = -pk + q$

$\Rightarrow ak - pk = b - q$

$$\therefore \qquad k = \frac{b-q}{a-p}$$

39. *(c)* We know that,

$$\frac{(x-1)(x-2)\left(x^2-9x+14\right)}{(x-7)\left(x^2-3x+2\right)}$$

$$= \frac{(x-1)(x-2)\left(x^2-7x-2x+14\right)}{(x-7)\left(x^2-2x-x+2\right)}$$

$$= \frac{(x-1)(x-2)(x-7)(x-2)}{(x-7)(x-2)(x-1)}$$

$= x - 2$

40. *(a)* Given, $x + y + z = 0$

$\Rightarrow x + y = -z,\ y + z = -x$ and $z + x = -y$

$$\therefore \frac{xyz}{(x+y)(y+z)(z+x)}$$

$$= \frac{xyz}{(-z)(-x)(-y)}$$

$$= \frac{xyz}{-xyz} = -1$$

41. *(c)* Here, $\dfrac{a_1}{a_2} = \dfrac{6}{9} = \dfrac{2}{3}$,

$\dfrac{b_1}{b_2} = \dfrac{5}{15/2} = \dfrac{2}{3}$ and $\dfrac{c_1}{c_2} = \dfrac{11}{21}$

$\Rightarrow \dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$

So, the system has no solution.

42. *(a)* I. $2x + 3y = 425$, $3x + 2y = 350$
Solving both the equations, we get
$x = 40$ and $y = 115$.
II. When $k = -2$

$$\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{2}{6} = \frac{1}{3}, \frac{c_1}{c_2} = \frac{-2}{5}$$

$$\because \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Hence, the given equations have no solution.
III. (2, 5) is not a solution of $2x + 5y = 13$
as $2(2) + 5(5) = 4 + 25 = 29 \neq 13$
Hence, only I is correct.

43. *(b)* Let the fare of ticket from station P to station Q is ₹ x and that from station P to station R is ₹ y.
By given condition, $x + y = 42$
and $5x + 10y = 350$
On solving Eqs. (i) and (ii), we get
$x = 14$ and $y = 28$
Hence, fare from station P to station Q is ₹ 14.

44. *(d)* Given, $\sqrt{2}x - \sqrt{3}y = 0$
and $\sqrt{7}x + \sqrt{2}y = 0$
Here, $a_1 = \sqrt{2}$, $b_1 = -\sqrt{3}$
$a_2 = \sqrt{7}$ and $b_2 = \sqrt{2}$
As the equations are homogeneous equations and also
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$. So, equation has one solution,
$\therefore x = y = 0 \Rightarrow x + y = 0$
Hence, the value of $x + y$ is zero.

45. *(c)* Let number of 10 paise coins be x and number of 50 paise coins be y.
According to the questions,
Then, $x + y = 17$ …(i)
and $10x + 50y = 450$ …(ii)
From Eq. (ii), $x + 5y = 45$ …(iii)
On subtracting Eq. (i) from Eq. (iii), we get
$(x + 5y) - (x + y) = 45 - 17$

$$\Rightarrow 4y = 28 \Rightarrow y = \frac{28}{4} = 7$$

Number of 10 paise coins
$= x = 17 - y = 17 - 7 = 10$

46. *(d)* a and c have the same sign opposite to that of b.

47. *(b)* Here, $2x + 1 \geqslant 7 \Rightarrow 2x \geqslant 7 - 1$
$\Rightarrow \quad 2x \geqslant 6 \Rightarrow x \geqslant 3$

48. *(c)* I. Every quadratic equation has two roots, which may or may not be real.
II. $x^2 - 4x + 2 = 0$ has integral coefficients but does not have integral roots.
III. Since, discriminant $= b^2 - 4ac$
If a and c have opposite sign, then
$b^2 - 4ac \geqslant 0$
$\therefore$ The quadratic equation has real roots

49. *(a)* Given, $\sqrt{\frac{x}{x+3}} - \sqrt{\frac{x+3}{x}} = -\frac{3}{2}$

Let, $y = \sqrt{\frac{x}{x+3}}$, then $\sqrt{\frac{x+3}{x}} = \frac{1}{y}$

$$y - \frac{1}{y} = -\frac{3}{2} \Rightarrow 2y^2 - 2 = -3y$$

$\Rightarrow \quad 2y^2 + 3y - 2 = 0$
$\Rightarrow \quad 2y^2 + 4y - y - 2 = 0$
$\Rightarrow \quad 2y(y + 2) - 1(y + 2) = 0$
$\Rightarrow \quad (2y - 1)(y + 2) = 0$
If, $\quad (2y - 1) = 0$

