HINTS & SOLUTION

1. *(a)* By BODMAS rule, $10 \div \overline{4+6} \times 4$

$= 10 \div (4+6) \times 4$

$= 10 \div 10 \times 4$

$= 1 \times 4 = 4$

2. *(b)* If k is any even positive integer, then $(k^2 + 2k)$ is divisible by 8 but may not be divisible by 24.

Let $k = 2m$, $m \in N$, then

$k^2 + k.2 = 4m^2 + 4m = 4m(m+1)$ which is divisible by 8.

3. *(d)* Since, $a < b \Rightarrow a - b < 0$. Also, $c < 0$

$\therefore (a-b)c > 0 \Rightarrow ac - bc > 0 \Rightarrow ac > bc$

4. *(d)* All are true.

5. *(b)* We know that, between any two rational numbers, there are an infinite number of rational and irrational numbers. Hence, only statement II is correct.

6. *(c)* Let $S_n = an(n-1)$, then

$S_{n-1} = a(n-1)(n-2)$

$\therefore T_n = S_n - S_{n-1} = 2a(n-1)$

$T_n^2 = 4a^2(n-1)^2$

$\therefore Sum = \sum T_n^2 = 4a^2 \frac{(n-1)(n)(2n-1)}{6}$

$= \frac{2a^2 n(n-1)(2n-1)}{3}$

7. *(a)* $\because a, x, y, x, b$ are in AP.

$\therefore x + y + z = 3\left(\frac{a+b}{2}\right)$

$\Rightarrow 15 = \left(\frac{a+b}{2}\right)$

$\Rightarrow a + b = 10$ …(i)

Also, a, x, y, z, b are in HP.

$\Rightarrow \frac{1}{a}, \frac{1}{x}, \frac{1}{y}, \frac{1}{z}, \frac{1}{b}$ are in AP.

$\Rightarrow \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 3\left(\frac{a+b}{2ab}\right)$

$\Rightarrow \frac{5}{3} = \frac{3 \times 10}{2ab}$ $[\because a + b = 10]$

$\Rightarrow ab = 9$ …(ii)

On solving Eqs. (i) and (ii), we get

$a = 1, b = 9$ or $b = 1, a = 9$

8. *(a)* I. $S_n = \frac{n(n+1)}{2} = 861$

$\Rightarrow n^2 + n - 861 \times 2 = 0$

$\Rightarrow (n+42)(n-41) = 0$

$\Rightarrow n = -42, 41$

Hence, statement I is correct.

II. Given, $S_n = S_{-(n+1)}$

If, $S_n = m$, then we have two values of n if and only if m is positive integer.

Hence, statement II is incorrect.

9. *(a)* For integers a, b and c,

if HCF $(a, b) = 1$ and HCF $(a, c) = 1$ then,

HCF $(a, b\,c) = 1$

10. *(d)* It is always 1

Illustrations Let $a = 21$ and $b = 35$

Then, HCF (21, 35) = 7

$\therefore \text{HCF}\left(\frac{21}{7}, \frac{35}{7}\right) = \text{HCF}(3,5) = 1$

11. *(c)* Here, say $a = 2^3 \times 3 \times 5$ and

$b = 2^4 \times 5 \times 7$, then

LCM $= 2^4 \times 3 \times 5 \times 7$

12. *(c)* Given, 0.232323…= $0.\overline{23}$

(which is a recurring decimal) = $\frac{23}{99}$

13. *(c)* $7.2 - \frac{7.2}{100}$

$\Rightarrow 7.2 - 0.72 = 6.48$

14. *(a)* Let fraction be x, then

$x^2 = 227.798649$

$\Rightarrow x = \sqrt{227.798649} = 15.093$

15. *(d)*

$\sqrt{9 - 2\sqrt{14}} = \sqrt{7 + 2 - 2 \times \sqrt{7} \times \sqrt{2}}$

$= \sqrt{(\sqrt{7} - \sqrt{2})^2} = \sqrt{7} - \sqrt{2}$

16. *(b)*Given,

$\sqrt{343 + \sqrt{307 + \sqrt{273 + \sqrt{241 + \sqrt{225}}}}}$

$= \sqrt{343 + \sqrt{307 + \sqrt{273 + \sqrt{241 + 15}}}}$

$= \sqrt{343 + \sqrt{307 + \sqrt{273 + 16}}}$

$= \sqrt{343 + \sqrt{307 + 17}}$

$= \sqrt{343 + 18} = \sqrt{361} = 19$

17. *(c)* Given, total number of tress = 17956

∴ Number of trees in each row

$= \sqrt{17956} = 134$

18. *(c)* Here, $\frac{\sqrt{7} + \sqrt{5}}{\sqrt{7} - \sqrt{5}}$

$= \frac{\sqrt{7} + \sqrt{5}}{\sqrt{7} - \sqrt{5}} \times \frac{\sqrt{7} + \sqrt{5}}{\sqrt{7} + \sqrt{5}} = \frac{(\sqrt{7} + \sqrt{5})^2}{7 - 5}$

$= \frac{7 + 5 + 2\sqrt{35}}{2} = \frac{12 + 2\sqrt{35}}{2}$

$[\because (a+b)^2 = a^2 + b^2 + 2ab]$

$= 6 + \sqrt{35} = 6 + 5.9160 = 11.9160$

19. *(a)* Distance between car and scooter

= 30 km

Relative speed

y = 60 − 50 = 10 km/h

So, the time taken by scooter to overtake the car $= \frac{30}{10} = 3$ h

20. *(d)* Distance travelled in 1h = 48 km

∴ Distance travelled in 50 min

$= \frac{48}{60} \times 50 = 40$ km

Time to be reduced $= \frac{40}{60}$h

∴ Required speed

$= \frac{40}{40/60} = \frac{40 \times 60}{40} = 60$ km/h

21. *(c)* Let the length of each train be I m.

