HINTS & SOLUTION

1. *(b)* $(a + b)$ always represent a natural number $\forall\, a, b \in N$.

2. *(c)* Consider, $2, 4 \in N$
So, $\sqrt{4} = 2$, a natural number
and $\sqrt{2}$ = irrational number.

3. *(b)* Difference of sums of even and odd places digit of 1254934
$= (1 + 5 + 9 + 4) - (2 + 4 + 3)$
$= 19 - 9 = 10$
This number will be divisible by 11, after adding x, if $x = 1$.
Also, the sum of digits of 1254934
$= 1 + 2 + 5 + 4 + 9 + 3 + 4 = 28$
1254934 will be divisible by 3, after adding y, if $y = -1$

4. *(c)* When we divide a positive integer by another positive integer, the resultant will be a rational number i.e. in the form of p/q, where p and q are positive integers and $q \neq 0$.

5. *(d)* On division of $(19)^n$ by 20, we get remainder either 19 or 1.
Since, last digit of $(19)^{100}$ is 1.
$\therefore$ Remainder of $\dfrac{(19)^{100}}{20}$ is 1.

6. *(c)* $\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} + \ldots n$ terms

$$= \left(1 - \frac{1}{2}\right) + \left(1 - \frac{1}{4}\right) + \left(1 - \frac{1}{8}\right) + \ldots + \left(1 - \frac{1}{2^n}\right)$$

$$= n - \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{2^n}\right)$$

$$= n - \frac{1}{2}\left(\frac{1 - (1/2)^n}{1 - 1/2}\right)$$

$$= n + 2^{-n} - 1$$

7. *(b)* When 'n' is even.
Let $n = 2m$, then

$$= 1^2 - 2^2 + 3^2 - 4^2 + 5^2 - \ldots$$

$$= (1^2 - 2^2) + (3^2 - 4^2) + (5^2 - 6^2) + \ldots + (2m-1)^2 - (2m)^2$$

$$= (1+2)(1-1) + (3+4)(-1) + (5+6)(-1) + \ldots + (2m-1+2m)(-1)$$

$$= -(1 + 2 + 3 + 4 + \ldots + 2m)$$

$$= \frac{-2m(2m+1)}{2} = \frac{-n(n+1)}{2}$$

8. *(b)* HCF $\left(\frac{3}{2}, \frac{9}{7}, \frac{15}{14}\right)$

$$= \frac{\text{HCF}(3, 9, 15)}{\text{LCM}(2, 7, 14)} = \frac{3}{14}$$

9. *(d)* Here, we know that

$144 = 12 \times 2 \times 2 \times 3$

and $192 = 12 \times 2 \times 2 \times 2 \times 2$

By taking option (d), $48 = 12 \times 2 \times 2$
Hence, the value of x will not be 48 otherwise the HCF of given numbers becomes 48.

10. *(c)* LCM of 11 and 13 will be (11×13). Hence, if a number is exactly divisible by 11 and 13, then the same number must be exactly divisible by their LCM
i.e. (11×13).

11. *(a)* $175 \times 1.24 = 2.17$

$\Rightarrow 175 \times 124 = 217$

12. *(d)* Here, $\left[\dfrac{(0.1)^2 - (0.01)^2}{0.0001} + 1\right]$

$= \dfrac{0.01 - 0.0001}{0.0001} + 1 = \left(\dfrac{0.0099}{0.0001} + 1\right)$

$= (99 + 1) = 100$

13. *(c)* $\dfrac{2}{5} = 0.4, \dfrac{5}{6} = 0.8\bar{3}, \dfrac{11}{12} = 0.91\bar{6}$

and $\dfrac{7}{8} = 0.875$

Clearly, the greatest fraction is $0.91\bar{6}$ i.e. $\dfrac{11}{12}$.

14. *(c)* Given, $\sqrt{\dfrac{x}{49}} = \dfrac{4}{7}$ or $\dfrac{\sqrt{x}}{7} = \dfrac{4}{7}$

Now, on squaring both sides, we get

$(\sqrt{x})^2 = (4)^2 \Rightarrow x = 16$

Hence, the value of x is 16.

15. *(d)* Since $15 = 3 \times 5 = 3 \times 6^B$

As, $3 = \dfrac{6}{2} = \dfrac{6^1}{6^A} = 6^{1-A}$

$\therefore\ 15 = 3 \times 6^B = 6^{1-A} \times 6^B$

$\Rightarrow 6^Q = 6^{1-A} \times 6^B$

$\Rightarrow 6^Q = 6^{1-A+B}$

$\Rightarrow Q = 1 - A + B$

16. *(c)* Let the speed of two trains be x km/h and y km/h, respectively. Then, the time taken by first train to cover 110 km = Time taken by second train to cover 100 km.

Thus, $\dfrac{110}{x} = \dfrac{100}{y} \Rightarrow \dfrac{x}{y} = \dfrac{110}{100}$

$\therefore\ x : y = 11 : 10$

17. *(a)* Distance travelled by x in 15 min

$= 60 \times \dfrac{15}{60} = 15$ km

Distance travelled by y in 10 min

$= 48 \times \dfrac{10}{60} = 8$ km

Difference = (15 − 8) km = 7 km

Hence, at 3:15 pm they are 7 km apart.