$\Rightarrow y = \frac{1}{2} \Rightarrow y = \sqrt{\frac{x}{x+3}} \Rightarrow \sqrt{\frac{x}{x+3}} =$

On squaring both sides, we get

$\frac{x}{x+3} = \frac{1}{4}$

$\Rightarrow 4x = x + 3 \Rightarrow x = 1$

or $y + 2 = 0 \Rightarrow y = -2$

Since, y cannot be negative.

Hence, $x = 1$ is the required solution.

50. *(a)* I. A is a finite set.

II. As all elements of A are integers, so A is subset of integers.

III. $\{1, 2\}$ is a proper subset of A.

IV. $A \neq \phi$

So, I, II and III are true.

51. *(b)* As we know, sin x is increasing from 0 to 90°.

$\therefore \sin y > \sin x$

52. *(d)* We know that, the value of $\cos\theta$ is decreasing in the interval $0 \leq \theta \leq 90°$

$\therefore \cos 1° > \cos 89° \Rightarrow p > q$

Also, cos 1° is close to 1 and cos 89° is close to 0.

53. *(b)* Given, $\sin 3\theta = \cos(\theta - 2°)$

$\Rightarrow \sin 3\theta = \sin[90° - (\theta - 2°)]$

$\Rightarrow \quad 3\theta = 90° - \theta + 2°$

$\Rightarrow \quad 4\theta = 92° \Rightarrow \theta = \frac{92°}{4} = 23°$

54. *(d)* We know that, in a cyclic quadrilateral sum of opposite angle is 180°.

$\therefore \quad A + C = 180° \qquad \ldots\text{(i)}$

and $\quad B + D = 180° \qquad \ldots\text{(ii)}$

$\therefore \quad \cos A + \cos B + \cos C + \cos D$

$= \cos A + \cos B + \cos(180° - A) + \cos(180° - B)$

$= \cos A + \cos B - \cos A - \cos B = 0$

$[\because \cos(180° - \theta) = -\cos\theta]$

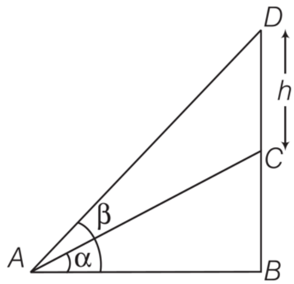
55. *(c)* $\frac{\sin x}{1 + \cos x} = \frac{1 - \cos x}{\sin x}$

The above identity is possible for all values of x except multiples of 180°. Since, for $x = 180°$, $\sin x = 0$ and $1 + \cos x = 0$.

56. *(d)* Let BC be the tower and CD be the flagstaff.

$\therefore \angle BAC = \alpha$ and $\angle BAD = \beta$

In right angled ΔABC,



$\tan\alpha = \frac{BC}{AB} \qquad \ldots\text{(i)}$

and in right angled ΔABD,

$\frac{BD}{AB} = \frac{BC + h}{AB} = \tan\beta \qquad \ldots\text{(ii)}$

On dividing Eq. (ii) by Eq. (i), we get

$\frac{BC + h}{AB} = \frac{\tan\beta}{\tan\alpha}$

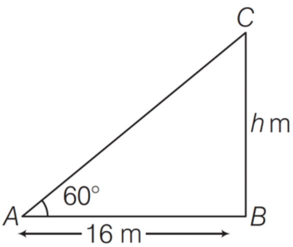
$\Rightarrow (BC + h)\tan\alpha = BC\tan\beta$

$\Rightarrow BC(\tan\beta - \tan\alpha) = h\tan\alpha$

$\therefore \quad BC = \frac{h\tan\alpha}{\tan\beta - \tan\alpha}$

57. *(c)* Let the height of the tree be h m.

$\therefore BC = h$ m and $AB = 16$ m



In right angled ΔABD,

$$\tan 60^\circ = \frac{BC}{AB} = \frac{h}{16}$$

$$\Rightarrow \sqrt{3} = \frac{h}{16}$$

$$\Rightarrow h = 16\sqrt{3} \text{ m}$$

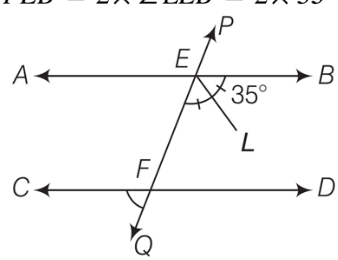
Hence, the height of the tree is $16\sqrt{3}$ m.