⇒ Speed of first train $= \left(\frac{I}{4}\right)$ m/s

and speed of second train $= \left(\frac{I}{5}\right)$ m/s

As, both trains are moving in opposite direction.

Time taken to cross each other

$= \frac{I + I}{\frac{I}{4} + \frac{I}{5}}$

$= \left(\frac{2I}{\frac{9I}{20}}\right)$ s $= \left(\frac{20 \times 2}{9}\right) = \frac{40}{9}$ s

22. *(b)* Speed of train = 48 km/h

$$= \left(48 \times \frac{5}{18}\right) \text{m/s}$$

Let the length of train be x m

$$x = 48 \times \frac{5}{18} \times 9$$

$$x = 120 \text{ m}$$

Length of the train is 120 m.

23. *(b)* One day's work of $A = \frac{1}{8}$

One day's work of $B = \frac{1}{12}$

3 day's work of $A = \frac{3}{8}$

Remaining work of $A = 1 - \frac{3}{8}$

$$= \frac{5}{8}$$

One day's work of A and B together

$$= \frac{1}{8} + \frac{1}{12} = \frac{3+2}{24} = \frac{5}{24}$$

Number of days to finish the work

$$= \frac{5}{8} \div \frac{5}{24} = 3 \text{ days}$$

24. *(b)* Here, $x = 25$ days and $y = 25$

$$\therefore \text{Required days } = \frac{100x}{100+y}$$

$$= \frac{100 \times 25}{100+25}$$

$$= \frac{2500}{125} = 20 \text{ days}$$

25. *(a)* Here, $x = 15$ days, $y = 20$ days and $z = 25$ days and $K =$ ₹ 4700

$$\text{Share of } C = ₹ \left(\frac{kxy}{xy + yz + zx}\right)$$

$$= ₹ \frac{4700 \times 15 \times 20}{15 \times 20 + 20 \times 25 + 25 \times 15}$$

$$= ₹ \frac{4700 \times 15 \times 20}{1175} = ₹ 1200$$

26. *(a)* B's one min's work = ($A + B + C$)'s one min's work −($A + C$)'s one min's work

$$= \frac{1}{30} - \frac{1}{45} = \frac{6-4}{180} = \frac{2}{180} = \frac{1}{90}$$

C's one min's work = ($A + B + C$)'s one min work − ($A + B$)'s one min work

$$= \frac{1}{30} - \frac{1}{40} = \frac{4-3}{120} = \frac{1}{120}$$

A' one min's work = ($A + B$)'s one min work − B's one min work

$$= \frac{1}{40} - \frac{1}{90} = \frac{9-4}{360}$$

$$= \frac{5}{360} = \frac{1}{72}$$

Hence, A, B and C alone can finish the work in 72 min, 90 min and 120 min, respectively.

27. *(c)* Given, 90% of A = 30% of B

$$\frac{90A}{100} = \frac{30B}{100}$$

$$\Rightarrow \frac{A}{B} = \frac{3}{9} \Rightarrow B = 3A$$

Now, $B = x\%$ of A, $3A = \frac{xA}{100}$

$$\therefore x = 300$$

Hence, the value of x is 300.

28. *(d)* Let total number of staff be 100.

Female staff = 40

Male staff = (100 − 40) = 60

Votes casted by females

$$\frac{40}{100} \times 40 = 16$$

Votes casted by males = $\frac{60}{100} \times 60 = 36$

Votes casted by both males and females
= 16 + 36 = 52

∴ Percentage votes obtained = 52%.

29. *(a)* Let the salary of Kunal be ₹ 100, then saving = ₹ 30

Expenses = ₹ 70

New expenses = (100 + 30)% of ₹ 70
= ₹ 91

New saving = ₹ (100 − 91) = ₹ 9

He saves ₹ 9, his salary = ₹ 100

If he saves ₹ 1215.

Then, his salary = ₹ $\left(\frac{100}{9} \times 1215\right)$
= ₹ 13500

30. *(c)* Amount of sugar in 6 L of solution

$$= \frac{4}{100} \times 6 = 0.24 \text{ L}$$

After evaporation, sugar in 5 L = 0.24 L

∴ Percentage of sugar

$$= \left(\frac{0.24}{5} \times 100\right) = 4\frac{4}{5}\%$$

31. *(a)* Let the sum be ₹ x and the original rate r%, then

Simple interest = $\frac{x \times r \times 2}{100}$

Now, rate is increased by 3%.

∴ New rate = $(r + 3)$%

∵ Simple interest $= \frac{x \times (r+3) \times 2}{100}$

$$\therefore \frac{x \times (r+3) \times 2}{100} - \frac{x \times r \times 2}{100} = 72$$

$$\Rightarrow \frac{(xr+3x)2}{100} - \frac{2xr}{100} = 72$$

$$\Rightarrow \frac{2xr + 6x - 2xr}{100} = 72$$

∴ $x =$ ₹ 1200

32. *(d)* SI at 5% = $6P - P = 5P$

$$\therefore 5P = \frac{P \times 5 \times T}{100} \Rightarrow T = 100 \text{ yr}$$

Now, for new rate (R),

$$11P = \frac{P \times R \times 100}{100}$$

$$\therefore \quad R = 11\%$$

33. *(a)* Let the principal be ₹ x and the time be t yr.