So, statement I is true. As speed of x is greater than y. So, y will never overtake x. Thus, statement II is false.

18. *(a)* Length of train = Distance covered in 40 s at the rate of 36 km/h.

$\therefore$ Length of train

$= 40 \times 36 \times \dfrac{5}{18} = 400$ m

19. *(b)* Here, a = 40 days, b = 50 days, x = 20 and T = ?

$\therefore$ Required time $= \dfrac{(b - x)\,a}{a + b}$

$= \dfrac{(50 - 20) \times 40}{(40 + 50)} = \dfrac{30 \times 40}{90}$

$= \dfrac{40}{3} = 13\dfrac{1}{3}$ days

20. *(a)* By given condition,

$n \times 30 = n \times 10 + (n + 50) \times 16$

$\Rightarrow 20n = 16n + 800$

$\therefore\ n = \dfrac{800}{4} = 200$

21. *(b)* Let the sum (principal) be ₹ x.

$\therefore$ Simple interest = ₹ $\dfrac{x}{2}$

and $T = 6$ yr, $R = 10\%$ per annum

$\therefore\ \text{SI} = \dfrac{P \times R \times T}{100}$

$\Rightarrow \dfrac{x}{2} = \dfrac{x \times 10 \times 6}{100} \Rightarrow \dfrac{1}{2} = \dfrac{6}{10}$

Which is not true, so it is not a possible case.

22. *(a)* Let the principal be ₹ P

As, amount = $3P$ and $T = 25$ yr

$\therefore \quad SI = 3P - P = 2P$

$\because \quad \text{Rate} = \dfrac{100 \times SI}{\text{principal} \times T}$

$= \dfrac{100 \times 2P}{P \times 25} = 8\%$

23. *(c)* As rate of interest is charged half yearly,

So, rate $= \dfrac{13}{2}\%$ half yearly

time $\left(\dfrac{42}{12} \times 2\right)$ half yearly $= 7$ half yearly

$SI = \dfrac{20000 \times 12 \times 7}{100 \times 2} = ₹\ 9100$

$\therefore$ Amount$(A) = 20000 \times 9100$

$= ₹\ 29100$

24. *(d)* Let amount be ₹ x and rate of interest is R % annually.

According to the questions,

Amount after 1st yr = ₹ 1200

$x\left(1 + \dfrac{R}{100}\right) = 1200 \qquad \ldots\text{(i)}$

Amount after 3^{rd} yr = ₹ 1587

$x\left(1 + \dfrac{R}{100}\right)^3 = ₹\ 1587 \quad \ldots\text{(ii)}$

On dividing Eq. (ii) by Eq. (i), we get

$\left(1 + \dfrac{R}{100}\right)^2 = \dfrac{1587}{1200} = \dfrac{529}{400}$

$1 + \dfrac{R}{100} = \dfrac{23}{20} \Rightarrow \dfrac{R}{100} = \dfrac{3}{20}$

$\therefore R = 15\%$

Put $R = 15\%$ in Eq. (i),

$x\left(1 + \dfrac{15}{100}\right) = ₹\ 1200$

$\Rightarrow x = \dfrac{1200 \times 100}{115}$

$\therefore x = ₹\ 1043.478$

25. *(a)* Given, principal amount = ₹ P

Rate of interest, $R = \dfrac{r}{k}\%$ per annum and

Time, $T = nk$

$\because A = P\left(1 + \dfrac{R}{100}\right)^T$

$\therefore A = P\left(1 + \dfrac{r}{100k}\right)^{nk}$

26. *(c)* Let selling price of 1 dozen pencil be ₹ x.

$\therefore$ Selling price of 8 dozen pencils = ₹ $8x$ and profit = ₹ x

$\because$ Cost price of 8 dozen pencils

$= 8x - x = ₹\ 7x$

$\therefore$ Gain, percent

$\dfrac{x}{7x} \times 100\% = 14\dfrac{2}{7}\%$

27. *(b)* I. Let CP = x

Then, SP = $\dfrac{213}{200}x$

If SP = $\dfrac{213}{200}x + 1250$, then gain = 19%

$\Rightarrow 19 = \dfrac{\dfrac{213x}{200} + 1250 - x}{x} \times 100$

$\Rightarrow \dfrac{19x}{100} - \dfrac{213x}{200} = 1250$

$\Rightarrow x = 1250 \times 8 = 10000$

II. If SP = $\dfrac{213}{200}x + 2400$ and gain = 19% then

$$19 = \frac{\frac{213x}{200} + 2400 - x}{x} \times 100$$

$$\Rightarrow \frac{19x}{100} - \frac{13x}{200} = 2400$$

$$\Rightarrow x = 2400 \times 8 = 19200$$

Hence, I is correct but II is incorrect.