58. *(a)* Complementary angle of 12° 25’40”
= 90° −12° 25’ 40”
= 89° 59’ 60” − 12° 25’ 40”
= (89 − 12)° + (59’ – 25’) + (60 − 40)”
= 77° + 34’ + 20” = 77° 34’ 20”

59. *(a)* All the three statements are true.

60. *(a)* Given, $\angle LEB = 35^\circ$

$$\angle FEB = 2\times \angle LEB = 2\times 35^\circ = 70^\circ$$



$$\Rightarrow \angle AEF = \angle AEF + \angle BEF = 180^\circ$$

[straight line]

$$\Rightarrow \angle AEF = 180^\circ - 70^\circ = 110^\circ$$

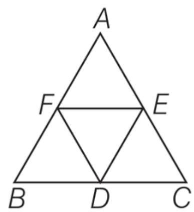
$$\Rightarrow \angle CFQ = \angle AEF = 110^\circ$$

[corresponding angles]

61. *(d)* Let angle be x and its complement be $90^\circ - x$.
According to the question,
$x = (90^\circ - x) + 14$
$\Rightarrow 2x = 104^\circ$

$$\Rightarrow x = \frac{104^\circ}{2} = 52^\circ$$

62. *(d)* As the line joining the mid-points of any two sides or a triangle is parallel to the third side and is half of the third side.



$$\therefore DE = \frac{1}{2}AB = \frac{1}{2}\times 10 = 5 \text{ cm}$$

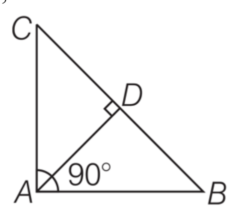
$$EF = \frac{1}{2}BC \Rightarrow BC = 2EF$$

$$= 2\times 3 = 6 \text{ cm}$$

$$\text{DF} = \frac{1}{2}\text{ AC} \Rightarrow \text{AC} = 2\times \text{DF}$$

$$= 2\times 4 = 8 \text{ cm}$$

63. *(d)* Hence, $AD^2 = BD\ .\ DC$



$$\Rightarrow \frac{AD}{BD} = \frac{DC}{AD}$$

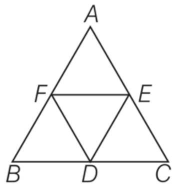
$\therefore\ \Delta ADB \sim \Delta CDA \Rightarrow \angle BAD = \angle ACD$
Now,
$\angle CAB = \angle CAD + \angle BAD$

$$= \angle CAD + \angle ACD = 90^\circ$$

So, ΔABC must be right angled triangle.

Hence, $BC^2 = AC^2 + AB^2$

64. *(a)* The line segments joining the mid-points of the sides of a triangle form four triangles each of which is similar to the original triangle.



Here, $\Delta BDF \sim \Delta ABC$

Also, ΔDEC, ΔDEF

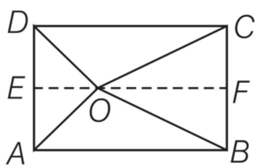
$\Delta AFE \sim \Delta ABC$

65. *(d)* Draw $EF \parallel AB$

In right angled ΔEOA and ΔOCF,

$$OA^2 = OE^2 + AE^2 \quad \ldots(i)$$

and $$OC^2 = OF^2 + CF^2 \quad \ldots(ii)$$



On adding Eqs. (i) and (ii), we get

$\therefore\ OA^2 + OC^2 = OE^2 + AE^2$

$+ OF^2 + CF^2 \quad \ldots(iii)$

In right angled ΔDEO and ΔOBF,

$$OD^2 = OE^2 + DE^2 \quad \ldots(iv)$$

and $$OB^2 = OF^2 + BF^2 \quad \ldots(v)$$

On adding Eqs. (iv) and (v), we get

$\Rightarrow$ Area of || gm $ABCD$

$\Rightarrow OD^2 + OB^2 = OE^2 + OF^2$

$+ DE^2 + BF^2 \quad \ldots(vi)$

As $FB = EA$ and $DE = CF$

$\therefore\quad OD^2 + OB^2 = OE^2$

$+ OF^2 + CF^2 + AE^2 \quad \ldots(vii)$

Here, from Eqs. (iii) and (vii), we get

$$OA^2 + OC^2 = OD^2 + OB^2$$

66. *(c)* Sum of all angles of a pentagon

$= (5-2) \times 180° = 3 \times 180° = 540°$

Let the angle be x , $2x$, $3x$, $5x$ and $9x$.