Rate = 10%

∵ Simple interest = $\frac{P \times R \times T}{100}$

According to the question,

0.125 × Principal = Simple interest

$$\therefore 0.125P = \frac{P \times 10 \times T}{100}$$

$$\Rightarrow \frac{125P}{100} = \frac{10 \times P \times T}{100}$$

$$\Rightarrow \frac{125}{100} = T$$

$$\Rightarrow T = \frac{5}{4} = 1\frac{1}{4} \text{ yr}$$

34. *(d)* Let the amount remaining to pay be ₹ x.

Price of house = ₹ $(x + 8000)$

$$\Rightarrow 9600 - \frac{x \times 4 \times 5}{100} = x$$

$$\Rightarrow \quad 9600 - \frac{x}{5} = x$$

$$\Rightarrow 9600 = \frac{6x}{5} \Rightarrow \frac{9600 \times 5}{6} = x$$

$\Rightarrow \quad x =$ ₹ 8000

∴ Cash price of the house
= ₹ (8000 + 8000) = ₹ 16000

35. *(a)* Let a man invest ₹ 1000 at a R%.
Now, rate is increased by 2%.
New rate = $(R + 2)$%
By given condition,

$$\frac{1000\times R\times 3}{100}+\frac{1500\times(R+2)\times 3}{100}=390$$

$$\Rightarrow 30R+45R+90=390$$

$$\Rightarrow 75R = 300 \Rightarrow R=4\%$$

36. *(a)* Let the value of machine 3 yr ago be ₹ x.
and given, P = ₹ 10935, R = 10% and n = 3 yr

$$\because x=P\left(1-\frac{R}{100}\right)^n$$

$$\therefore x\left(1-\frac{10}{100}\right)^3=10935$$

$$\Rightarrow x\left(\frac{90}{100}\right)^3=10935$$

$$\therefore x=\frac{10935\times 10\times 10\times 10}{9\times 9\times 9}=₹\ 15000$$

37. *(a)* Let the sum be ₹ x, then

$$x\left(1+\frac{R}{100}\right)^5=2x$$

$$\Rightarrow \left(1+\frac{R}{100}\right)^5 = 2 \quad \ldots\text{(i)}$$

The amount after 20 yr

$$x\left(1+\frac{R}{100}\right)^{20}=x\left[\left(1+\frac{R}{100}\right)^5\right]^4$$

$$= 2^4x=16x \text{ [from Eq. (i)]}$$

$$=16\times 10000= ₹\ 160000$$

[put x = ₹ 10000]

38. *(d)* Given, P = ₹ 5000,
$R_1=8\%$, $R_2=10\%$, $R_3=12\%$
and $n_1=n_2=n_3=$ 1 yr
∵ Amount

$$=P\left(1+\frac{R_1}{100}\right)^{n_1}\left(1+\frac{R_2}{100}\right)^{n_2}\left(1+\frac{R_3}{100}\right)^{n_3}$$

$$=\left[5000\times\left(1+\frac{8}{100}\right)\left(1+\frac{10}{100}\right)\left(1+\frac{12}{100}\right)\right]$$

$$=\left(5000\times\frac{27}{25}\times\frac{11}{10}\times\frac{28}{25}\right)$$

= ₹ 6652.80

∴ Compound interest = 6652.80 – 5000
= ₹ 1652.80

39. *(b)* Total gain = SP – CP
= (840 − 720) = ₹ 120

$$\therefore \text{Gain percent} = \frac{120}{720}\times 100=16\frac{2}{3}\%$$

40. *(c)* Here, true weight = 1000 and gain = 25%

$$\Rightarrow 25=\frac{1000-\text{false weight}}{\text{false weight}}\times 100$$

$$\Rightarrow \frac{\text{false weight}}{4}=1000-\text{false weight}$$

$$\Rightarrow \text{false weight}=1000\times\frac{4}{5}=800$$

41. *(b)* We know that,
Net percentage discount

$$=\frac{\text{Discount}}{\text{Cost price}}\times 100\%$$

$$=\frac{1}{4}\times 100\% = 25\%$$

42. *(d)* The cost price of table for person B

$$=2000+6\times\frac{2000}{100}$$

$$=2000+120= ₹\ 2120$$

Selling price for person B

$$= 2120 - \frac{2120 \times 5}{100}$$

$$= 2120 - 106 = ₹\ 2014$$

43. *(c)* Given, SP = ₹ 110 and loss = 12%

$$\therefore \text{CP} = ₹\left(\frac{100}{88} \times 110\right) = ₹\ 125$$

Now, CP = ₹ 125, gain required = 8%

$$\therefore \text{SP} = ₹\left(\frac{(100+8)}{100} \times 125\right) = ₹\ 135$$

44. *(c)* Let the ratio be x and $(x + 40)$.

Then, by given condition, $\frac{x}{x+40} = \frac{2}{7}$

$\Rightarrow 7x = 2x + 80 \Rightarrow x = 16$

So, the required ratio is 16 : 56.

45. *(a)* Number of people having characteristic

$X = 10 + 30 = 40$

Number of people having characteristic

$Y = 10 + 20 = 30$

Required ratio = 40:30 = 4:3.