28. *(c)* CP of 1 lemon = $\frac{1}{2}$

SP of 1 lemon = $\frac{3}{5}$

$\therefore$ Gain%

$$= \frac{\frac{3}{5} - \frac{1}{2}}{\frac{1}{2}} \times 100$$

$$= \frac{6-5}{10} \times 2 \times 100 = 20\%$$

29. *(d)* Given, list price of a video cassette = ₹ 100

Let the rate of discount be r %.

Selling price of 3 video cassette = ₹ 274.50

Selling price of 1 video cassette

$$= ₹ \frac{27450}{3} = ₹\ 91.50$$

$$\therefore 100 - \frac{r}{100} \times 100 = ₹\ 91.50$$

$$\Rightarrow 100 - 91.50 = r$$

$$\Rightarrow 8.50 = r$$

$\therefore$ Rate of discount $= 8.5\%$

30. *(c)* Let the number be x.

$$\therefore \frac{49+x}{68+x} = \frac{3}{4}$$

$$\Rightarrow 196 + 4x = 204 + 3x$$

$$\therefore \quad x = 8$$

31. *(b)* This is the solution with explanation for Let two numbers be $3x$ and $5x$, respectively.

According to the question,

$(3x - 9) : (5x - 9) :: 12 : 23$

$$\Rightarrow \frac{3x-9}{5x-9} = \frac{12}{23}$$

$$\Rightarrow 69x - 207 = 60x - 108$$

$$69x - 60x = 207 - 108$$

$$\Rightarrow 9x = 99$$

$$\therefore \quad x = 11$$

$\therefore$ Second number $= 5x = 5 \times 11 = 55$

32. *(d)* Let the amount be ₹ x.

In first condition, Q's part = $\frac{5x}{5+3} = \frac{5}{8}x$

In second condition, Q's part

$$= \frac{3x}{3+2} = \frac{3}{5}x$$

By given condition, $\frac{5}{8}x - \frac{3}{5}x = 10$

$$\Rightarrow \frac{x}{40} = 10 \quad \Rightarrow x = ₹\ 400$$

33. *(a)* $\because \log_a \sqrt{2} = \frac{1}{6} \Rightarrow a^{1/6} = \sqrt{2}$

$$\therefore \quad a = (\sqrt{2})^6$$

34. *(b)* $\frac{\log_{13}(10)}{\log_{169}(10)} = \frac{\log_{13}(10)}{\log_{13^2}(10)}$

$$= \frac{\log_{13} 10}{\frac{1}{2}\log_{13} 10} \left[\because \log_{a^b} c = \frac{1}{b}\log_a c\right]$$

$$= \frac{1}{1/2} = 2$$

35. *(d)* Given, $x^4 + \frac{1}{x^4} = 322$

$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 - 2 = 322$

$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = 324 = 18^2$

$\Rightarrow x^2 + \frac{1}{x^2} = 18$

$\Rightarrow \left(x - \frac{1}{x}\right)^2 + 2 = 18 \Rightarrow x - \frac{1}{x} = 4$

36. *(a)* $ax - b = 0 \Rightarrow x = b/a$
By remainder theorem, if $f(x)$ is divisible by $ax - b$, the remainder is $f(b/a)$.

37. *(d)* We have, $x + \frac{1}{x} = 4$

I.
$\left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3(x) \times \left(\frac{1}{x}\right)\left(x + \frac{1}{x}\right)$

$(4)^3 = x^3 + \frac{1}{x^3} + 3(4)$

$64 - 12 = x^3 + \frac{1}{x^3} \Rightarrow x^3 + \frac{1}{x^3} = 52$

II. $x + \frac{1}{x} = 4 \Rightarrow x^2 - 4x + 1 = 0$

$x = \frac{4 \pm \sqrt{16 - 4 \times 1 \times 1}}{2 \times 1}$

$= \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$

38. *(b)* $x^2 + y^2 + z^2 - 2xy + 2yz - 2zx$
$= (x - y - z)^2 = (5 - 3 - 2)^2 = 0$
$[\because x = 5,\ y = 3 \text{ and } z = 2]$

39. *(c)* LCM $= \frac{\text{Product of expressions}}{\text{HCF}}$

$= \frac{a \times b}{1} = ab$

40. *(a)* Since, $(x + 4)$ is HCF, so it will divide both the expressions i.e. $x = -4$ will make each one zero.
$\therefore\ 2(-4) + k(-4) - 12 = 0$
$\Rightarrow\ 32 - 12 = 4k$
$\therefore\ 20 = 4k \Rightarrow k = 5$

41. *(a)* We know $\frac{a+2}{a+3} - \frac{(a+1)}{(a+2)}$

$= \frac{(a+2)^2 - (a+1)(a+3)}{(a+3)(a+2)}$

$= \frac{a^2 + 4 + 4a - (a^2 + 4a + 3)}{a^2 + 5a + 6}$

$= \frac{1}{a^2 + 5a + 6}$

42. *(b)* Given, $pq + qr + rp = 0$

$\therefore \frac{p^2}{p^2 - qr} + \frac{q^2}{q^2 - rp} + \frac{r^2}{r^2 - pq}$

$= \frac{p^2}{p^2 + rp + pq} + \frac{q^2}{q^2 + pq + qr} + \frac{r^2}{r^2 + qr + rp}$