$\therefore x + 2x + 3x + 5x + 9x = 540°$

$$20x = 540° \quad \Rightarrow \quad x = 27°$$

$\therefore$ Largest angle $= 9x = 9 \times 27°$

$= 243°$

67. *(b)* I. All squares are parallelograms.

II. No parallelogram is a trapezium.

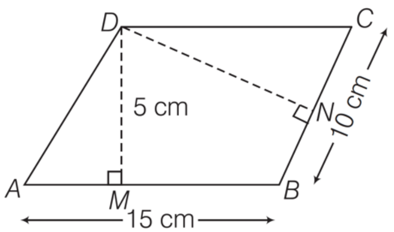
III. All squares are rhombuses and also rectangles.

IV. All rhombuses are parallelograms.

So, statements (III) and (IV) are correct.

68. *(b)* Area of parallelogram

$= \text{Base} \times \text{height} = 15 \times 5 = 75\ \text{cm}^2$



Area of parallelogram = Base $\times$ Height

$= 10 \times DN$

$\therefore\quad 10 \times DN = 75$

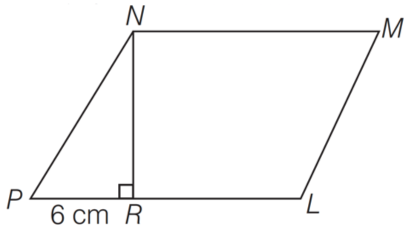
$$\Rightarrow\quad DN = \frac{75}{10} = 7.5\ \text{cm}$$

69. *(b)* Let h be the height of the parallelogram. Then, clearly $h < q$

$\therefore\ R = p \times h < p \times q = S$

$R < S$

70. *(b)* By given condition,



Area of parallelogram *LMNP*

$$= 6 \times \text{ Area of } \Delta\, NPR$$

$$\therefore\ NR \times PL = 6 \times \frac{1}{2} \times NR \times PR$$

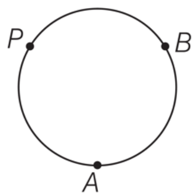
$$\Rightarrow\ PL = 3PR$$

$$\Rightarrow\ PR + RL = 3PR\ [\because\ PL = PR + RL]$$

$$\Rightarrow\ RL = 2PR = 2 \times 6 = 12 \text{ cm}$$

71. *(a)* Let P be the moving point.

Then, $PA^2 + PB^2 =$ constant

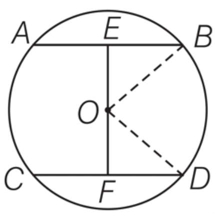


So, the locus of P is a circle.

72. *(d)* Here, $BE = \frac{1}{2} AB = \frac{16}{2}$ cm

$$= 8 \text{ cm}$$

$OB = OD = 17$ cm (radii)



In right angled $\Delta\, OEB$,

$$OE = \sqrt{OB^2 - BE^2}$$

$$= \sqrt{17^2 - 8^2} = \sqrt{289 - 64}$$

$$= \sqrt{225} = 15 \text{ cm}$$

$$\therefore\ OF = EF - OE$$

$$= (23 - 15) = 8 \text{ cm}$$

In right angled $\Delta\, OFD$,

$$FD = \sqrt{OD^2 - OF^2}$$

$$= \sqrt{17^2 - 8^2} = \sqrt{289 - 64}$$

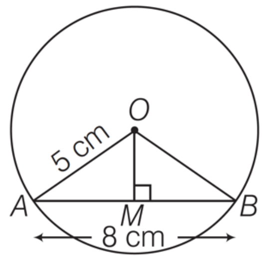
$$= \sqrt{225} = 15 \text{ cm}$$

$$\therefore\ CD = 2FD = 30 \text{ cm}$$

73. *(c)* Given, $OA = 5$ cm,

$$AM = \frac{1}{2} \times AB = \frac{8}{2}$$

$$\Rightarrow AM = 4 \text{ cm}$$



In right angled $\Delta\, OMA$,

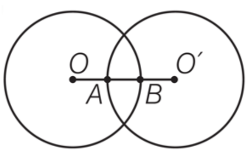
$$OM^2 = OA^2 - AM^2$$

[by pythagoras theorem]

$$= 5^2 - 4^2 = 25 - 16 = 9$$

$$\Rightarrow\ OM = 3 \text{ cm}$$

74. *(d)* $OB = 4$ cm



$$O'A = 2 \text{ cm},\ OO' = 3 \text{ cm}$$

As, $OO' \neq OB + O'A$

So, circle does not touch each other externally.