46. *(d)* Given, $\frac{x^3 + 3x}{3x^2 + 1} = \frac{341}{91}$

By componendo and dividendo rule,

$$\frac{x^3 + 3x + 3x^3 + 1}{x^3 + 3x - 3x^2 - 1} = \frac{341 + 91}{341 - 91}$$

$$= \frac{432}{250}$$

$$\Rightarrow \frac{(x+1)^3}{(x-1)^3} = \frac{216}{125}$$

$$\left[\because (a+b)^3 = a^3 + b^3 + 3ab(a+b)\right]$$

On cube roots both sides, we get

$$\Rightarrow \frac{x+1}{x-1} = \frac{6}{5} \Rightarrow 5(x+1) = 6(x-1)$$

$\therefore x = 11$

47. *(b)* Given, $x \propto \frac{y}{z^2} \Rightarrow x = \frac{ky}{z^2}$

$\because x = 10, y = 4$ and $z = 14$

$$\therefore 10 = \frac{k.4}{196} \Rightarrow k = \frac{1960}{4} = 490$$

Now, $z = 7$ and $y = 16$, then

$$x = \frac{490 \times 16}{7 \times 7} = 160$$

48. *(c)* Given, $p + r = 2q \Rightarrow 2 = \frac{p+r}{q}$

and $\frac{1}{q} + \frac{1}{s} = \frac{2}{r}$

$$\Rightarrow \frac{1}{q} + \frac{1}{s} = \frac{p+r}{qr}$$

$$\Rightarrow \frac{s+q}{sq} = \frac{p+r}{qr}$$

$$\Rightarrow r(s+q) = s(p+r)$$

$$\Rightarrow rq = sp \Rightarrow \frac{p}{q} = \frac{r}{s}$$

$$\therefore \quad p : q = r : s$$

49. *(d)* Let the number of passengers travelling by class I and class II be x and $50x$ respectively.

Then, amount collected from class I and II will be ₹ $3 \times x$ and ₹ $50x$ respectively.

Given, $3x + 50x = 1325$

$53x = 1325 \Rightarrow x = 25$

$\therefore$ Amount collected from class II

$= 50 \times 25 =$ ₹ 1250

50. *(c)* I. 4 leaps of cat = 3 leaps of dog

$\Rightarrow$ 1 leap of cat = 3/4 leap of dog

Cat takes 5 leaps for every 4 leaps of dog.

$\therefore$ Required Ratio

= (5 × cat's leap) : (4 × dog's leap)

$= \left(5 \times \frac{3}{4} \text{ dog's leap}\right) : (4 \times \text{dog's leap})$

$= 15 : 16$

Thus, $\frac{\text{Speed of cat}}{\text{Speed of dog}} = \frac{15}{16}$

II. $\frac{\text{Distance (cat)}}{\text{Distance (dog)}} = \frac{s(\text{cat}) \times t}{s(\text{dog}) \times t}$

$= \frac{s(\text{cat}) \times 30}{s(\text{dog}) \times 30} = \left(\frac{s(\text{cat})}{s(\text{dog})} = \frac{15}{16}\right)$

Thus, both statements I and II are correct.

51. *(b)* $\log_{100} 0.1 = \log_{10^2}\left(\frac{1}{10}\right)$

$= \frac{1}{2}\log_{10}\left(\frac{1}{10}\right) = \frac{1}{2}\log_{10}(10)^{-1}$

$= -\frac{1}{2}\log_{10} 10 = -\frac{1}{2}$

52. *(c)* $\log_y x \log_z y \log_x z$

$= \frac{\log x}{\log y} \times \frac{\log y}{\log z} \times \frac{\log z}{\log x} = 1$

$\left[\because \log_a b = \frac{\log b}{\log a}\right]$

53. *(b)* Given, $10^x = 1.73$, $x = \log_{10} 1.73$

$= \log_{10} 1730 - \log_{10} 1000$

$= \log_{10} 1730 - \log_{10} 10^3$

$= 3.2380 - 3 = 0.2380$

54. *(c)* $\log_5 10 = \frac{\log_{10} 10}{\log_{10} 5} = \frac{1}{0.70}$

$= 1.43143$

$[\because \log_{10} 10 = 1]$

55. *(b)* $(a^2 - b^2 - 4ac + 4c^2)$

$= (a^2 - 4ac + 4c^2) - b^2$

$[(a^2) - 2(2c)(a) + (2c)^2] - b^2$

$= (a - 2c)^2 - b^2$

$= (a - 2c - b)(a - 2c + b)$

56. *(a)* Since, $x^{1/3} + y^{1/3} + z^{1/3} = 0$

$\therefore (x^{1/3})^3 + (y^{1/3})^3 + (z^{1/3})^3 - 3x^{1/3}y^{1/3}z^{1/3} = 0$

$\Rightarrow x + y + z - 3(xyz)^{1/3} = 0$

$\Rightarrow x + y = z = 3(xyz)^{1/3}$

$\Rightarrow (x + y + z)^3 = 27xyz$

57. *(c)* Let $f(x) = 9x^2 + 3px + 6q$

Given, $f(-1/3) = -3/4$

$\Rightarrow 9\left(-\frac{1}{3}\right)^2 + 3p\left(-\frac{1}{3}\right) + 6q = -3/4$

$\Rightarrow 1 - p + 6q = -3/4$

$\Rightarrow 24q - 4p + 7 = 0 \qquad \ldots\text{(i)}$

Let $g(x) = qx^2 + 4px + 7$

Since, $(x + 1)$ is a factor of $g(x)$

$\therefore g(-1) = 0 \Rightarrow q - 4p + 7 = 0 \quad \ldots\text{(ii)}$

On solving Eq. (i) and Eq. (ii), we get

$q = 0$ and $p = 7/4$

58. *(a)* $\left(a^2 + a + \frac{1}{4}\right) = a^2 + \frac{1}{2}a + \frac{1}{2}a + \frac{1}{4}$