$= \frac{p^2}{p(p + r + q)} + \frac{q^2}{q(p + q + r)} + \frac{r^2}{r(p + q + r)}$

$= \frac{p + q + r}{p + q + r} = 1$

43. *(b)* Given,
$\frac{1}{x+1} + \frac{2}{y+2} + \frac{1009}{z + 1009} = 1$

$$\Rightarrow \frac{1}{x+1}-1+\frac{2}{y+2}-1+\frac{1009}{z+1009}-1 = 1-3$$

$$\Rightarrow -\frac{x}{x+1}-\frac{y}{y+2}-\frac{z}{z+1009}=-2$$

$$\therefore \frac{x}{x+1}+\frac{y}{y+2}+\frac{z}{z+1009}=2$$

44. *(d)* Given,

$$\frac{1}{(1-a)(1-b)}+\frac{a^2}{(1-a)(b-a)}-\frac{b^2}{(b-1)(a-b)}$$

$$=\frac{(b-a)+a^2(1-b)-b^2(1-a)}{(1-a)(1-b)(b-a)}$$

$$=\frac{b-a+a^2-a^2b-b^2+ab^2}{(1-a)(1-b)(b-a)}$$

$$=\frac{(b-a)+\left(a^2-b^2\right)+ab(b-a)}{(1-a)(1-b)(b-a)}$$

$$=\frac{1-a-b+ab}{1-b-a+ab}=1$$

45. *(c)* A pair of linear equations in two variables has a unique solution if their graphs intersect in one point.

46. *(a)* Given, $px + q = 0$ and $rx + s = 0$

$$\Rightarrow x=\frac{-q}{p} \text{ and } x=\frac{-s}{r}$$

So, $\frac{-q}{p}=\frac{-s}{r} \Rightarrow ps = qr$

47. *(d)* Since, given system of equations $x + 2y - 3 = 0$ and $5x + ky + 7 = 0$ has no solution.

Then, $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

$$\therefore \frac{1}{5}=\frac{2}{k}\neq\frac{-3}{7} \Rightarrow k=10$$

48. *(b)* Three lines $a_1x+b_1y+c_1=0$

$a_2x+b_2y+c_2=0$

and $a_3x+b_3y+c_3=0$

which are non-parallel and non-collinear they have only one solution if they meet in a common point in this case these lines are called concurrent lines.

49. *(c)* $a^2b^2x^2-\left(a^2+b^2\right)x+1=0$

$\Rightarrow a^2x\left(b^2x-1\right)-1\left(b^2x-1\right)=0$

$\Rightarrow \left(b^2x-1\right)\left(a^2x-1\right)=0$

So, either $a^2x-1=0 \Rightarrow a^2x=1$

or $b^2x-1=0$ or $b^2x=1$

$\Rightarrow x=1/a^2$ or $x=1/b^2$

50. *(b)* Here, $\alpha+\beta=\left(1+a^2\right)$

and $\alpha\beta=\frac{1}{2}\left(a^4+a^2+1\right)$

$\because \alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

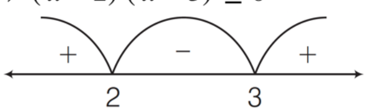
$=\left(1+a^2\right)^2-\left(a^4+a^2+1\right)$

$=1+a^4+2a^2-a^4-a^2-1$

$\therefore \alpha^2+\beta^2=a^2$

51. *(a)* $x^2-5x+6\geq 0$

$\Rightarrow (x-2)(x-3)\geq 0$



$\Rightarrow x\leq 2$ or $x\geq 3$

$\Rightarrow x\in[-\infty,2]$ or $[3,\infty]$

$\Rightarrow x\in[-\infty,2]\cup[3,\infty]$

52. *(b)* $B-A=\{7, 8, 9, 12\}-\{2, 6, 8, 9\}$

$=\{7, 12\}$

53. *(a)* $B \cup \{1, 2\} = \{1, 2, 3, 5, 9\}$

$\Rightarrow B = \{3, 5, 9\}$ is the required smallest set

54. *(d)* All sets are infinite set.

55. *(a)* I. There are infinite points lie on a line segment so, it is an infinite set.

II. Number of birds in zoo are countable so, it is a finite set.

III. It is not a well-defined set.

Hence, only I is correct.

56. *(b)* Angle traced by hour hand in 1 hr = 30°

Angle traced by hour hand in

$\frac{91}{12}\text{hrs} = \left(30 \times \frac{91}{12}\right)^\circ = 227^\circ\, 30'$

Angle traced by minute hand in 1min = 6°

Angle traced by minute hand in

$35 \text{ min} = (6 \times 35)^\circ = 210^\circ$

$\therefore$ Required angle = 227°30' − 210°

= 17°30'

57. *(b)* $\operatorname{cosec}^2\theta - 2 + \sin^2\theta$

$= (\sin\theta - \operatorname{cosec}\theta)^2$

Hence, it is always non-negative.