Also, $OO' \neq OB - O'A$

So, circle does not touch internally, hence they intersect each other at two distinct points.

75. *(b)* Let the length and breadth of rectangular field be $5x$ and $3x$, respectively.

$l = 5x$ and $b = 3x$

Perimeter of rectangular field = $2(l + b)$

$2(5x + 3x) = 240$

$\Rightarrow 8x = 120 \Rightarrow x = 15$

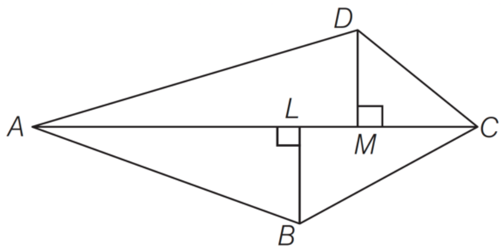
$\therefore l = 5x = 5 \times 15 = 75$m

and $b = 3x = 3 \times 15 = 45$m

Now, area of rectangle = $l \times b = 75 \times 45$

$= 3375 \text{ m}^2$

76. *(d)* Here, AC = 128 m, BL = 22.7 m, DM = 17.3 m



$\therefore$ Area of the field

$$= \frac{1}{2}[AC(BL + DM)]$$

$$= \frac{1}{2} \times 128(22.7 + 17.3)$$

$$= 64 \times 40 = 2560 \text{ m}^2$$

77. *(c)* Area of trapezium = 1/ 2 (Sum of parallel sides)

$\times$ Distance between them

$$= \frac{1}{2}(25 + 15) \times 7 = 140 \text{ cm}^2$$

78. *(c)* We have, $a = l \times w$ …(i)

and $p = 2(l + w)$ … (ii)

Putting the value of l from Eq. (i) in (ii), we get

$wp - 2w^2 = 2a$

Now, $p^2 - 8a = [2(l + w)^2] - 8lw$

$= 4(l^2 + w^2 + 2lw) - 8lw$

$= 4l^2 + 4w^2 + 8lw - 8lw$

$= 4(l^2 + w^2)$

Hence, the statement I and III are correct.

79. *(d)* Angle inscribed by minute hand in 60 min = 360°.

Angle inscribed in 35 min = $\frac{360°}{60} \times 35$

$= 210°$

given, $r = 12$ cm

$\therefore$ Area swept by the minute-hand in 35 min

= Area of sector with $r = 12$ cm and $\theta = 210°$

$$= \frac{22}{7} \times \left(12 \times 12 \times \frac{210}{360}\right) \text{cm}^2$$

$$= 264 \text{ cm}^2$$

80. *(d)* Here, $2\pi rh = 264 \text{ m}^2$

and $\pi r^2 h = 924 \text{ m}^3$ (given)

$$\therefore \frac{\pi r^2 h}{2\pi rh} = \frac{924}{264}$$

$$\Rightarrow \frac{r}{2} = \frac{7}{2} \Rightarrow r = 7 \text{ m}\backslash$$

$\therefore$ Diameter of pillar

$$= r \times 2 = 7 \times 2 = 14 \text{ m}$$

81. *(c)* Volume of sphere = Volume of cylinder

$$\therefore \frac{4}{3}\pi R^3 = \pi \times 3 \times 3 \times 32$$

$$\Rightarrow R^3 = 3 \times 3 \times 3 \times 8$$

$$\Rightarrow R = 6 \text{ cm}$$

82. *(c)* I. Coterminous edges are those who have same boundaries and for a cuboid, these may be considered as length, breadth and height. So, given statement is true
II. The surface area of a cuboid is twice the sum of the products of lengths of its coterminous edges taken two at a time.
Hence, both statements I and II are correct.