$= a\left(a + \frac{1}{2}\right) + \frac{1}{2}\left(a + \frac{1}{2}\right)$

$= \left(a + \frac{1}{2}\right) + \left(a + \frac{1}{2}\right) = \left(a + \frac{1}{2}\right)^2$

59. *(b)* $\because (a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$

$(10)^2 = (a^2 + b^2 + c^2) + 2(31)$

$a^2 + b^2 + c^2 = 100 - 62$

$\Rightarrow a^2 + b^2 + c^2 = 38$

60. *(d)* Let $f(x) = a_0 + a_1x + a_2x^2 + \ldots + a_nx^n$
Since, $(x - 1)$ is a factor of $f(x)$.
Put $x = 1$ in $f(x)$, then
$f(1) = a_0 + a_1 + a_2 + \ldots + a_n$
$\Rightarrow 1 = a_0 + a_1 + a_2 + \ldots. + a_n$
$\therefore 1 - a_0 - a_2 - \ldots = a_1 + a_3 + \ldots$

61. *(a)* LCM = product of the largest power of each factor
$= x^2 (x - 1)(x - 2)(x + 3)$

62. *(a)* I. $x^2 - 6x + 9 = (x - 3)(x - 3)$
and $x^3 - 27 = x^3 - (3)^3$
$= (x - 3)\left(x^2 + 3x + 9\right)$
$\therefore$ HCF $= x - 3$
Hence, it is true.
II. LCM of $10x^2yz$, $15xyz$, $20xy^2z^2$ is $60x^2y^2z^2$.
Hence, it is false.
III. $6x^2 - 7x - 3 = (2x - 3)(3x + 1)$
and $2x^2 + 11x - 21$
$= (x + 7)(2x - 3)$
Hence, HCF $= (2x - 3)$, it is also true.
Hence, the statement I and III are correct.

63. *(a)* $A = (x+3)^2(x-2)(x+1)^2$ and
$B = (x+1)^2(x+3)(x+4)$
$\therefore$ HCF of polynomials
$= (x+1)^2(x+3)$

64. *(d)* $(x + 1)$ is the HCF of
$Ax^2 + Bx + C$ and $Bx^2 + Ax + C$
$\therefore A(-1)^2 + B(-1) + C = 0$
$\Rightarrow A - B + C = 0$
$\Rightarrow C = B - A$
and $B(-1)^2 + A(-1) + C = 0$
$\Rightarrow B - A + C = 0$
$\Rightarrow C = A - B$
$\therefore C = 0$

65. *(c)* Given, $a = \dfrac{1+x}{2-x}$.

So, $\dfrac{1}{a+1} + \dfrac{2a+1}{a^2-1} = \dfrac{3a}{a^2-1}$

$$= \frac{3\left(\frac{1+x}{2-x}\right)}{\left(\frac{1+x}{2-x}\right)^2 - 1}$$

$$= \frac{3(1+x)(2-x)}{1 + x^2 + 2x - \left(4 + x^2 - 4x\right)}$$

$$= \frac{3(1+x)(2-x)}{6x-3}$$

$$= \frac{(1+x)(2-x)}{(2x-1)}$$

66. *(b)* Here, reciprocal of $\dfrac{x-3}{x^2+1}$ is $\dfrac{x^2+1}{(x-3)}$

So,

$$\frac{x-3}{x^2+1} + \frac{x^2+1}{x-3} = \frac{(x-3)^2 + \left(x^2+1\right)^2}{\left(x^2+1\right)(x-3)}$$

$$= \frac{x^2 + 9 - 6x + x^4 + 1 + 2x^2}{x^3 - 3x^2 + x - 3}$$

$$= \frac{x^4 + 3x^2 - 6x + 10}{x^3 - 3x^2 + x - 3}$$

67. *(d)* Put $x = 2$ and $y = 1$in each equation
I. $2x + 5y = 9 \Rightarrow 2(2) + 5(1) = 9$
$9 = 9$, it is true.
II. $5x + 3y = 14$
$\Rightarrow 5(2) + 3(1) = 14$

$\Rightarrow$ 13 = 14, it is false.

III. $2x + 3y = 7 \Rightarrow 2(2) + 3(1) = 7$

$7 = 7$, it is true.

IV. $2x - 3y = 1 \Rightarrow 2(2) - 3(1) = 1$

$1 = 1$, it is true.

So, $x = 2$ and $y = 1$ is a solution of I, III and IV.

68. *(a)* We have, $25x - 19 - [3 - \{4x - 5\}] = 3x - (6x - 5)$

$\Rightarrow 25x - 19 - [3 - 4x + 5] = 3x - 6x + 5$

$\Rightarrow 25x - 19 + 4x - 8 = -3x + 5$

$\Rightarrow 29x + 3x = 5 + 27$

$\Rightarrow 32x = 32 \Rightarrow x = \frac{32}{32} = 1$

$\Rightarrow x = 1$

69. *(c)* The graph of $ax + by = c$, $dx + ey = f$ will be coincident, if the system has infinite number of solutions.
So, statement II is false.
Thus, statements I and III are correct.