58. *(b)* Given, $\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 2$

$\therefore \sin^2\theta + \cos^2\theta = 2\sin\theta\cos\theta$

$\Rightarrow \sin 2\theta = 1 = \sin 90^\circ$

$\Rightarrow 2\theta = 90^\circ$

$\Rightarrow \theta = 45^\circ$

59. *(a)* The squares of the tangents of the angles 30°, 45° and 60° are in G.P.

$\Rightarrow \tan^2 30^\circ, \tan^2 45^\circ, \tan^2 60^\circ$ are in G.P. $\Rightarrow \left(\frac{1}{\sqrt{3}}\right)^2, (1)^2, (\sqrt{3})^2$ are in G.P. $\Rightarrow \frac{1}{3}$, 1, 3 are in G.P.

which is true as $1^2 = \frac{1}{3} \times 3 \Rightarrow 1 = 1$s

60. *(d)* I. $1^\circ = \frac{\pi}{180}$ radian $= \frac{3.14}{180} = 0.017$

$= 0.02$ radians (approx)

which is less than 0.03 radians.

II. $1^\circ = \frac{180}{\pi}$ degree

$= \frac{180}{3.14} = 57.32$ degree

which is greater than 45°.

Hence, both statements are true.

61. *(c)* $\cos(180^\circ + A) + \cos(180^\circ + B) + \cos(180^\circ + C) + \cos(180^\circ + D)$

$= -\cos A - \cos B - \cos C - \cos D$

$= -\cos A - \cos B - \cos(180^\circ - A) - \cos(180^\circ - B)$

$(\because A + C = B + D = 180^\circ)$

$= -\cos A - \cos B + \cos A + \cos B = 0$

62. *(a)* We know that, for $0^\circ < \theta < 90^\circ$, there exist only one θ such that $\sin\theta = a$.

63. *(a)* $\tan(45^\circ + x) = \frac{\tan 45^\circ + \tan x}{1 - \tan 45^\circ \tan x}$

$= \frac{1 + \tan x}{1 - \tan x}$

$\tan(45^\circ - x) = \frac{\tan 45^\circ - \tan x}{1 + \tan 45^\circ \tan x}$

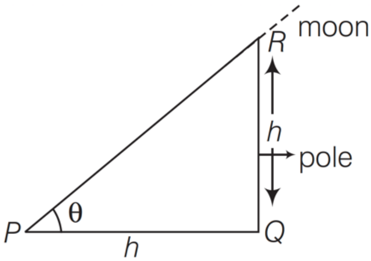
$= \frac{1 - \tan x}{1 + \tan x}$

$$\therefore \frac{\tan(45^\circ + x)}{\tan(45^\circ - x)} = \frac{1-\tan x}{\frac{1-\tan x}{1+\tan x}}$$

$$= \left[\frac{1+\tan x}{1-\tan x}\right]^2$$

64. *(b)* Let the height of pole be h m, then
$PQ = RQ = h$

In right angled $\Delta\, PRQ$,



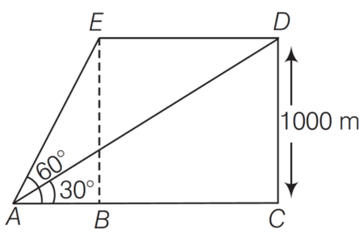
$$\tan\theta = \frac{RQ}{PQ} = \frac{h}{h} = 1$$

$$\Rightarrow \tan\theta = 1 = \tan 45^\circ$$

$$\therefore \theta = 45^\circ$$

Hence, the angle of elevation of moon is 45°.

65. *(b)* Let the radar base is at point A. The plane is at point D in the first sweep and at point E in the second sweep. The distance it covers in the one minute interval is DE.



From the figure,

In right angled $\Delta\, ADC$, we get

$$\tan 30^\circ = \frac{DC}{AC} = \frac{1000}{AC} \Rightarrow AC = \frac{1000}{\tan 30^\circ}$$

Similarly, in $\Delta\, EAB$, we get

$$\tan 60^\circ = \frac{EB}{AB} = \frac{1000}{AB} \Rightarrow AB = \frac{1000}{\tan 60^\circ}$$

Total distance covered by plane in 1 min, then

$DE = AC - AB$

$$DE = \frac{1000}{\tan 30^\circ} - \frac{1000}{\tan 60^\circ}$$

$$= 1000\sqrt{3} - \frac{1000}{\sqrt{3}}$$

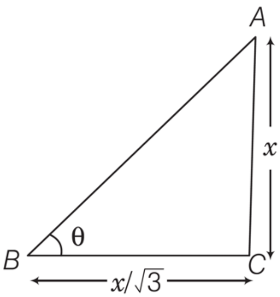
$$= 1154.70 \text{ m}$$

The speed of the plane is given by

s = distance covered/time taken

$= DE / 60 = 19.25$ m/s.