83. *(a)* Let the radius of cone and sphere be r and height of cone be h_1
By given condition,
Volume of cone = Volume of sphere

$$\therefore \frac{1}{3}\pi r^2 h_1 = \frac{4}{3}\pi (r)^3$$

$$\Rightarrow \frac{h_1}{2r} = \frac{2}{1}$$

$or,\ 2:1$

84. *(d)* Let $r_1 = k$ and $r_2 = nk$
Since, $V_1 = V_2$

$$\therefore \frac{1}{3}\pi r_1^2 h_1 = \pi r_2^2 h_2$$

$$\Rightarrow \frac{1}{3}\pi k^2 \times h_1 = \pi n^2 k^2 h_2$$

$$\Rightarrow h_1 = 3n^2 h_2$$

85. *(d)* Let side of a cube be 'a' unit.
Then, radius of sphere is $\frac{a}{2}$ unit.

$$\therefore \frac{\text{Volume of cube}}{\text{Volume of sphere}} = \frac{(a)^3}{\frac{4\pi}{3}\left(\frac{a}{2}\right)^3}$$

$$= \frac{6}{\pi}$$

86. *(c)* Frequency polygon can be drawn by joining the mid-points of the respective tops in histogram.

87. *(b)* Median is the middle item of the series arranged in ascending or descending order.

88. *(b)* Given, average score of 50 students = 44
Total score = 44 × 50 = 2200
Correct score of 50 students = (2200 − 73 + 23) = 2150

$$\text{Correct average score} = \frac{2150}{50} = 43$$

89. *(c)* We know that, histogram is an equivalent graphical representation of the frequency distribution and is suitable for continuous random variables where the total frequency of an interval is evenly distributed over the interval. Hence, both the given statements are correct.

90. *(b)* Let the lower limit be x. Then, the upper limit of class interval = $x + 10$
∵ mid-value = 42

$$\therefore \frac{x + (x + 10)}{2} = 42$$

$$\Rightarrow 2x + 10 = 84$$

$$\Rightarrow 2x = 74 \Rightarrow x = 37$$

∴ Lower limit = 37
Upper limit = 37+10 = 47

91. *(a)* As p is prime, so p/a or p/b.

92. *(c)* The total number of three-digit numbers with unit digit 7 and divisible by 11 are 187, 297, 407, 517, 627, 737, 847, 957.
∴ Total numbers = 8

93. *(b)* Apply hit and trial method from the given option. As, here when $a = 11$, $b = 2$, then

$$3\frac{7}{a} \times b\frac{3}{15} = 3\frac{7}{11} \times 2\frac{3}{15}$$

$$= \frac{40}{11} \times \frac{33}{15} = 8$$

94. *(c)* The HCF of (21, 42, 56) is 7.

$\therefore$ The minimum number of rows

$$= \frac{21}{7} + \frac{42}{7} + \frac{56}{7}$$

$$= 3 + 6 + 8 = 17$$

95. *(a)* Given, $x = 2^3 \times 3^2 \times 5^4$

and $y = 2^2 \times 3^2 \times 5 \times 7$

$\therefore$ HCF $= 2^2 \times 3^2 \times 5$

$= 4 \times 9 \times 5 = 180$

96. *(a)* Here, $\frac{6}{8} = 0.75, \frac{7}{9} = 0.\bar{7}, \frac{5}{6}$

$$= 0.8\bar{3}, \frac{11}{13} = 0.846$$

$$So,\ 0.75 < 0.\bar{7} < 0.8\bar{3} < 0.846$$

97. *(c)* Since, 1.16666…and 1.454545…are recurring numbers and we know that, recurring numbers represent rational numbers.

Hence, statements I and IV are rational numbers.

98. *(b)* Given, $a = 3$, $b = 9$ and $c = 10$

$$\therefore \sqrt{13 + a} + \sqrt{112 + b} + \sqrt{c - 1}$$

$$= \sqrt{13 + 3} + \sqrt{112 + 9} + \sqrt{10 - 1}$$

$$= \sqrt{16} + \sqrt{121} + \sqrt{9}$$

$$= 4 + 11 + 3 = 18$$

99. *(a)* Let speed be v km/h.

We know that,

Distance = Speed $\times$ Time

$$6 \times 50 = 5 \times v$$

$$\Rightarrow v = \frac{6 \times 50}{5} = 60 \text{ km/h}$$

100. *(b)* Let distance between A and B be x km.

By given condition,

$$\frac{x}{12} - \frac{x}{18} = 2$$

$$\Rightarrow 6x = 2 \times 18 \times 12$$

$$\therefore x = \frac{2 \times 18 \times 12}{6} = 72 \text{ km}$$