70. *(a)* Given, $x^2 - 8x + p = 0$

Sum of roots $\alpha + \beta = 8$ and product of roots $\alpha\beta = p$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ …(i)

$\Rightarrow 40 = (8)^2 - 2(p)$ $[\because \alpha^2 + \beta^2 = 40]$

$\Rightarrow 40 - 64 = -2p$

$\Rightarrow -24 = -2p$

$\Rightarrow p = 12$

71. *(c)* As, $\sqrt{x+4} = x - 2$

On squaring both sides, we get

$(x + 4) = (x - 2)^2$

$\Rightarrow x + 4 = x^2 + 4 - 4x$

$\Rightarrow x^2 - 5x = 0 \Rightarrow x = 0,\ x = 5$

But for $x = 0$, $\sqrt{0+4} = 0 - 2$

$\sqrt{4} \neq -2$

So, $x = 5$ is the only solution.

72. *(d)* Here, roots are α and $\alpha + 1$.

$\therefore \alpha + (\alpha + 1) = l$ [sum of roots]

$\Rightarrow 2\alpha = l - 1 \Rightarrow \alpha = \frac{l-1}{2}$

Also, $\alpha(\alpha + 1) = m$ or $\alpha^2 + \alpha = m$

$\Rightarrow \left(\frac{l-1}{2}\right)^2 + \left(\frac{l-1}{2}\right) = m$

$\Rightarrow (l-1)^2 + 2(l-1) = 4m$

$\Rightarrow l^2 - 1 = 4m \Rightarrow l^2 = 4m + 1$

73. *(d)* Given equation, $x^2 - 3x + 2 = 0$

$\Rightarrow x^2 - 2x - x + 2 = 0$

$\Rightarrow (x - 2)(x - 1) = 0 \Rightarrow x = 2, 1$

Let $\alpha = 1$ and $\beta = 2$

$\therefore \alpha + 1 = 2$ and $(\beta + 1) = 3$

Now, sum of roots = 2 + 3 = 5

and product of roots = $2 \times 3 = 6$

Required equation = x^2 − (sum of roots) + product of roots = 0

$\Rightarrow x^2 - 5x + 6 = 0$

Hence, the equation is neither I nor II.

74. *(c)* Here, $2x + 3 \geq 8 \Rightarrow 2x \geq 8 - 3$

$\Rightarrow 2x \geq 5 \Rightarrow x \geq \frac{5}{2}$

Again, $3x + 1 \leq 12$

$\Rightarrow 3x \leq 11 \Rightarrow x \leq \frac{11}{3}$

By combining values, we get

$\frac{5}{2} \leq x \leq \frac{11}{3}$

75. *(b)* Let the smaller part = x and greater part $= 16 - x$

By given condition,

$$2(16 - x)^2 - x^2 = 164$$

$$\Rightarrow 2(256 + x^2 - 32x) - x^2 = 164$$

$$\Rightarrow x^2 - 64x + 348 = 0$$

$$\Rightarrow (x - 58)(x - 6) = 0$$

$$\Rightarrow x = 58, x = 6$$

Here, $x \neq 58$

$\therefore$ $x = 6$

and, hence larger part

$= 16 - x = 16 - 6 = 10$

76. *(a)* Given, A = {B, O,W, L}

B = {B, O,W, L, E}

C = {B, O,W, L, E}

$\therefore A \subset B$ and $B = C$

77. *(c)* I. $A = \{0\}$ II. $B = \{2\}$

III. $C = \{\ \}$; $\pm$ 4 is not an odd integer

Here, only III is empty set.

78. *(b)* $(B \cup C) = \{1, 2, 3, 4, 5, 6, 7\}$

$$\therefore A \cap (B \cup C) = \{1, 2, 3, 4\} \cap \{1, 2, 3, 4, 5, 6, 7\} = \{1, 2, 3, 4\}$$

79. *(b)* Let $A = \{2, 4, 16, 256, \ldots\}$

for $n = 0$, $2^{2^0} = 2^1 = 2$

for $n = 1$, $2^{2^1} = 2^2 = 4$

for $n = 2$, $2^{2^2} = 2^4 = 16$

Thus,

$A = \{x \in N \mid x = 2^{2^n}, n = 0, 1, 2, \ldots\}$

80. *(c)* A = diagonal equal and bisecting each other.

A is square or rectangle. and B diagonal bisecting each other at 90°.

So, $A \cap B$ = the set of squares.

81. *(b)* We know that, π radian = 180°

$$\Rightarrow 1 \text{ radian} = \frac{180°}{\pi} = \frac{180°}{22} \times 7$$

$$= \frac{630°}{11} = 57\frac{3°}{11} = 57° + \frac{3 \times 60}{11} \text{ min}$$

$$= 57° + 16' + \frac{4}{11} \text{ min}$$

$$= 57° + 16' + \frac{4}{11} \times 60 \text{ s}$$

$$= 57° + 16' + 21.8''$$

$$= 57°16'21.8'' = 57°16'22''$$

82. *(b)* Given, $\tan A = 1 = \tan 45°$

$\Rightarrow A = 45°$ and $\tan B = \sqrt{3} = \tan 60°$

$\therefore B = 60°$

Now, $\cos A \cos B - \sin A$ sin i

$= \cos 45° \cos 60° - \sin 45° \sin 60°$

$$= \frac{1}{\sqrt{2}} \times \frac{1}{2} - \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} = \frac{1 - \sqrt{3}}{2\sqrt{2}}$$

83. *(c)* Given,

I. $\text{RHS} = \cos^2\theta(1 + \tan\theta)(1 - \tan\theta)$

$$= \cos^2\theta(1 - \tan^2\theta)$$

$$= \cos^2\theta\left(\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}\right)$$

$$= \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta} = \text{LHS}$$

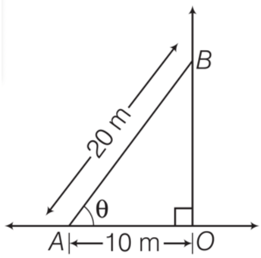
II. Given,

$$\text{LHS} = \frac{1 + \sin\theta}{1 - \sin\theta} = \frac{(1 + \sin\theta)^2}{1 - \sin^2\theta}$$

$$= \left(\frac{1 + \sin\theta}{\cos\theta}\right)$$

$$= (sec\ \theta + \tan\theta)^2$$

84. *(c)* Let θ be the inclination of the ladder to the horizontal.