66. *(c)* Let θ be the angle of elevation,



In right angled $\Delta\, ABC$,

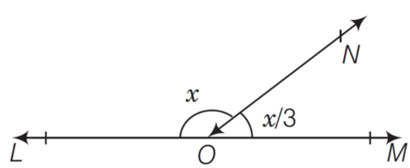
$$\tan\theta = \frac{AC}{BC} = \frac{x}{\frac{x}{\sqrt{3}}} = \frac{\sqrt{3}x}{x} = \sqrt{3}$$

Here, $\tan\theta = \sqrt{3} = \tan 60^\circ$

$\therefore \theta = 60^\circ \quad \left[\because \tan 60^\circ = \sqrt{3}\right]$

67. *(a)* Given that, $\angle MON = \frac{1}{3}\angle LON$

Let $\angle LON = x$, then , $\angle MON = \frac{x}{3}$



We know that,

$\angle LON + \angle MON = 180°$ [linear pair]

$\Rightarrow x + \frac{x}{3} = 180° \Rightarrow x = \frac{180° \times 3}{4} = 135°$

thus, $\angle MON = \frac{x}{3} = \frac{135°}{3} = 45°$

68. *(c)* Both the statements I and II are correct.

69. *(c)* Let the other side be b and p.

$\therefore \frac{1}{2} b \times p = 6 \Rightarrow b \times p = 12$

$\Rightarrow b = \frac{12}{p}$

Also, by Pythagoras theorem

$h^2 = b^2 + p^2$

$5^2 = \left(\frac{12}{p}\right)^2 + p^2$

$\Rightarrow 25 = \frac{144}{p^2} + p^2$

$25p^2 = 144 + p^4$

$\Rightarrow p^4 - 25p^2 + 144 = 0$

$\Rightarrow p^4 - 16p^2 - 9p^2 + 144 = 0$

$\Rightarrow (p^2 - 9)(p^2 - 16) = 0$

$\Rightarrow p = 3$ or $p = 4$

Hence, other sides are 3 cm and 4 cm.

70. *(d)* In ΔPQR,

$\angle P + \angle Q + \angle R = 180°$

$\Rightarrow a + 3a + b = 180°$

$\Rightarrow 4a + b = 180°$ ...(i)

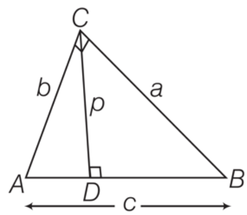
Given, $-5a + 3b = 30°$ ...(ii)

Solving Eqs.(i) and (ii), we get $a = 30°$ and $b = 60°$

$\therefore \angle P = 30°, \angle Q = 90°$ and $\angle R = 60°$

So, ΔPQR is right angled triangle.

71. *(b)* Since, c is the base and p is the altitude of ΔABC.



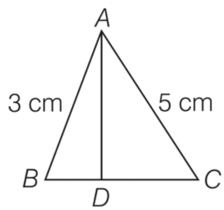
Here, area of $\Delta ABC = \frac{1}{2} pc$ ...(i)

Also, area of $\Delta ABC = \frac{1}{2} ab$ ...(ii)

From Eqs. (i) and (ii), we get

$\frac{1}{2} pc = \frac{1}{2} ab \Rightarrow pc = ab$

72. *(c)* We know that, The sum of any two sides of a triangle is greater than twice the median drawn to the third side.



i.e. $(AB + AC) > 2AD$

$\Rightarrow (3 + 5) > 2AD \Rightarrow AD < 4$

Hence, AD is always less than 4 cm.

73. *(d)* Number of diagonals of a polygon of

8 sides $= \frac{n(n-1)}{2} - n$

$= \frac{8(8-1)}{2} - 8 = 20$

74. *(c)* By condition,

$\frac{\text{Interior angle of a regular polygon}}{\text{Exterior angle a regular polygon}}$

$= \frac{5}{1}$

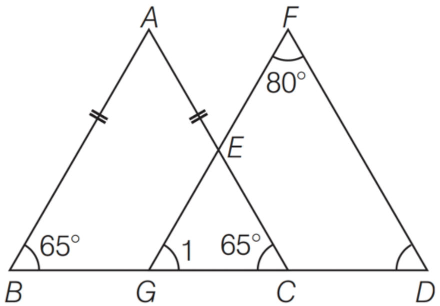
$$\Rightarrow \frac{\frac{(n-2)}{n}\times 180°}{\frac{360°}{n}} = \frac{5}{1}$$

$$\Rightarrow \frac{(n-2)}{2} = \frac{5}{1}$$

$$\Rightarrow n-2=10 \Rightarrow n=12$$

75. *(d)* Here,

$$\angle B = \angle C = 65° \ [\because AB = AC]$$



$\angle 1 = \angle B = 65°$

(corresponding angles)

In ΔFGD,

$$\angle 1 + \angle F + \angle D = 180°$$

(by angle sum of property of a triangle)

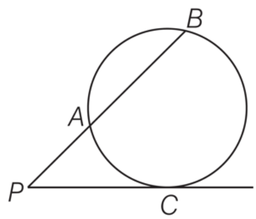
$\Rightarrow 65° + 80° + \angle D = 180°$

$\Rightarrow \quad \angle D = 35°$

76. *(c)* The perpendicular bisectors of the sides of a triangle passes through the same point. The perpendicular bisectors are concurrent and point is called the circumcentre.