Now, in right angled ΔAOB,

$$\cos\theta = \frac{AO}{AB} = \frac{10}{20} = \frac{1}{2}$$

$\Rightarrow \cos\theta = \cos 60°$

$\therefore \theta = 60°$

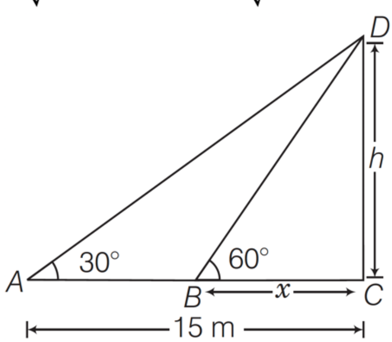
Hence, the angle of inclination of the ladder is 60°.

85. *(c)* Let the height of the tower be h m and length of the shadow (BC) be x m.

In right angled ΔACD,

$$\tan 30° = \frac{CD}{AC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{15} \Rightarrow h = \frac{15}{\sqrt{3}} \text{ m} \quad \ldots\text{(i)}$$



and in right angled ΔBCD,

$$\tan 60° = \frac{CD}{BC} \Rightarrow \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow \frac{h}{\sqrt{3}} = x$$

$$\therefore x = \frac{15}{3} = 5 \text{ m}$$

[from Eq. (i)]

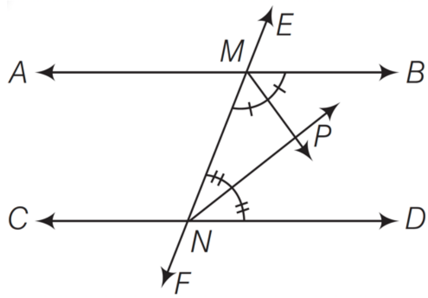
Hence, the length of shadow is 5 m. When sun's altitude is 60°.

86. *(a)* Given, $\angle PMN = \frac{1}{2}\angle BMN$

and $\angle PNM = \frac{1}{2}\angle DNM$

As, $\angle BMN + \angle DNM = 180°$

[angles on the same side of transversal]



$\therefore$ In ΔMPN, $\angle PMN + \angle PNM = 90°$

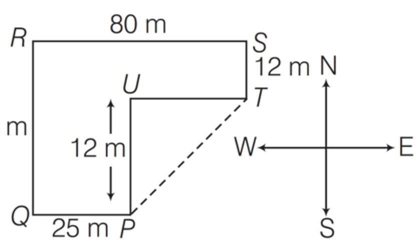
$\Rightarrow \angle MPN = 180° - (\angle PMN + \angle PNM)$

[angle sum property]

$\therefore \angle MPN = 180° - 90° = 90°$

87. *(c)* The least number of straight lines for a bounded plane figure is 3.

88. *(d)* Let P be the starting point of his run, then PT is the distance between the starting and the finishing point.



$\therefore PU = RQ - ST = 60 - 12 = 48$ m

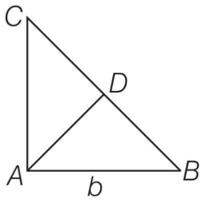
and $UT = RS - QP = 80 - 25 = 55$ m

$\therefore$ In ΔPUT, $PT^2 = (PU)^2 + (TU)^2$

$\therefore\ PT = \sqrt{(48)^2 + (55)^2} = \sqrt{2304 + 3025}$

$= \sqrt{5329} = 73$ m

89. *(a)* In $\Delta\ ABC$,



Area of $\Delta ABC = \frac{1}{2} \times \text{Base} \times \text{Altitude}$

$$\Delta = \frac{1}{2} b \times AC,\ AC = \frac{2\Delta}{b}$$

In $\Delta\ ABC$, Using Pythagoras theorem,

$$AC^2 + AB^2 = BC^2$$

$$\Rightarrow BC = \sqrt{\frac{4\ \Delta^2}{b^2} + b^2}$$

Again in $\Delta\ ABC$, area of $\Delta\ ABC$

$$\Delta = \frac{1}{2} \times BC \times AD$$

$$\Rightarrow AD = \frac{2\Delta}{\sqrt{\frac{4\Delta^2 + b^4}{b^2}}} = \frac{2\Delta b}{\sqrt{4\Delta^2 + b^2}}$$

90. *(c)* Each interior angle of a regular polygon

$$= \frac{(n-2) \times 180°}{n}$$

$$\therefore \frac{(n-2) \times 180°}{n} = 150° \qquad \text{(given)}$$

$$(n-2)180 = n \times 150$$

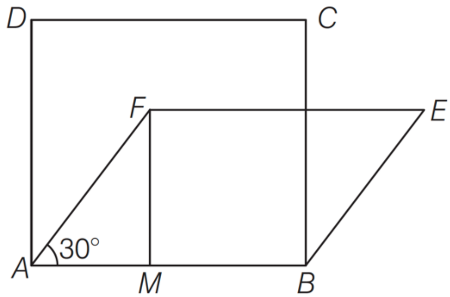
$$\Rightarrow 30n = 360 \Rightarrow n = \frac{360}{30}$$

$$\Rightarrow n = 12$$

91. *(b)* $ABCD$ is square and $ABEF$ is a rhombus.

$$\frac{FM}{AF} = \sin 30° = \frac{1}{2}$$

$$\therefore FM = \frac{AF}{2},\ AF = AB$$



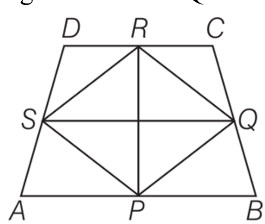
Area of square $= a^2 \quad (AB = AD = a)$

Area of rhombus $= \frac{a \times a}{2} \quad \left(FM = \frac{a}{2}\right)$

(Area of rhombus = base $\times$ height)

$$\therefore \frac{\text{Area of square}}{\text{Area of rhombus}} = \frac{2}{1}$$

92. *(c)* PQRS can be shown parallelogram, so the diagonal PR and SQ bisect each other.



II. Area (ΔRSQ)

$$= \frac{1}{2} \text{ Area of } (SQCD) \qquad \ldots\text{(i)}$$

and area $\Delta(PSQ) = \frac{1}{2}$ Area $(ABQS)$

$\ldots$(ii)

On adding Eqs. (i) and (ii), we get

Area (RSQ) + Area $(PSQ) = \frac{1}{2}$

[Area $(SQCD)$ + Area $(ABQS)$]

$\Rightarrow$ Area ($PQRS$) = $\frac{1}{2}$ Area ($ABCD$)

Hence, both statements are true.

93. *(b)* Given, each interior angle = 140°

Then, each exterior angle

$= (180° - 140°) = 40°$

Number of sides

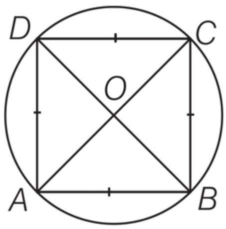
$$= \frac{360°}{\text{Each exterior angle}} = \frac{360°}{40°} = 9$$

Hence, the number of vertices of polygon is 9.

94. *(d)* A square has four equal side

$\therefore$ Each side subtends the same angle at the centre O.

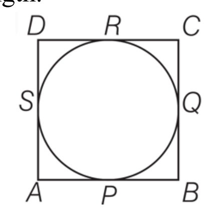
Let angle subtended be $x°$.



So, $4x° = 360°$

$$x° = \frac{360°}{4} = 90°$$

95. *(c)* We know that, two tangents drawn from an external point to a circle are equal in length.



$\therefore$ $AP = AS$ …(i)

$BP = BQ$ …(ii)

$CR = CQ$ …(iii)

$DR = DS$ …(iv)

On adding Eqs. (i), (ii), (iii) and (iv), we get

$(AP + BP) + (CR + DR)$

$= (AS + DS) + (BQ + CQ)$

$\Rightarrow AB + CD = AD + BC$

96. *(d)* As, the tangents drawn from an external point to a circle are equal.

$\therefore$ $AP = AR$ …(i)

$BQ = BP$ …(ii)

and $CR = QC$ ...(iii)

On adding Eqs. (i), (ii) and (iii),we get

$AP + BQ + CR = BP + QC + RA$

and perimeter of

$$\Delta ABC = AB + BC + CA$$
$$= (AP + PB) + (BQ + QC) + (CR + RA)$$
$$= (AP + BQ) + (BQ + CR) + (CR + AP)$$
$$= 2(AP + BQ + CR)$$

$$\therefore \quad AP + BQ + CR = \frac{1}{2}[\text{perimeter of } \Delta\, ABC]$$

Hence, the statement I and II are correct.

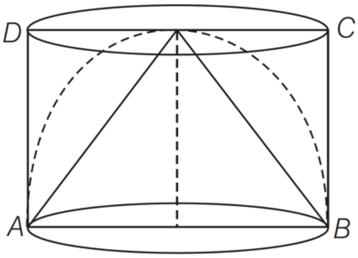
97. *(a)* Area of square $= \frac{1}{2} \times (\text{Diagonal})^2$

$$= \frac{1}{2} \times 50 \times 50$$
$$= 1250 \text{ m}^2$$

98. *(b)* From the information given in the question and the figure it is clear that

Radius of the hemisphere = radius of cone = height of cone = height of cylinder. Let it be r .

Then, ratio of volume of cylinder, hemisphere and cone.



$= \pi r^3 : \frac{2}{3}\pi r^3 : \frac{1}{3}\pi r^3$

$= 1 : \frac{2}{3} : \frac{1}{3} = 3 : 2 : 1$

99. *(c)* The height of the bar is not proportional to the frequency of the class.

100. *(a)* Given distribution is 1, 3, 5, 7, 9, x, 15, 17, 19, 21.

Number of terms = 10 (even)

∴ Meridian =

$$\frac{\text{value of } \frac{10}{2}\text{th term} + \text{Value of } \left(\frac{10}{2}+1\right)\text{th term}}{2}$$

$$10 = \frac{\text{Value of 5th term} + \text{Value of 6th term}}{2}$$

$$\Rightarrow 10 = \frac{9+x}{2} \Rightarrow 20 = 9 + x$$

$$\Rightarrow x = 11$$