77. *(a)* If a secant to a circle intersect circle at points A and B and PC is a tangent to circle, then

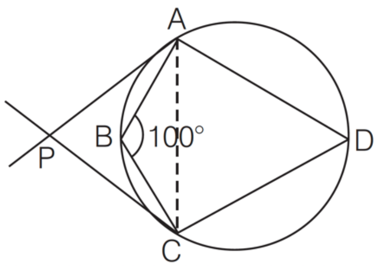
$$PC^2 = PA \times PB$$



which is equivalent to saying that area of rectangle with PA and PB as sides is equal to the area of square with PC as sides.

78. *(a)* A line perpendicular to the given line, passing through the given point is the required locus.

79. *(b)* We know that, the sum of opposite angles of a cyclic quadrilateral is always 180°.



$\therefore \angle B + \angle D = 180°$

$\Rightarrow 100 + \angle D = 180° \Rightarrow \angle D = 180°$

$\therefore \angle ACP = \angle PAC = 80°$

[by theorem of alternate interior segment]

In ΔPAC,

$$\angle P + \angle PAC + \angle PCA = 180°$$

[by angle sum property of a triangle]

$\Rightarrow \angle P + 80° + 80° = 180°$

$\Rightarrow \angle P = 180° - 160° = 20°$

80. *(b)* The locus obtained is the circumference of the circle concentric with the given circle and having radius equal to the distance of the chords from the centre.

81. *(c)* Side of the greatest square tile = GCM of the length and breadth of the room = GCM of 10.5 and 3 is 1.5 m

Area of room = 10.5 × 3 m^2

∴ Number of tiles needed

$$= \frac{l \times b}{(\text{H.C.F of } l \,\&\, b)^2}$$

$$= \frac{10.5 \times 3}{2.25} = 14 \text{ tiles}$$

82. *(a)* Given, inner circumference

$$= 2\pi r = 440 \text{ m}$$

$$\Rightarrow r = \frac{440}{2 \times 22} \times 7 = 70 \text{ m}$$

Width of track = 14 m

∵ Radius of outer circle = (70 + 14) m = 84 m

∴ Diameter of outer circle

= 2 × 84 = 168 m

83. *(c)* Distance covered in one revolution

$$= \frac{11 \times 1000 \times 100}{5000} = 220 \text{ cm}$$

∴ The circumference of the wheel = 220 cm

Let the diameter be 'D'.

Then, $\pi D = 220 \Rightarrow \frac{22}{7} \times D = 220$

$$\therefore D = \frac{220 \times 7}{22} = 70 \text{ cm}$$

84. *(d)* I. Let R be the radius of circumcircle

Area of circumcircle = πR^2

$\Rightarrow 9\pi = \pi R^2 \Rightarrow R = 3$ cm

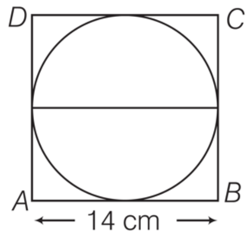
II. $\frac{\text{Inradius of polygon}}{\text{circumradius of polygon}} = \frac{r}{R}$

$$= \frac{\frac{1}{2} a \cot\left(\frac{180°}{n}\right)}{\frac{1}{2} a \operatorname{cosec}\left(\frac{180°}{n}\right)} = \cos\left(\frac{180°}{n}\right)$$

$$= r = R\cos\left(\frac{180}{12}\right)^{\circ} = 3\cos 15°$$

85. *(d)* Given, diameter of circle = Side of square = 14 cm

∴ $r = 7$ cm



Area of circle $= \pi r^2 = \frac{22}{7} \times 7 \times 7$

$= 154 \text{ cm}^2$

86. *(c)* Let radius of cylinder = x and radius of cone = $2x$

Height of each = h

Required ratio $= \frac{\text{Volume of cone}}{\text{Volume of cylinder}}$

$$= \frac{\frac{1}{3}\pi 4x^2 h}{\pi x^2 h} = \frac{4}{3} = 4 : 3$$

87. *(b)* Given, R = 3 cm, r = 2 cm, h = 10 cm

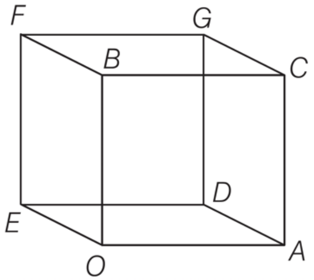
Total surface area = $2\pi (R + r)(R + h - r)$

$= 2\pi (3 + 2)(3 + 10 - 2) = 110\pi \text{ cm}^2$

88. *(d)* I. The distance between vertices B and C is 1 cm.

II. The distance between A and B is

$\sqrt{1^2 + 1^2} = \sqrt{2}$ cm



III. The distance between vertices B and D is $\sqrt{1^2+1^2+1^2} = \sqrt{3}$ cm

Hence, the statements I and II, III are correct.

89. *(d)* Area of the field = Length × Breadth

$$= 12 \times 10 = 120 \text{ m}^2$$

Area of the pit's surface = $5 \times 4 = 20\text{m}^2$

Area on which the Earth is to be spread

$$= 120 - 20 = 100 \text{ cm}^2$$

Volume of Earth dug out

$$= 5 \times 4 \times 2 = 40 \text{ cm}^3$$

Level of field raised $= \frac{40}{100} = \frac{2}{5}$ m

$$= \frac{2}{5} \times 100 = 40 \text{ cm}$$

90. *(d)* Total surface area = Curved surface area of cylinder + Curved surface area of cone + Top surface area of cylinder

$$= 2\pi rh + \pi rl + \pi r^2 = \pi[2rh + r^2 + rl]$$

$$= \pi\left[2 \times 3 \times 4 + 3^2 + 3\sqrt{3^2 + 4^2}\right]$$

$$= 48\pi \text{ cm}^2$$

91. *(a)* Geometric mean of 40, 50 and x

$$= (40 \times 50 \times x)^{1/3}$$

$$\Rightarrow (40 \times 50 \times x)^{1/3} = 10 \quad (\text{given})$$

$$\Rightarrow 40 \times 50 \times x = 10^3$$

$$\Rightarrow x = \frac{1000}{40 \times 50} = \frac{1}{2}$$

Hence, the value of x is $\frac{1}{2}$.

92. *(b)* Range of the data = 120 − 71 = 49

$$\therefore \text{Class size} = \frac{\text{Range}}{\text{Number of classes}}$$

$$= \frac{49}{7} = 7$$

The class are 71-78, 78-85, 85-92, 92-97 etc.

So, limits of second class interval are 78 and 85.

93. *(b)* Collar sizes of 200 shirts sold in a week, here mode is a suitable measure of central tendency.

94. *(d)* The volume of rainfall in certain geographical area, recorded every month for 24 consecutive months.

95. *(b)* Given, $n_1 = 45, \bar{x}_1 = 2, n_2 = 40$

$\bar{x}_2 = 2.5$ and $n_3 = 15, \bar{x}_3 = 2$

Required mean

$$= \frac{n_1\bar{x}_1 + n_2\bar{x}_2 + n_3\bar{x}_3}{n_1 + n_2 + n_3}$$

$$= \frac{45 \times 2 + 40 \times 2.5 + 15 \times 2}{45 + 40 + 15}$$

$$= \frac{90 + 100 + 30}{45 + 40 + 15} = \frac{220}{100} = 2.2$$

Hence, the mean of the combined distribution is 2.2.

96. *(d)* All statements are true.

97. *(c)* $\sqrt[3]{\sqrt{x}} = x/5$

On cubing both sides, we get

$$\sqrt{x} = x^3/5^3$$

On squaring both sides, we get

$$x = x^6/5^6 \Rightarrow 5^6 = \frac{x^6}{x} = x^5$$

$$\Rightarrow x^5 = 5^6 \quad \Rightarrow x = \sqrt[5]{5^6}$$

98. *(a)* $x^2 - x - 2 = 0$

$(x-2)(x+1) = 0 \Rightarrow x = 2, -1$

So, both the roots are integers.

99. *(a)* Let S be the set of people who speak Spanish and F be the set of people who speak French

$\therefore \quad n(S) = 20, n(F) = 50, n(S \cap F) = 10$

$$\Rightarrow n(S \cup F) = n(S) + n(F) - n(S \cap F) = 20 + 50 - 10 = 60$$

100. *(d)* I. $\sin\theta . \sin(60° + \theta) . \sin(60° - \theta)$

$$= \sin\theta\left[\sin^2 60° - \sin^2\theta\right]$$

$$= \sin\theta\left[\frac{3}{4} - \sin^2\theta\right]$$

$$= \frac{1}{4}\left[3\sin\theta - 4\sin^3\theta\right] = \frac{1}{4}\sin 3\theta$$

II. $\cos\theta . \sin(30° + \theta) . \sin(30° - \theta)$

$$= \cos\theta\left[\sin^2 60° - \sin^2\theta\right]$$

$$= \cos\theta\left[\frac{1}{4} - \left(1 - \cos^2\theta\right)\right]$$

$$= \frac{1}{4}\left[4\cos^3\theta - 3\cos\theta\right] = \frac{1}{4}\cos 3\theta$$

III. $\sin\theta . \cos(30° + \theta) . \cos(30° - \theta)$

$$= \sin\theta\left[\cos^2 30° - \sin^2\theta\right]$$

$$= \sin\theta\left[\frac{3}{4} - \sin^2\theta\right] = \frac{1}{4}\sin 3\theta$$

Hence, all three statements are correct.